ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 8

Source Transformation and Mesh Analysis

Announcements

- Recommended Reading:
  – Textbook Chapter 4

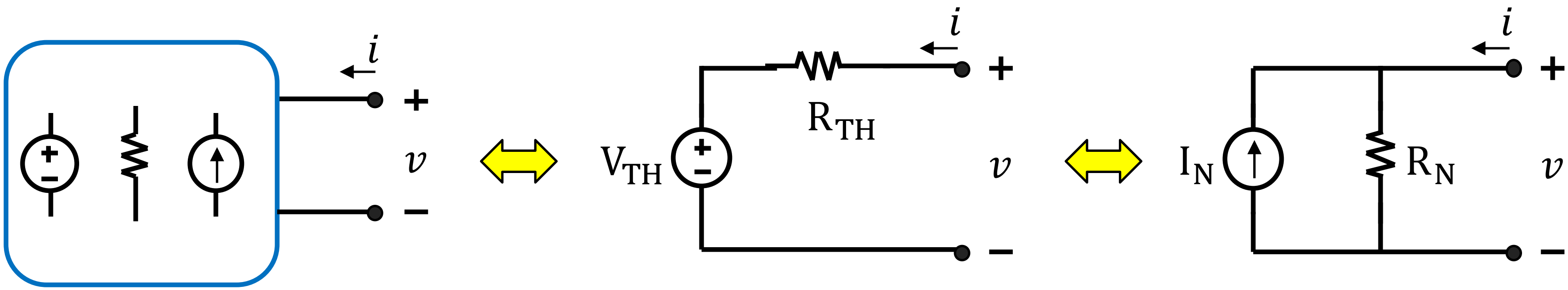
- Upcoming due dates:
  – Lab report 1 due by 11:59 pm on Friday February 8, 2019
  – Prelab 2 due by 12:20 pm on Tuesday February 12, 2019
  – Homework 2 due by 11:59 pm on Friday February 15, 2019
  – Lab report 2 due by 11:59 pm on Friday February 22, 2019

- Lab 2 is next week (starting Tuesday February 12, 2019)

- Prelim 1 on Thursday February 21, 2019 from 7:30 – 9 pm in 203 Phillips

Thevenin and Norton Equivalent Summary

- We can represent any linear network (with linear resistors, linear dependent sources and independent sources) seen from a particular port with simple equivalent models (i.e., models with the same i-v characteristics at that port)

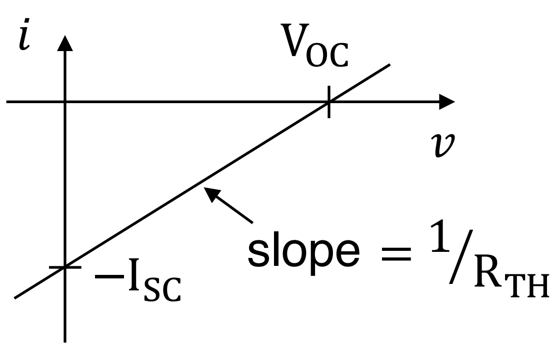


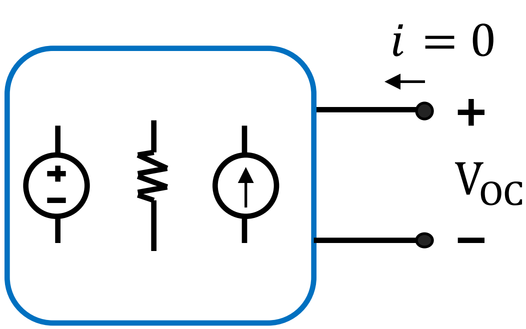
Arbitrary Linear Circuit

Thevenin Equivalent

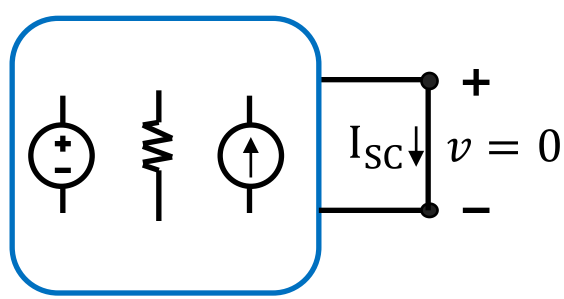
Norton Equivalent

$R_{TH} = R_N$





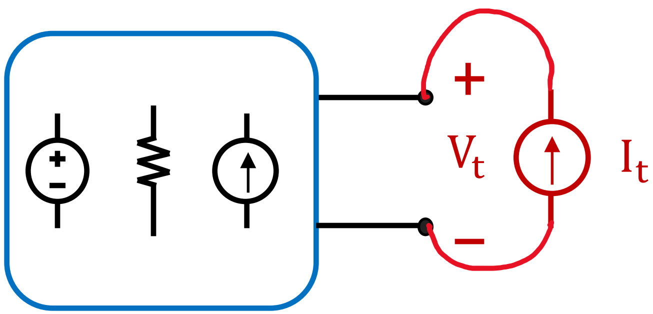
$V_{TH} = V_{OC}$



$I_N = I_{SC}$

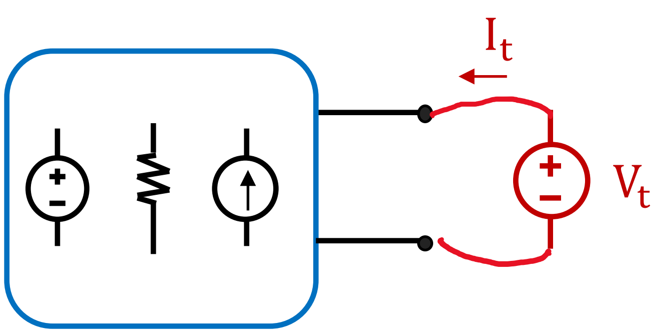
Thevenin and Norton Equivalent Summary (Cont.)

- Thevenin and Norton Resistances are the same and can be found several ways:
  1) $R_{TH} = R_N = {}^{V_{OC}}/_{I_{SC}}$ (Does not work if both V_{OC} and I_{SC} are zero)
  2) Set all independent sources in the network to zero and find equivalent resistance looking into the "dead" circuit:
     a) Sometimes can use series and parallel combinations to determine R_{TH} (Does not always work, especially if circuit has dependent sources)
     b) Use Test Source Method (will always work)



Apply I_t

Determine V_t



Apply V_t

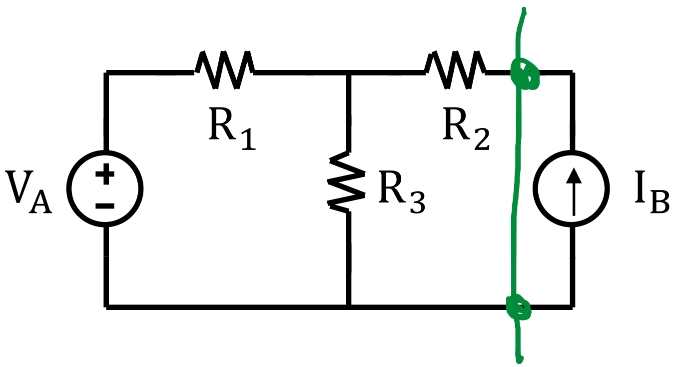
Determine I_t

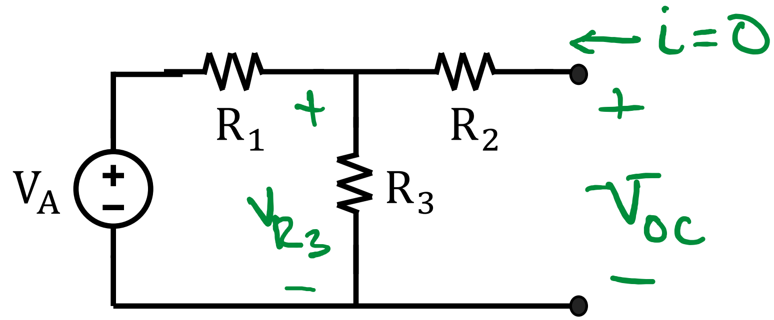
$$R_{TH} = {}^{V_t}/_{I_t}$$

Thevenin Equivalent Circuit Example 2

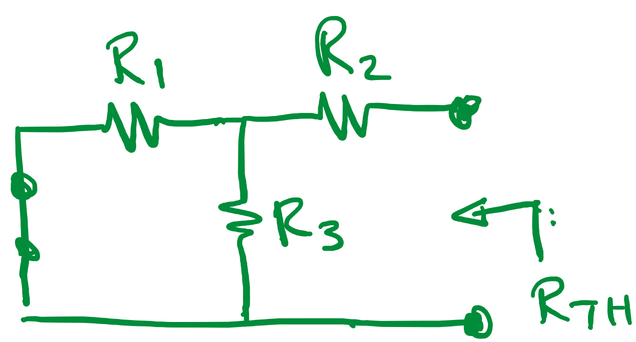
Thevenin and Norton equivalent circuit depends on where we look at the circuit from (i.e., it depends on the port)

Now consider the part of the circuit (of example 1) as seen by the current source





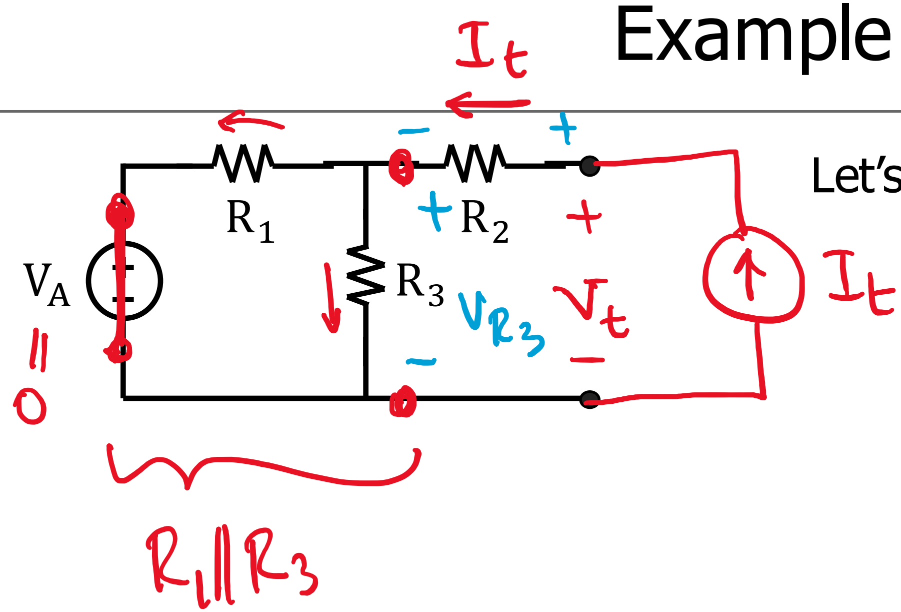
$$V_{TH} = V_{oc} = \frac{R_3}{R_1+R_3} V_A$$



$$R_{TH} = R_2 + (R_1 \| R_3)$$

$$= R_2 + \frac{R_1 R_3}{R_1+R_3}$$

Example 2 (Cont.)



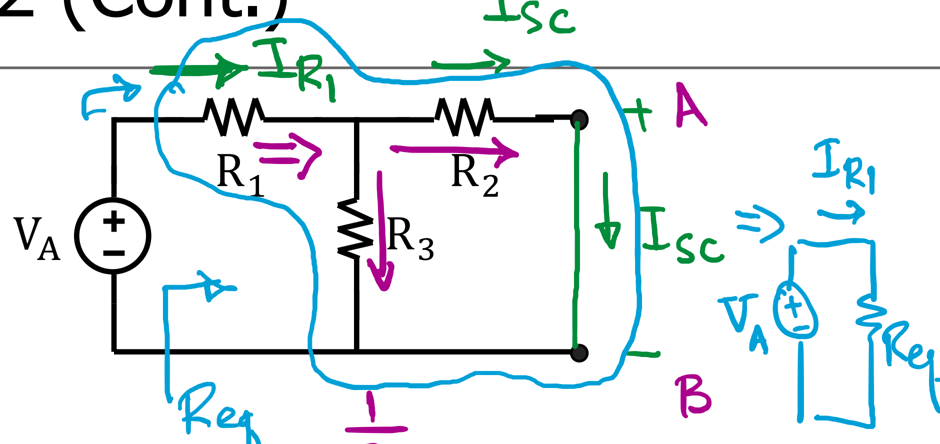
Let's confirm R_{TH} using the Test Source method

$$V_{R_3} = (R_1 \| R_3) I_t$$

$$V_t = V_{R_3} + R_2 I_t = \frac{R_1 R_3}{R_1 + R_3} I_t + R_2 I_t$$

$$R_{TH} = \frac{V_t}{I_t} = \frac{R_1 R_3}{R_1 + R_3} + R_2$$

Example 2 (Cont.)



How about Norton Equivalent

$$R_N = R_{TH} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

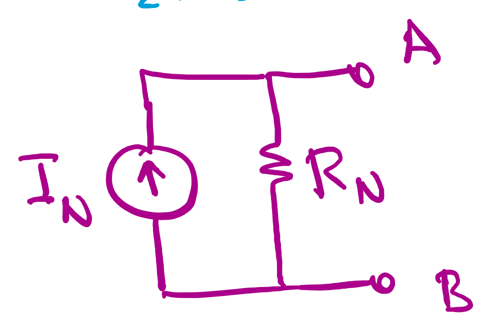
$$I_{sc} = \frac{R_3}{R_2 + R_3} I_{R_1}$$

$$I_{sc} = \frac{\frac{1}{R_2}}{\frac{1}{R_2} + \frac{1}{R_3}} I_{R_1}$$

$$I_{R_1} = \frac{V_A}{R_1 + R_2 \| R_3} \Rightarrow I_{sc} = \frac{V_A}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} \cdot \frac{R_3}{R_2 + R_3}$$

$$\Rightarrow I_{sc} = \frac{R_3 V_A}{R_2 R_3 + R_1 (R_2 + R_3)}$$

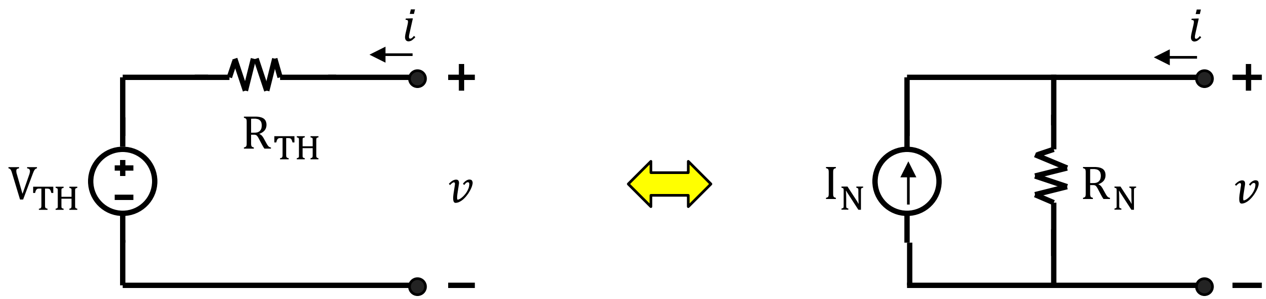
$$I_{sc} = I_N$$



We could have also found I_N using $I_N = V_{TH}/R_{TH}$

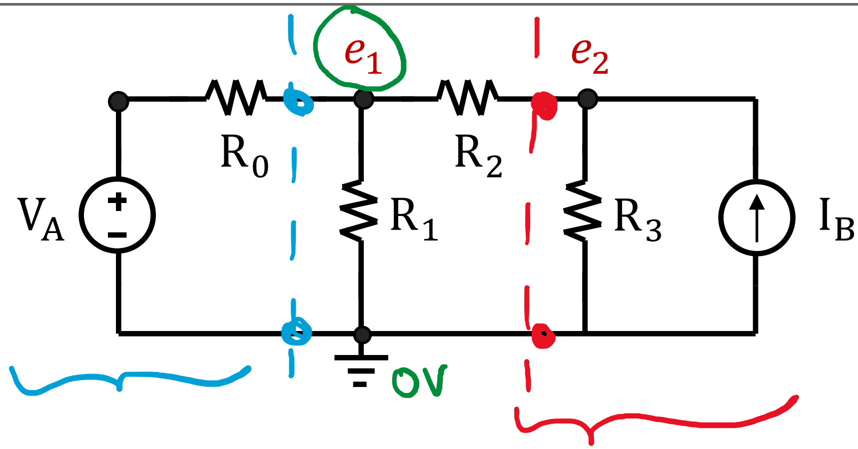
Source Transformation

- For the rest of the circuit connected at their port the following two subcircuits are equivalent



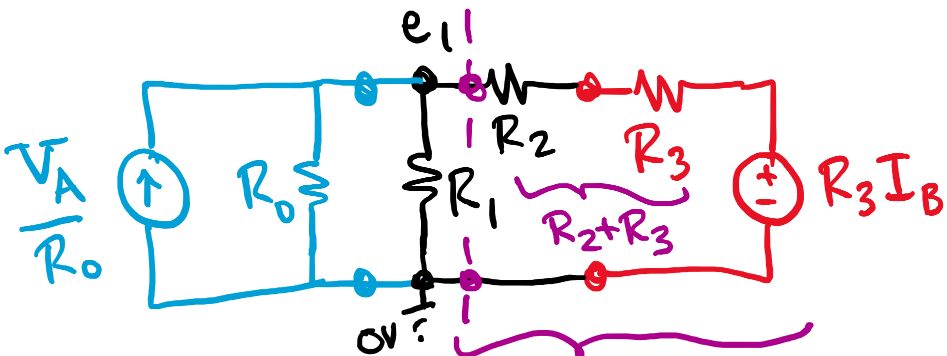
We can use this fact to aid in circuit analysis using simplification

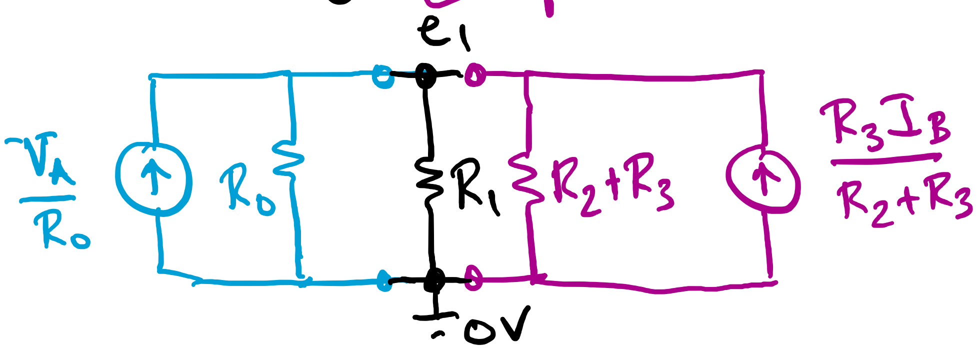
Circuit Analysis Utilizing Source Transformations

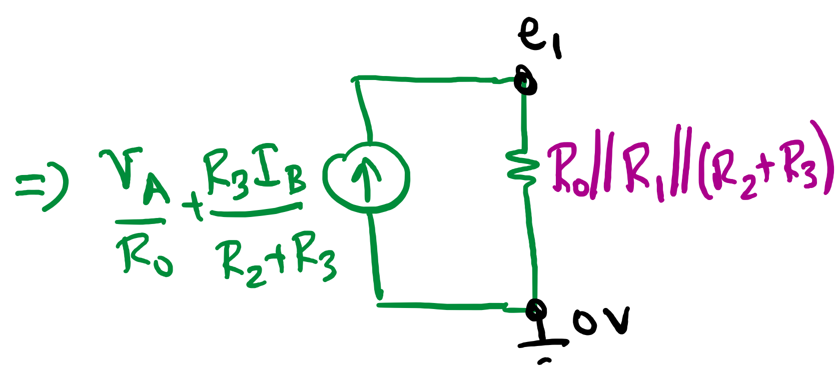


Suppose we want to find e_1

$$e_1 = \left(R_0 \| R_1 \| (R_2+R_3)\right)\left(\frac{V_A}{R_0} + \frac{R_3 I_B}{R_2+R_3}\right)$$

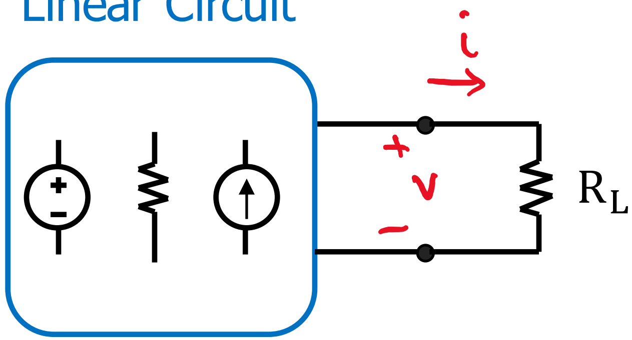


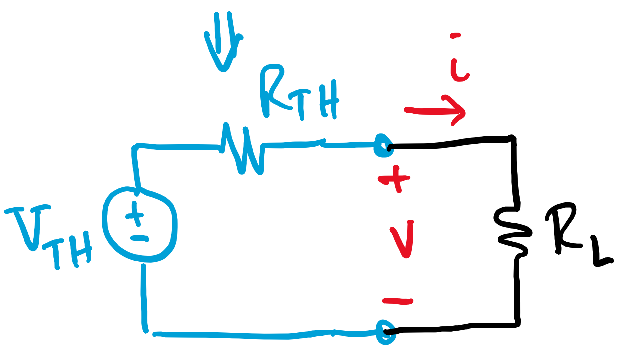




Maximum Power Transfer

Linear Circuit





What value of R_L will draw maximum power?

$$P = vi$$

$$v = \frac{R_L}{R_L + R_{TH}} \cdot V_{TH} \qquad i = \frac{V_{TH}}{R_{TH} + R_L}$$

if $R_L = 0 \Rightarrow v = 0 \Rightarrow P = 0$

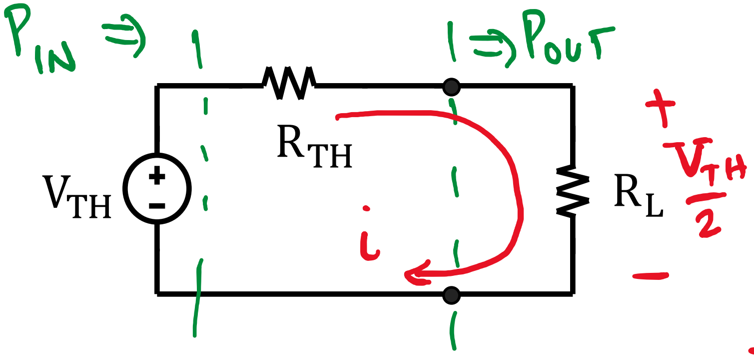
if $R_L = \infty \Rightarrow i = 0 \Rightarrow P = 0$

$$P = \frac{V_{TH}^2 R_L}{(R_L + R_{TH})^2}$$

$$\frac{dP}{dR_L} = V_{TH}^2 \left[R_L(-2)(R_L + R_{TH})^{-3} + (R_L + R_{TH})^{-2} \right] = V_{TH}^2 \left[\frac{\cancel{R_L} + R_{TH} - \cancel{2}R_L}{(R_L + R_{TH})^3} \right] = 0$$

$$\Rightarrow \boxed{R_L = R_{TH}} \qquad \Rightarrow P = \frac{V_{TH}^2}{4R_{TH}}$$

Maximum Power Transfer (Cont.)



$$R_L = R_{TH}$$

$$\eta = \frac{P_{OUT}}{P_{IN}} = \frac{\frac{V_{TH}}{2} i}{V_{TH}\, i} = \frac{1}{2}$$

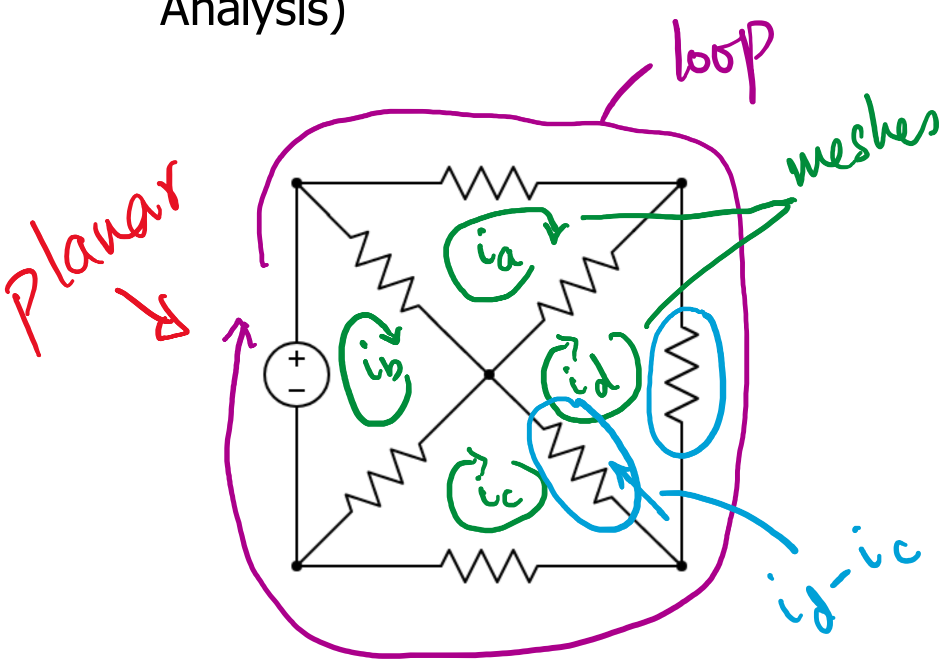
$$= 50\%$$

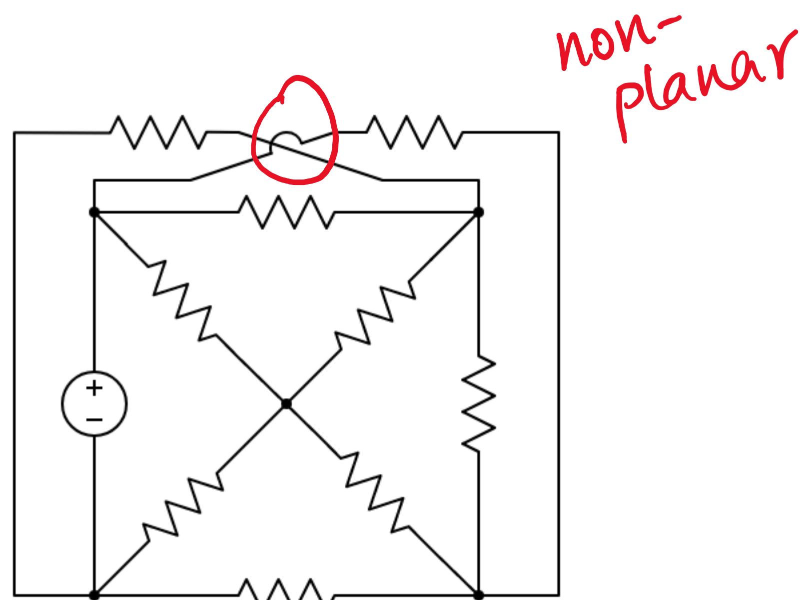
Efficiency

$$\frac{R_{TH}}{R_L} \to 0 \Rightarrow \eta = 1 \quad (100\%)$$

Mesh Analysis

- Another circuit analysis technique based on a combination of KVL, KCL and element constitutive relationships
  - Analysis is organized so that only (B-N+1) equations have to be solved
  - Only solve for mesh currents
  - Once mesh currents are known, determine branch currents and voltages
  - Mesh Analysis works only for planar circuits
  - Loop Analysis works for planar and nonplanar circuits (Dual of Node Analysis)



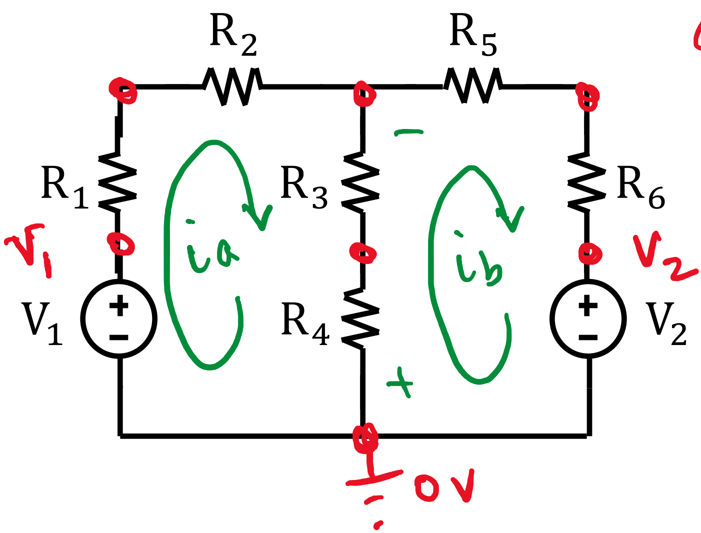


Mesh Analysis – Basic Procedure

1) Specify mesh currents ⇦ Implicitly imposes KCL
2) Write KVL for meshes with unknown mesh currents in terms of mesh currents and element constitutive relationships

   $B-(N-1)$
3) Solve for mesh currents
4) Back-solve for any required branch voltages and currents

Mesh Analysis – Example 1



4 unknown nodes

N−1 ←

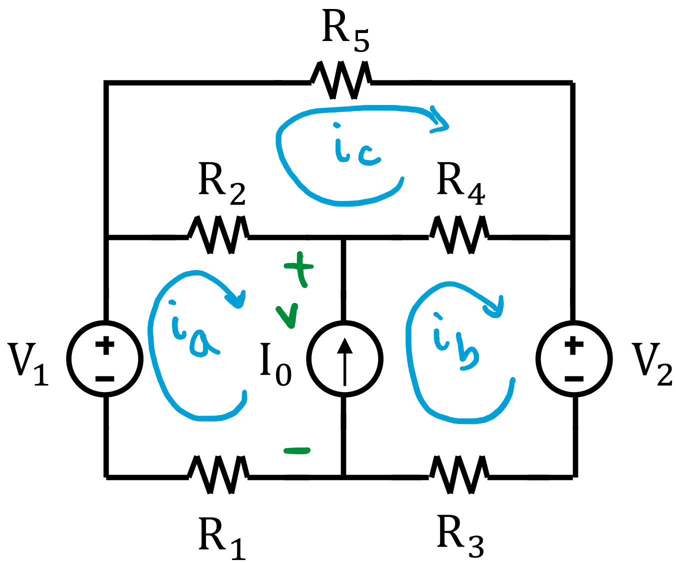
B−N+1 ←

2 unknown meshes

$$A: \quad -V_1 + (R_1+R_2)i_a + (R_3+R_4)(i_a - i_b) = 0$$

$$B: \quad V_2 + (R_3+R_4)(i_b - i_a) + (R_5+R_6)i_b = 0$$

Mesh Analysis – Example 2



Has a current source between two meshes

A: $-V_1 + R_2(i_a - i_c) + v + R_1 i_a = 0$

B: $V_2 + R_3 i_b - v + R_4(i_b - i_c) = 0$

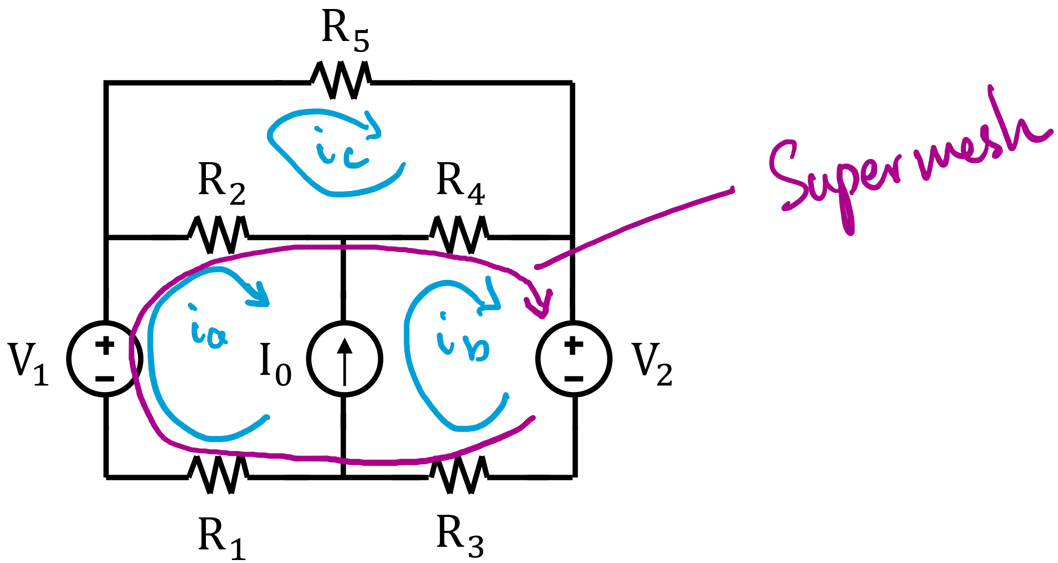
C: $R_5 i_c + R_4(i_c - i_b) + R_2(i_c - i_a) = 0$

4: $i_b - i_a = I_0$

4 unknowns

4 equations

Mesh Analysis – Example 2 – Supermesh Method



$$-V_1 + R_2(i_a - i_c) + R_4(i_b - i_c) + V_2 + R_3 i_b + R_1 i_a = 0$$

$$R_5 i_c + R_4(i_c - i_b) + R_2(i_c - i_a) = 0$$

$$i_b - i_a = I_0$$

Now only 3 unknowns and 3 equations

Circuit Analysis Techniques Summary

1 • KVL, KCL and Element Constitutive Relationships (always works)
   – Brute force approach: 2B simultaneous equations

Never use

2 • Node Analysis (always works) ← Good work horse

3 • Mesh Analysis (works for planner circuits)
   – Loop Analysis can be used for non-planner circuits (always works)

Rarely use

4 • Superposition (works for linear circuits)

5 • Simplification Techniques (don't always work):
   – Series Combinations and Parallel Combinations
   – Voltage Divider and Current Divider
   – Source Transformations

Use whenever you can

6 • Graphical Load Line Analysis (always works)

Generally used when there is one non-linear element and analytical solution is difficult

1, 2, 3 & 6 work for both linear & non-linear circuits