ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 7

Thevenin and Norton Equivalent Circuits

Announcements

- Recommended Reading:
  - Textbook Chapter 4

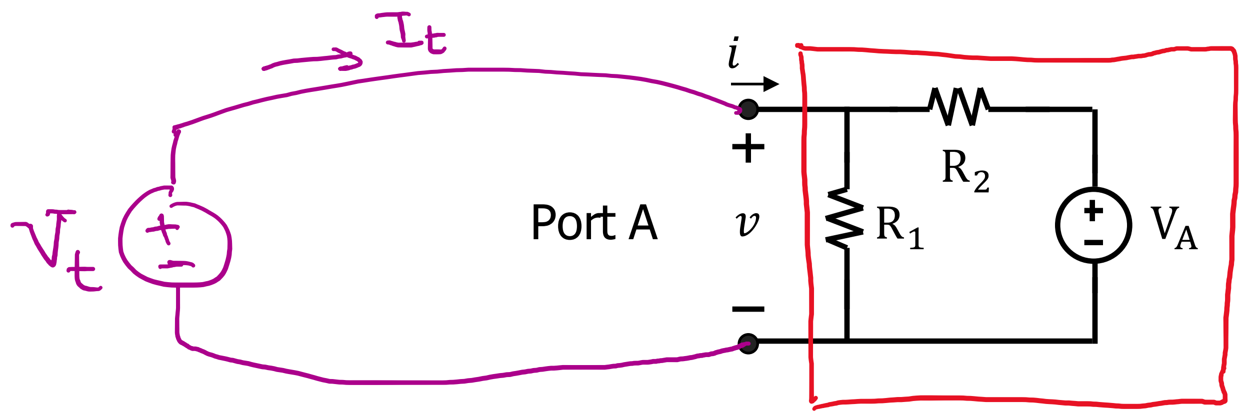
- Upcoming due dates:
  - Lab report 1 due by 11:59 pm on Friday February 8, 2019
  - Prelab 2 due by 12:20 pm on Tuesday February 12, 2019
  - Homework 2 due by 11:59 pm on Friday February 15, 2019
  - Lab report 2 due by 11:59 pm on Friday February 22, 2019

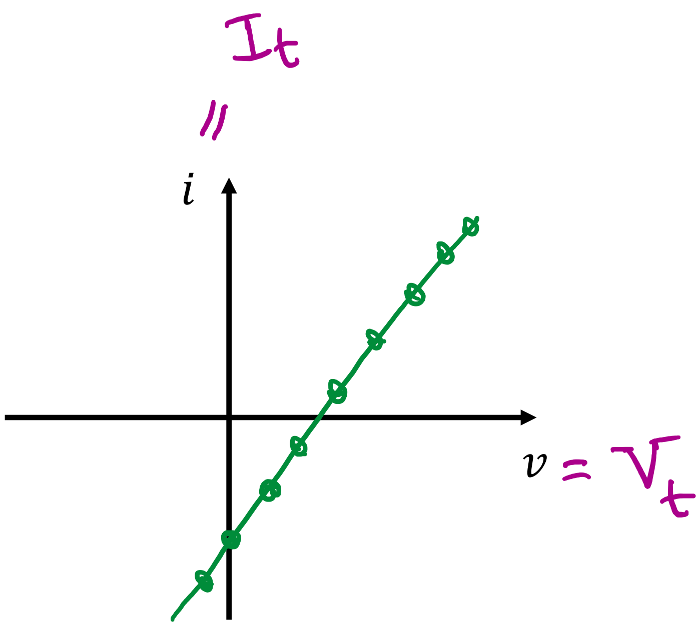
- Lab 2 is next week (starting Tuesday February 12, 2019)

- Prelim 1 on Thursday February 21, 2019 from 7:30 – 9 pm in 203 Phillips

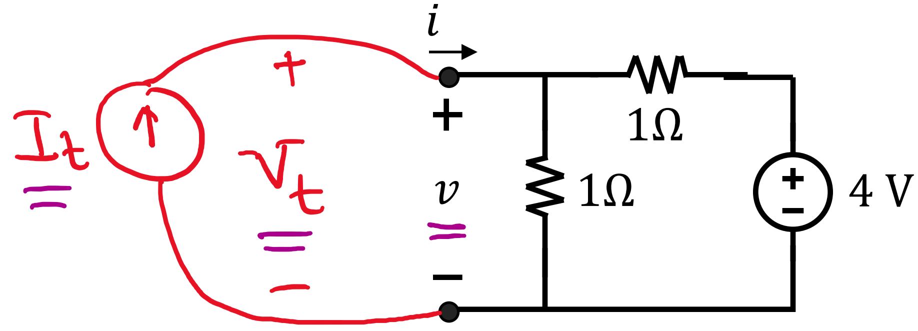
i-v Relationship of a Subcircuit

Determining i-v relationship using a Test Source

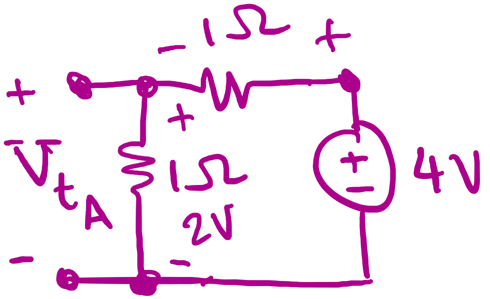




Example of i-v Relationship of a Subcircuit

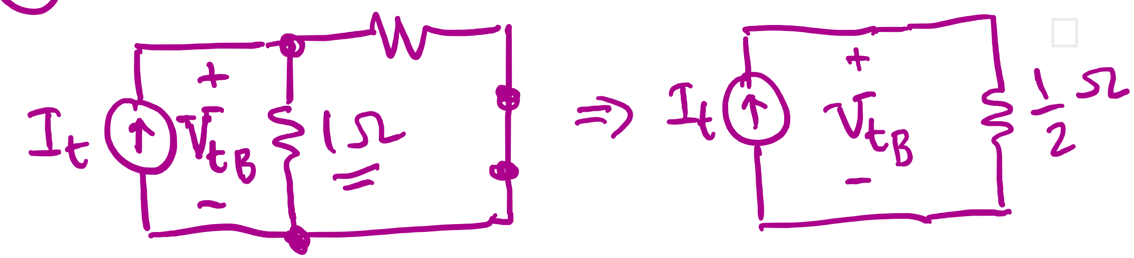


(A) $I_t = 0$ 2V



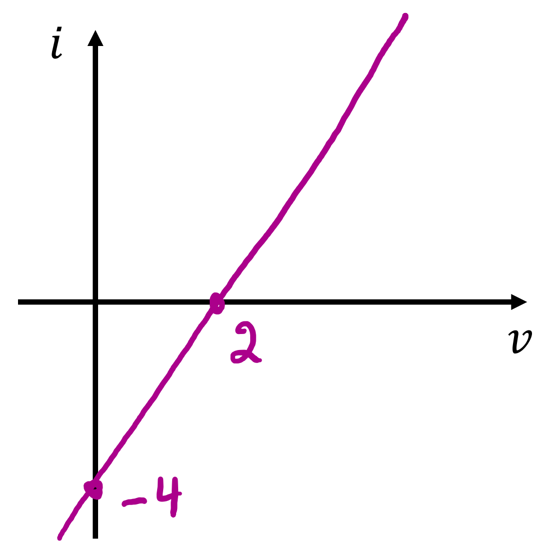
$$V_{t_A} = \frac{1}{1+1} \cdot 4 = 2\,V$$

(B)

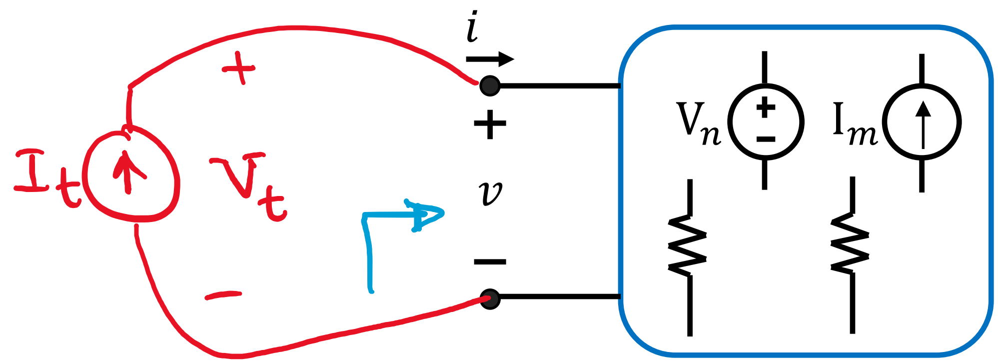


$$V_{t_B} = \frac{1}{2} I_t$$

$$V_t = V_{t_A} + V_{t_B} = 2 + \frac{I_t}{2} \Rightarrow v = 2 + \frac{i}{2}$$



i-v Relationship of Arbitrary Linear Subcircuit



Arbitrary Linear Circuit

By superposition

$$\underbrace{\sum_{n=1}^{N} \alpha_n V_n + \sum_{m=1}^{M} r_m I_m}_{V_{oc} \equiv \underline{\underline{V_{TH}}}} + \underline{\underline{R_{TH}}} I_t = V_t$$

$$V_t = V_{TH} + R_{TH} I_t$$

$$\boxed{v = V_{TH} + R_{TH}\, i}$$

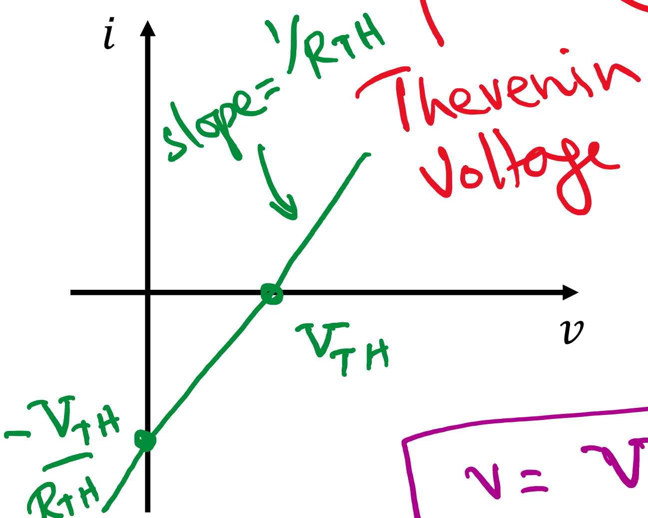
Port Model of Arbitrary Linear Subcircuit

$$v = \underbrace{\sum_n \alpha_n V_n + \sum_m r_m I_m}_{V_{TH}} + R_{TH} i$$

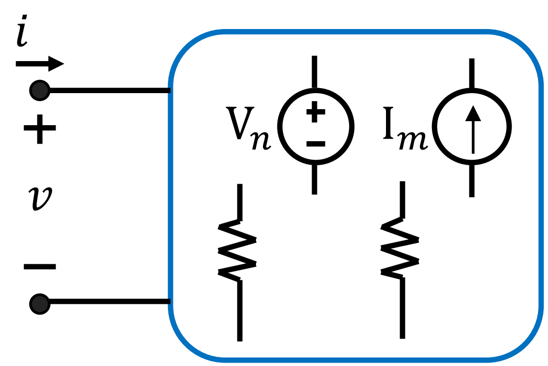
$$\boxed{v = V_{TH} + R_{TH} i}$$

Thevenin Voltage

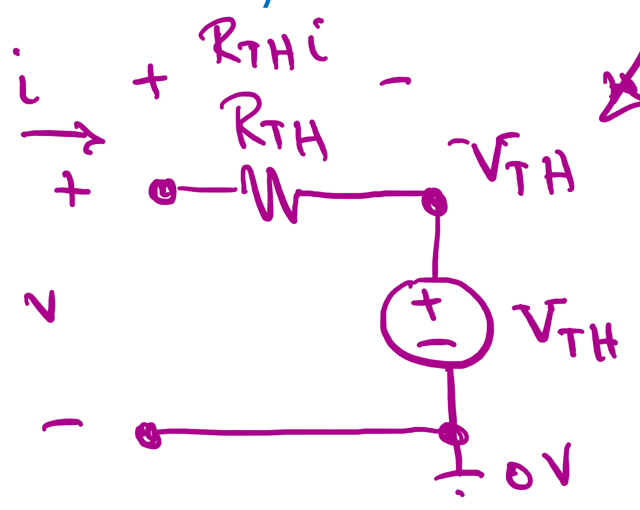
Thevenin Resistance



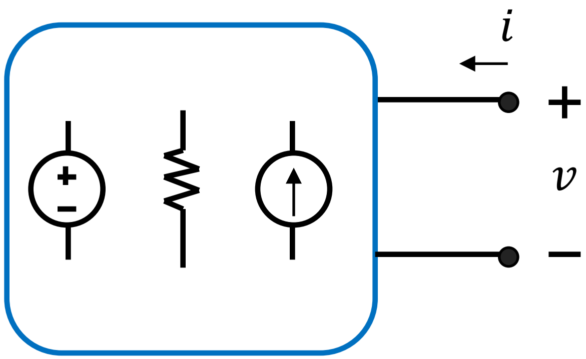
$$\boxed{v = V_{TH} + R_{TH} i}$$



Arbitrary Linear Circuit

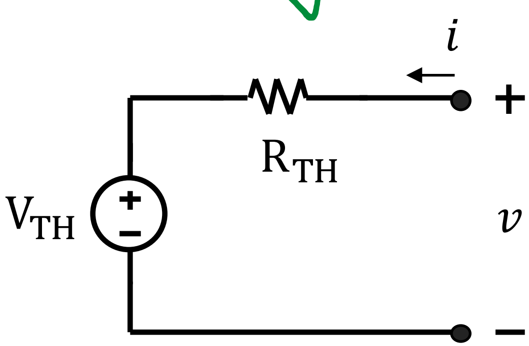


Thevenin Equivalent Circuit



Arbitrary Linear Circuit

$$v = \underbrace{\sum_n \alpha_n \mathrm{V}_n + \sum_m r_m \mathrm{I}_m}_{\mathrm{V_{TH}}} + \mathrm{R_{TH}}\, i$$



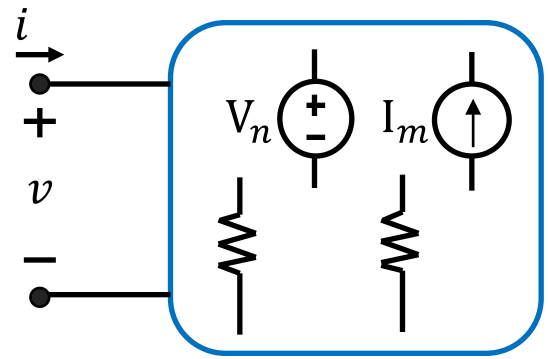
Thevenin Equivalent

$$v = \mathrm{V_{TH}} + \mathrm{R_{TH}}\, i$$

Norton Equivalent Circuit

The i-v relationship of an arbitrary linear subcircuit can also be modeled using a current source in parallel with a resistor

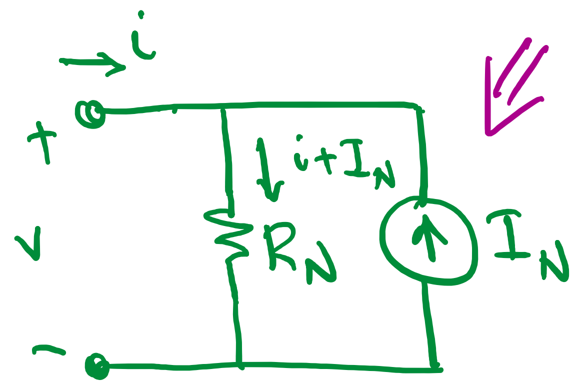
$$v = V_{TH} + R_{TH}\, i$$



Arbitrary Linear Circuit

$$v = (i + I_N) R_N$$

$$v = I_N R_N + R_N i$$

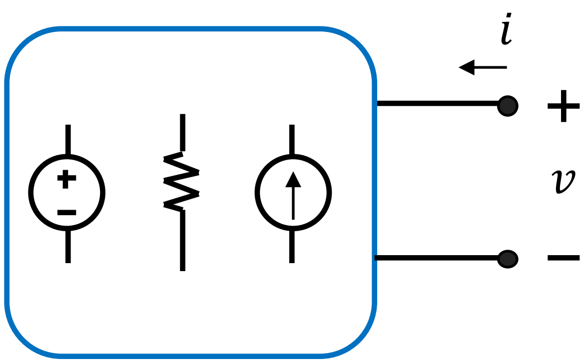


$R_N = R_{TH}$ & $I_N R_N = V_{TH} \Rightarrow I_N = V_{TH}/R_{TH}$

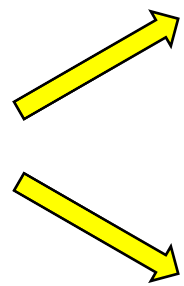
Norton Equivalent

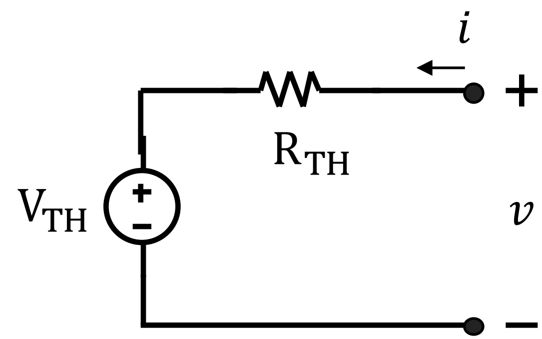
Thevenin and Norton Equivalent

- Superposition allows us to replace a circuit, or part of a circuit (i.e., subcircuit), by a very simple equivalent model – Very powerful technique
- Only works for linear circuits



Arbitrary Linear Circuit



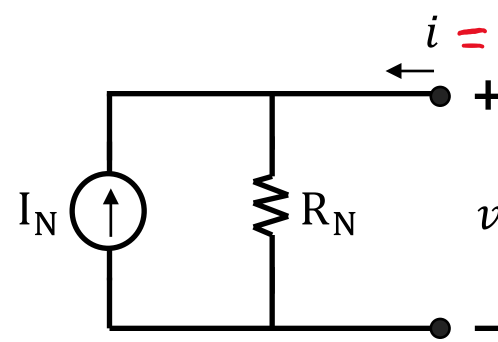


Power

$i = 0$

$P_{diss} = 0$

Thevenin Equivalent



$i = 0$

$P_{diss} = I_N^2 R_N$

Norton Equivalent

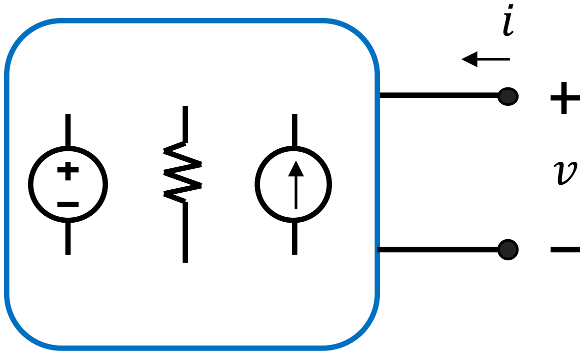
Thevenin and Norton Equivalent - Comments

- Makes sense, since expect i-v relationship at a port of a linear circuit to be a straight line and straight line can be modeled using two parameters: slope ($\mathrm{R_{TH}}$) and intercept ($\mathrm{V_{TH}}$ or $\mathrm{I_N}$)

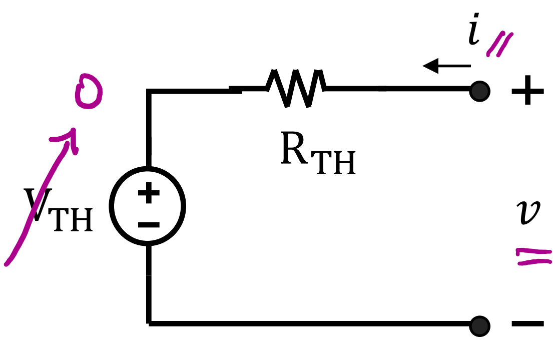
- Very useful when analyzing large circuits – we are typically only interested in the details of part of the circuit – the rest can be modeled with Thevenin or Norton (either is equally good)

- Thevenin and Norton only model terminal i-v characteristics (at a port).
  - Cannot be used to extract information about what is going on inside – including power dissipation

Finding Thevenin and Norton Equivalents



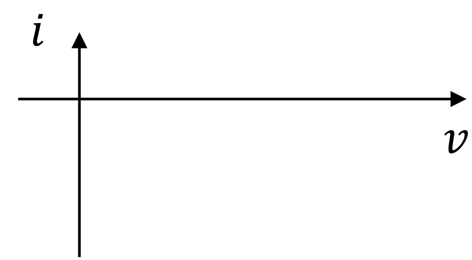
Arbitrary Linear Circuit



Thevenin Equivalent

$$v = \underbrace{\sum_n \alpha_n V_n + \sum_m r_m I_m}_{V_{TH} = V_{OC}} + R_{TH}\, i$$

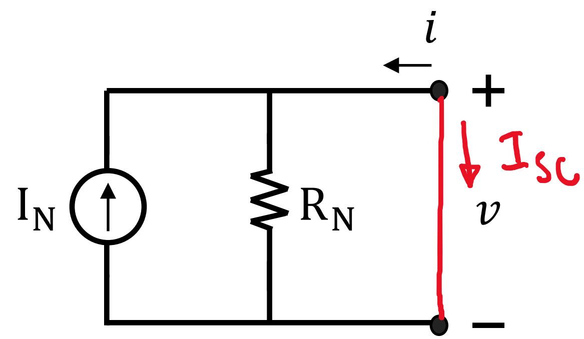
$$v = V_{TH} + R_{TH}\, i$$



- V_{TH} is equal to the "Open Circuit Voltage"
  - $V_{TH} = V_{OC} \equiv v|_{i=0}$
- R_{TH} is the resistance seen from that port with all independent sources set to zero
  - $\boxed{R_{TH} = \frac{v}{i}\Big|_{V_n=0,\, I_m=0}}$

Finding Thevenin and Norton Equivalents (Cont.)

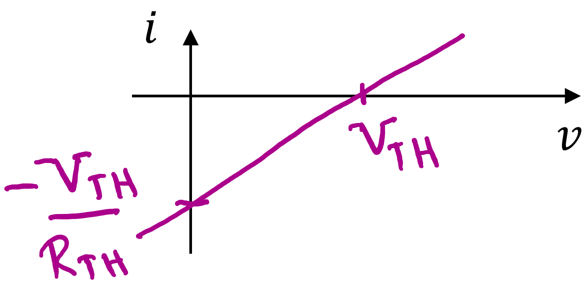
- For the Norton model to match the Thevenin model in terms of port i-v characteristics:
  - $R_N = R_{TH}$
  - $I_N = \frac{V_{TH}}{R_{TH}}$



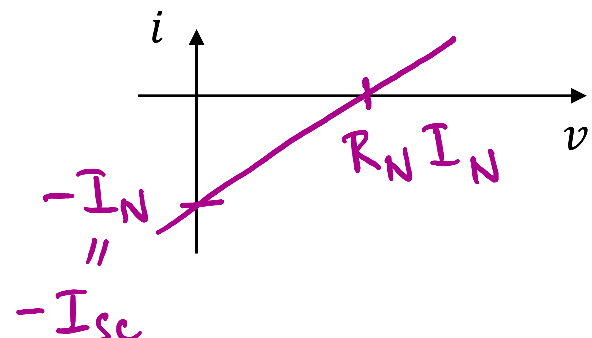
Norton Equivalent

- I_N is also the "Short Circuit Current"
  - $I_N = I_{SC} \equiv -i|_{v=0}$

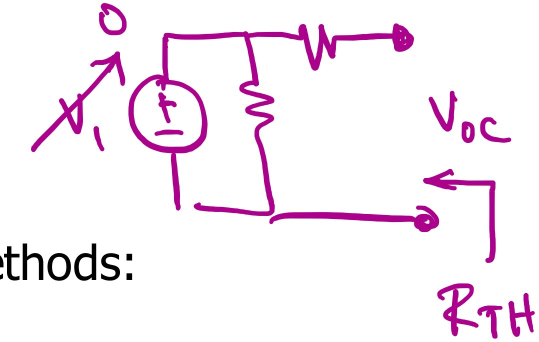
$$v = V_{TH} + R_{TH}i$$



$$v = R_N I_N + R_N i$$



Finding Thevenin and Norton Equivalents (Cont.)



- Note $R_{TH} = {}^{V_{TH}}/_{I_N} = {}^{V_{OC}}/_{I_{SC}}$

- Therefore to find R_{TH} can use one of the following methods:

1) Find V_{OC} and I_{SC} and take ratio (does not work if both are zero)

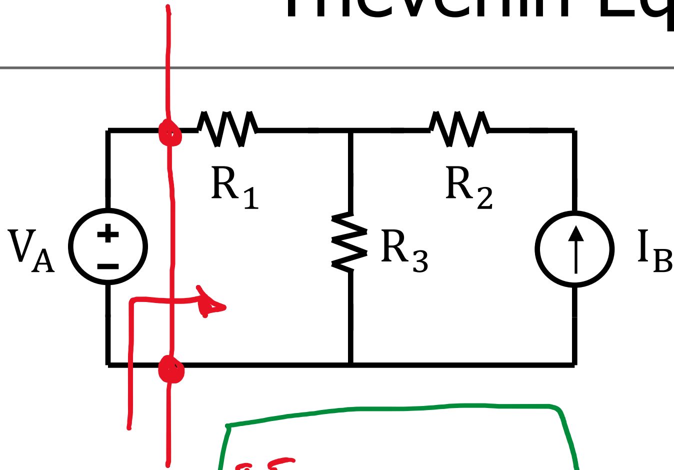
2) Set all independent sourses to zero and do resistor series/parallel combinations (does not always work, e.g., when you have dependent sources)

Fastest way if it works

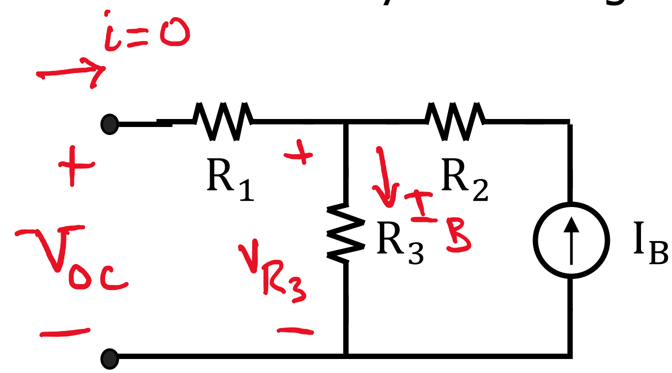
3) Set all independent sources to zero, apply a test source, find the co-variable and find resistance from ratio, $R_{TH} = {}^{V_t}/_{I_t}$

Always works

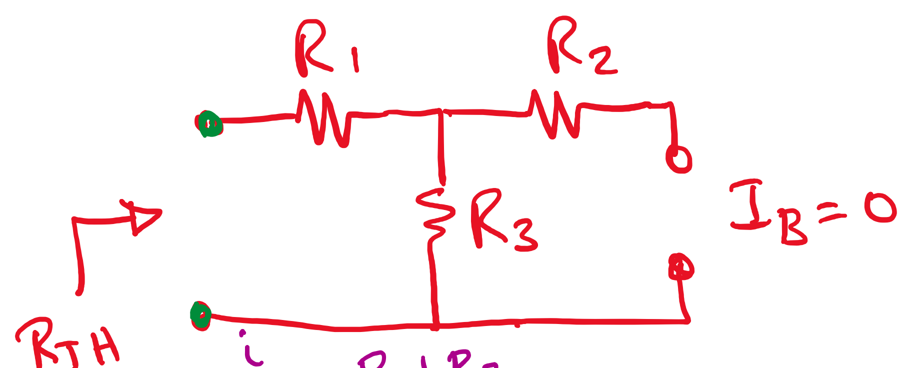
Thevenin Equivalent Circuit Example 1

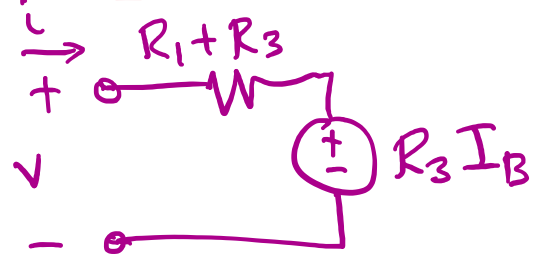


Let's find the Thevenin Equivalent Model of the part of the circuit seen by the voltage source



$$V_{TH} = V_{oc} = R_3 I_B$$

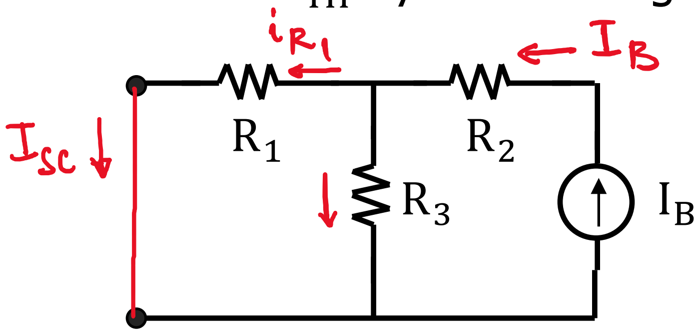




$$\boxed{R_{TH} = R_1 + R_3}$$

Example 1 (Cont.)

Let's confirm R_{TH} by also finding the Short Circuit Current



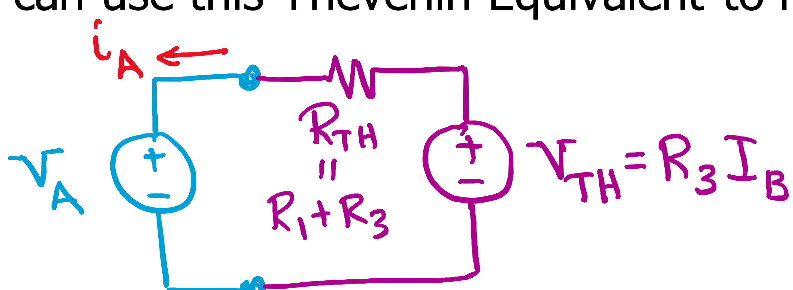
$$I_{sc} = i_{R_1} = \frac{R_3}{R_1 + R_3} \cdot I_B$$

Using Current Divider

$$R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{\cancel{R_3}\,\cancel{I_B}}{\frac{\cancel{R_3}\,\cancel{I_B}}{R_1 + R_3}} = R_1 + R_3$$

$$\Rightarrow \boxed{R_{TH} = R_1 + R_3}$$ Same as before

We can use this Thevenin Equivalent to find the current going into V_A

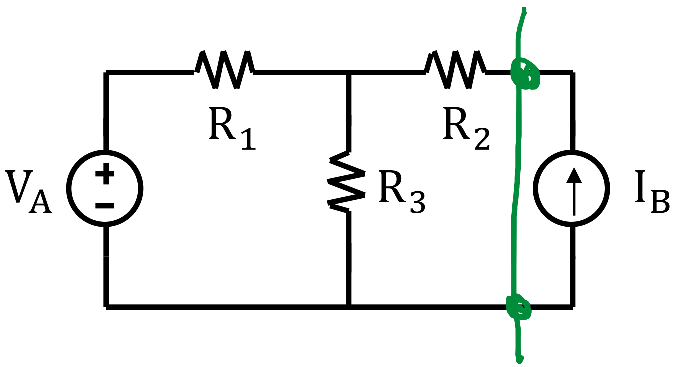


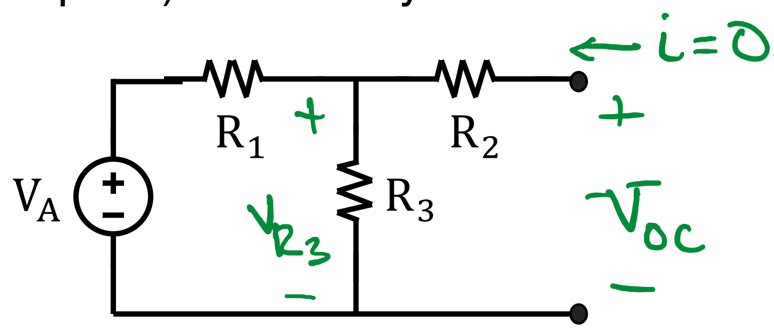
$$i_A = \frac{V_{TH} - V_A}{R_{TH}} = \frac{R_3 I_B - V_A}{R_1 + R_3}$$

Thevenin Equivalent Circuit Example 2

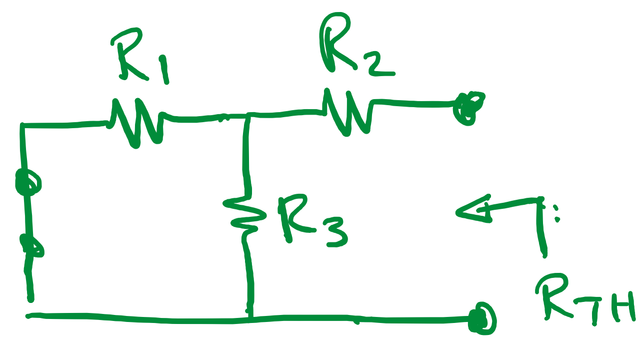
Thevenin and Norton equivalent circuit depends on where we look at the circuit from (i.e., it depends on the port)

Now consider the part of the circuit (of example 1) as seen by the current source





$$V_{TH} = V_{oc} = \frac{R_3}{R_1+R_3} V_A$$

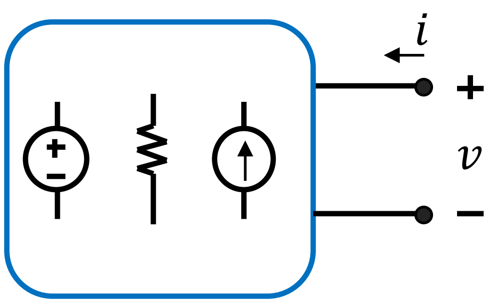


$$R_{TH} = R_2 + (R_1 \| R_3)$$

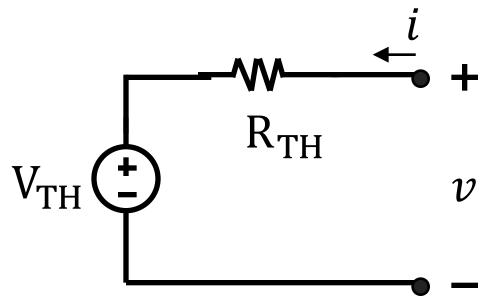
$$= R_2 + \frac{R_1 R_3}{R_1+R_3}$$

Thevenin and Norton Equivalent Summary

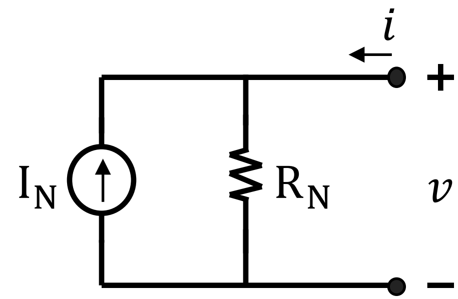
- We can represent any linear network (with linear resistors, linear dependent sources and independent sources) seen from a particular port with simple equivalent models (i.e., models with the same i-v characteristics at that port)



Arbitrary Linear Circuit

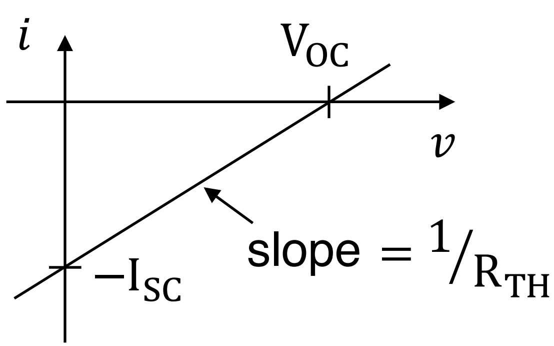


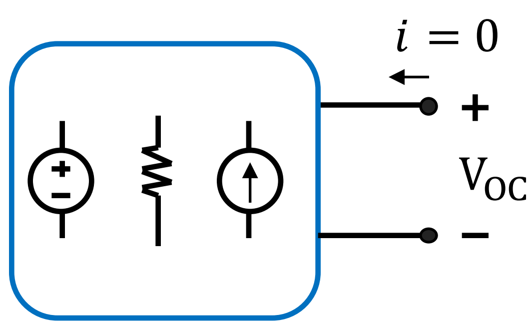
Thevenin Equivalent



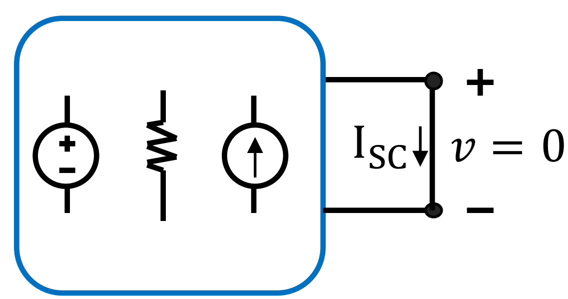
Norton Equivalent

$R_{TH} = R_N$





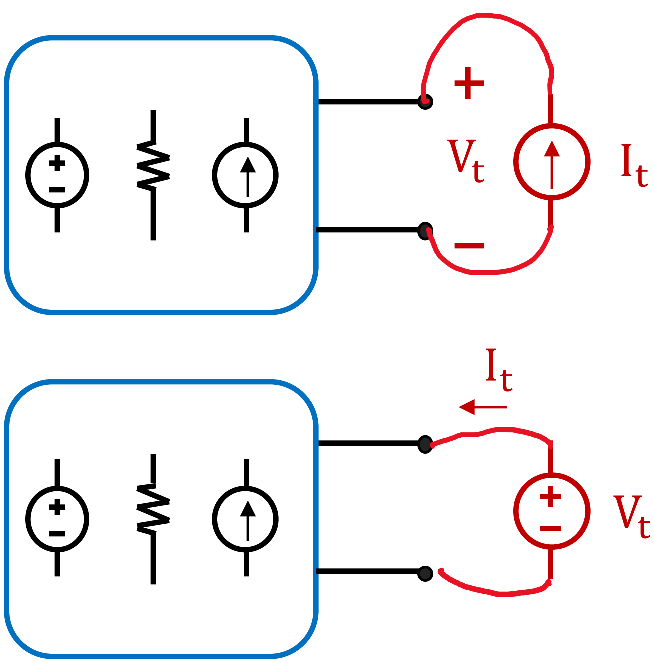
$V_{TH} = V_{OC}$



$I_N = I_{SC}$

Thevenin and Norton Equivalent Summary (Cont.)

- Thevenin and Norton Resistances are the same and can be found several ways:
  1) $R_{TH} = R_N = {}^{V_{OC}}/_{I_{SC}}$ (Does not work if both V_{OC} and I_{SC} are zero)
  2) Set all independent sources in the network to zero and find equivalent resistance looking into the "dead" circuit:
     a) Sometimes can use series and parallel combinations to determine R_{TH} (Does not always work, especially if circuit has dependent sources)
     b) Use Test Source Method (will always work)



Apply I_t

Determine V_t

$R_{TH} = {}^{V_t}/_{I_t}$

Apply V_t

Determine I_t