Node Analysis

① Make a neat drawing of circuit

② Mark and label every node

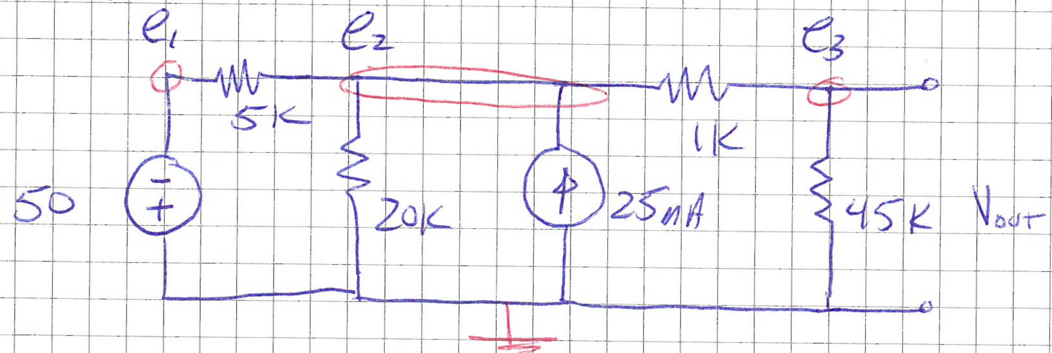
③ Select one node to be the reference (because voltages are always defined with respect to something)

④ Write KCL for each node (except the reference)

⑤ Solve for the node voltages

⑥ As needed, determine the currents.

Example 1



What is V_{OUT}?

3 nodes + reference.

$e_1 = -50V$

So we only need 2 eqs.

$$\frac{+50 - e_2}{5K} + \frac{0 - e_2}{20K} + 25mA + \frac{e_3 - e_2}{1K} = 0$$

$$\frac{e_2 - e_3}{1K} + \frac{0 - e_3}{45K} = 0$$

Consolidate

$$-e_2\left(\frac{1}{5K}+\frac{1}{20K}+\frac{1}{1K}\right)+e_3\left(\frac{1}{1K}\right)=\frac{50}{5K}-25\,mA=-15\,mA$$

$$e_2\left(\frac{1}{1K}\right)-e_3\left(\frac{1}{1K}+\frac{1}{45K}\right)=0$$

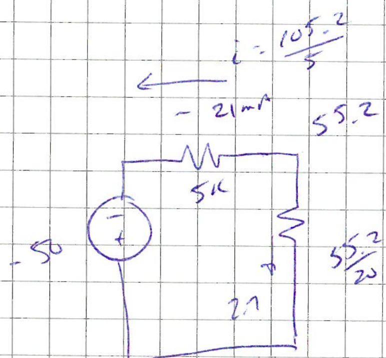
Clear the denominator

$$-e_2(4+1+20)+e_3\cdot 20=-300$$

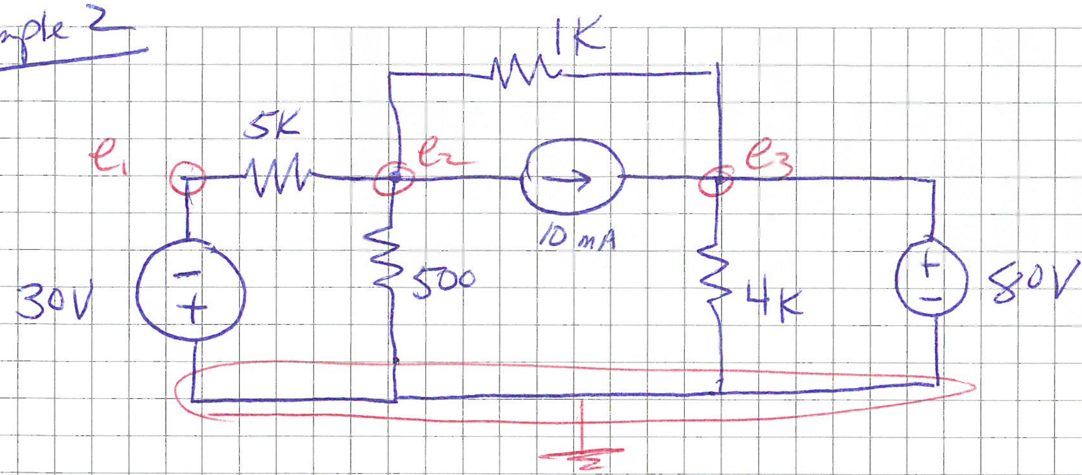
$$e_2\cdot 45-e_3\cdot 46=0$$ (Voltage divider!)

Solve 2 eqs

$e_2 = 55.2 \quad e_3 = 54$



Example 2



$e_1 = -30$

$e_3 = 80$

e_2 $$\frac{-30 - e_2}{5K} + \frac{0 - e_2}{500} + \frac{80 - e_2}{1000} - 10mA = 0$$

Consolidate

$$-e_2\left[\frac{1}{5000} + \frac{1}{500} + \frac{1}{1000}\right] = 10mA + \frac{30}{5000} - \frac{80}{1000}$$

$$= 10mA + 6mA - 80mA$$

$$= 64mA$$

Clear fractions by multiplying by 5000

$$-e_2\left[1 + 10 + 5\right] = 5 \cdot 64$$

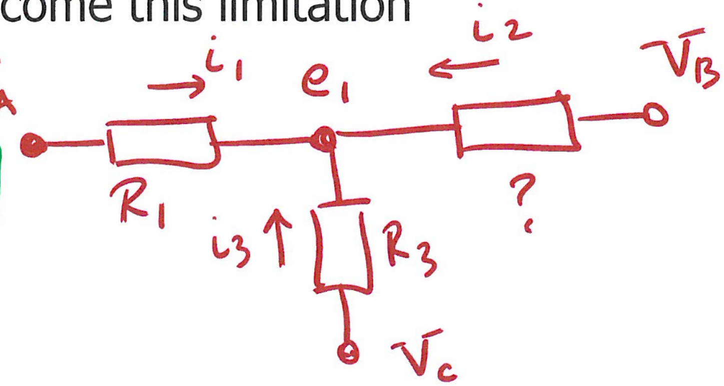
$$e_2 = -\frac{5 \cdot 64}{16} = \underline{-20V}$$

Notice the 4K resistor had no impact - it was parallel to a voltage source.

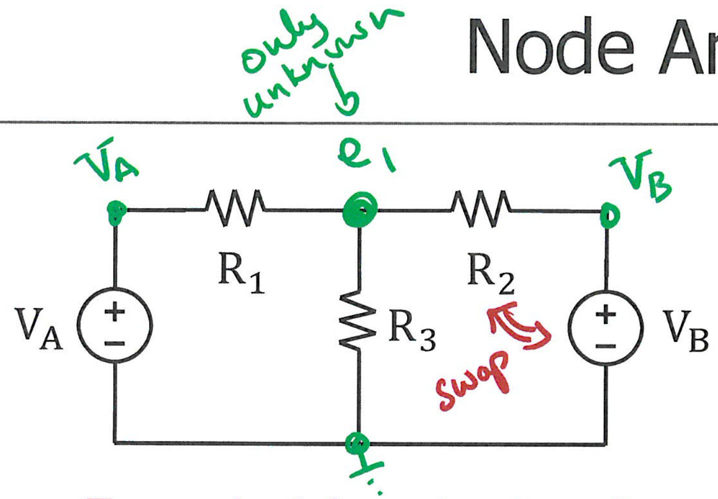
Standard Node Analysis Limitations

- There are a few situations where standard Nodal Analysis does not work
  - One of these situations is when the circuit has a floating independent voltage source (i.e., voltage source is not connected to the ground node either directly or via another independent voltage source)
  - Standard node analysis does not work since the current of an independent voltage source is not a function of the voltage across it – hence it is not possible to write KCL in terms of node voltages and constitutive relationships
  - There are two ways to overcome this limitation

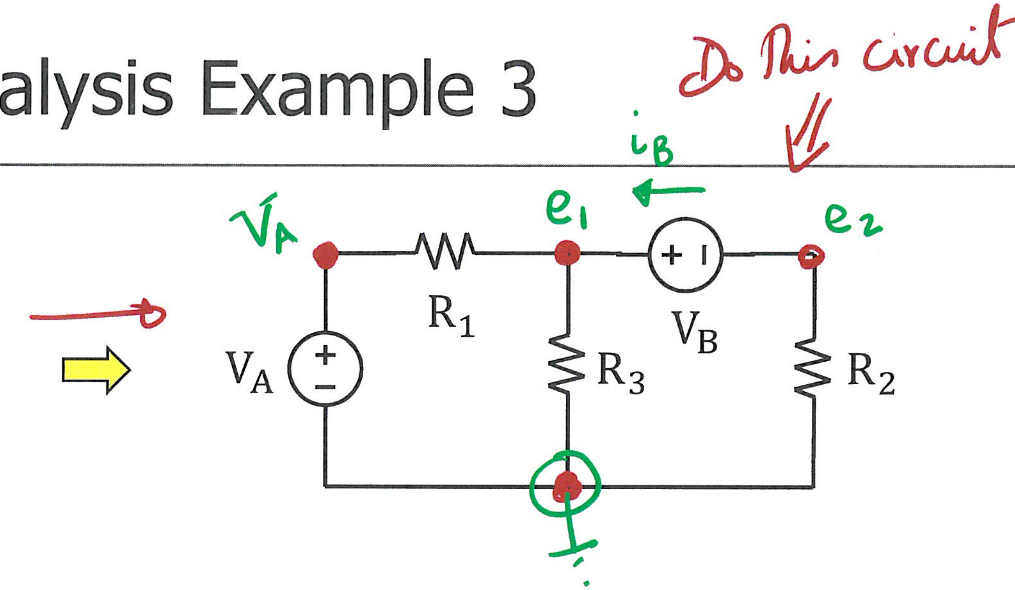
$$\frac{V_A - e_1}{R_1} + \frac{V_C - e_1}{R_3} + ? = 0$$



Node Analysis Example 3



Example 1 from Lecture 5

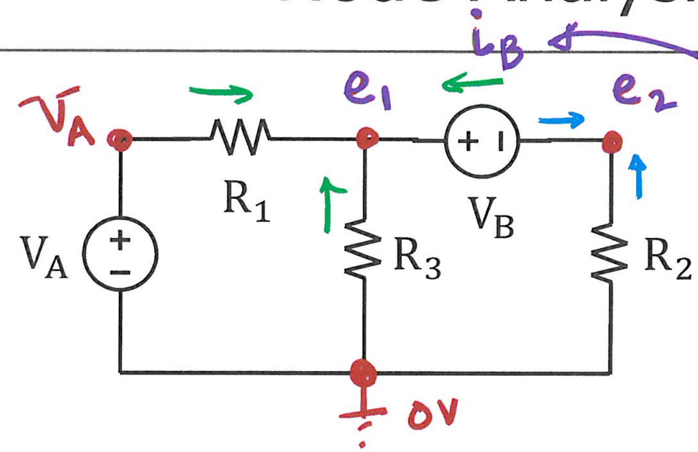


Don't know how to write i_B in terms of e_1 and e_2

Therefore, Can't apply Basic Node Analysis

Will have to Modify Node Analysis

Node Analysis Example 3 – Method 1



Modified Node Analysis (used by computer programs such as SPICE)

Create new unknown i_B

Have 3 unknowns e_1, e_2, i_B

KCL @ e_1

$$\frac{V_A - e_1}{R_1} + \frac{0 - e_1}{R_3} + i_B = 0 \quad \text{— (1)}$$

KCL @ e_2

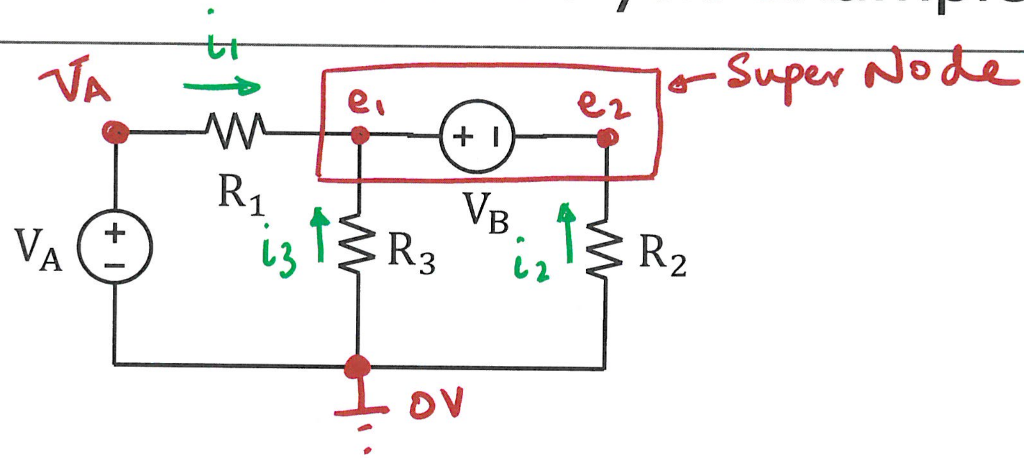
$$\frac{0 - e_2}{R_2} - i_B = 0 \quad \text{— (2)}$$

3rd Eqn comes from constitutive relationship of V_B ⇒

$$e_1 - e_2 = V_B \quad \text{— (3)}$$

Node Analysis Example 3 – Method 2

Super Node Method



Apply KCL to the Super Node

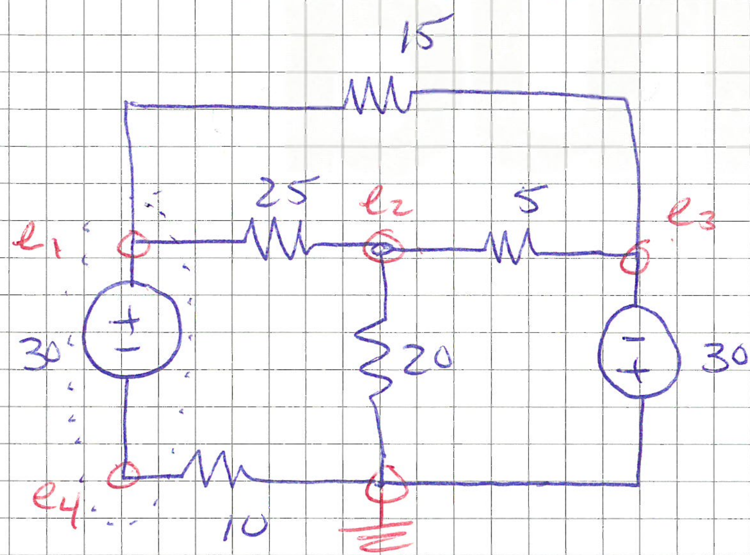
$i_1 + i_2 + i_3 = 0 \Rightarrow$

$$\frac{V_A - e_1}{R_1} + \frac{0 - e_2}{R_2} + \frac{0 - e_1}{R_3} = 0 \quad \text{— (1)}$$

2nd Eqn: $e_1 - e_2 = V_B$ — (2)

Constitutive Relationship of V_B ⇒

Solve for e_1 & e_2



① Pick nodes   5 nodes   set one to Ground

② Note $e_3 = -30V$.

③ Define super node $(e_1 - e_4)$

④ Write KCL

$(e_1 - e_4)$: $$\frac{0 - e_4}{10} + \frac{e_2 - e_1}{25} + \frac{-30 - e_1}{15} = 0$$

e_2: $$\frac{e_1 - e_2}{25} + \frac{0 - e_2}{20} + \frac{-30 - e_2}{5} = 0$$

~~$e_3 = 30$~~

Finally $$e_1 = e_4 + 30$$

} 3 eq.