ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 39

Power in AC Systems

Announcements

- Recommended Reading:
  - Textbook Chapter 10

- Upcoming due dates:
  - Lab report 6 due by 11:59 pm on Friday May 3, 2019
  - Homework 6 due by 11:59 pm on Tuesday May 7, 2019

Power System

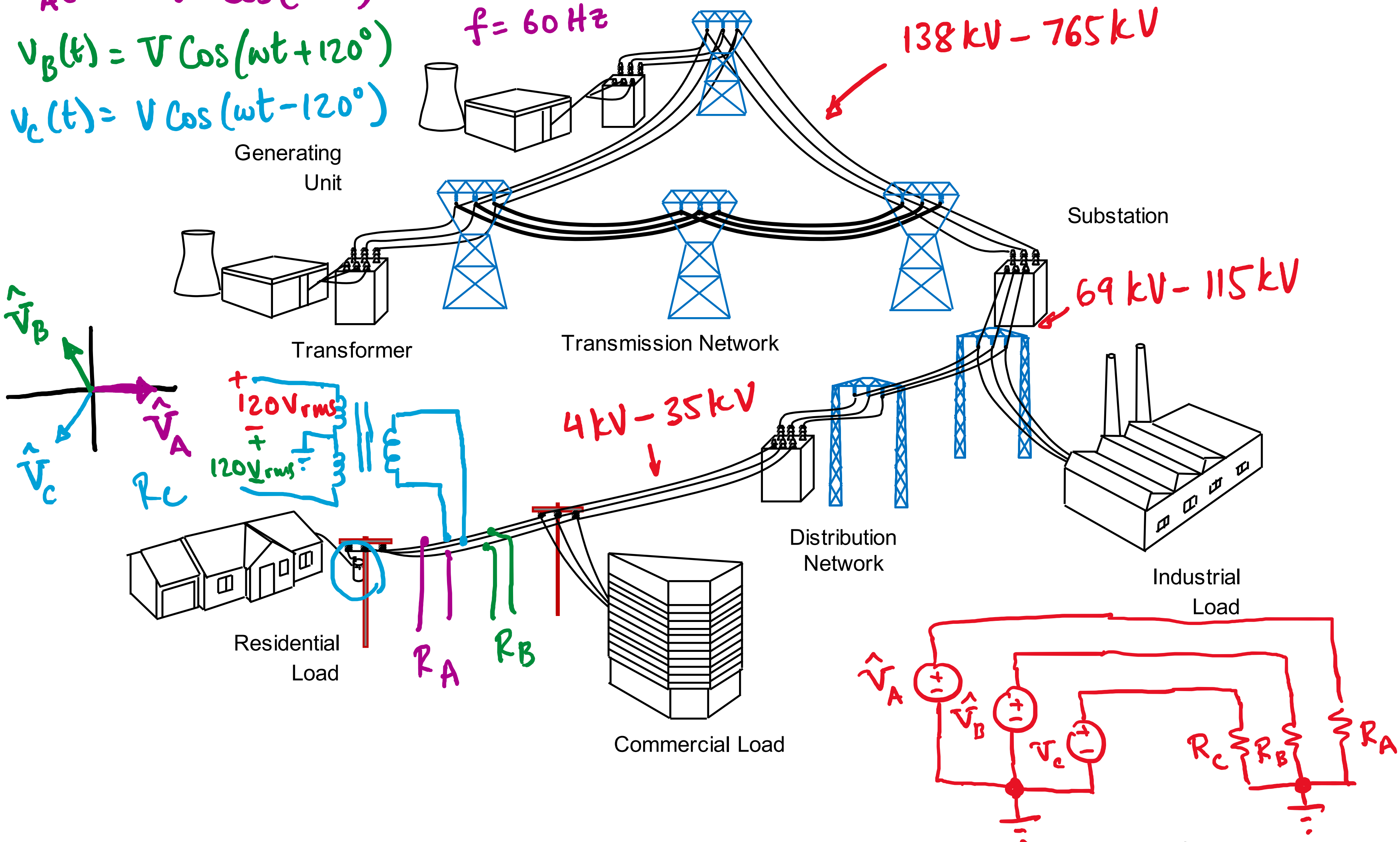
$v_A(t) = V \cos(\omega t)$

$v_B(t) = V \cos(\omega t + 120^\circ)$

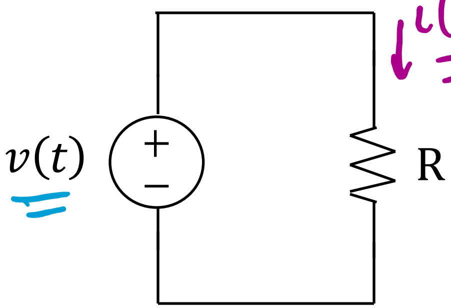
$v_C(t) = V \cos(\omega t - 120^\circ)$

$\omega = 2\pi f$

$f = 60\,Hz$



Instantaneous Power and Average Power



$i(t) = \frac{V_A}{R}\cos(\omega t)$

$v(t) = V_A \cos(\omega t)$

Trigonometric Identity

$\cos(2\theta) = 2\cos^2(\theta) - 1$

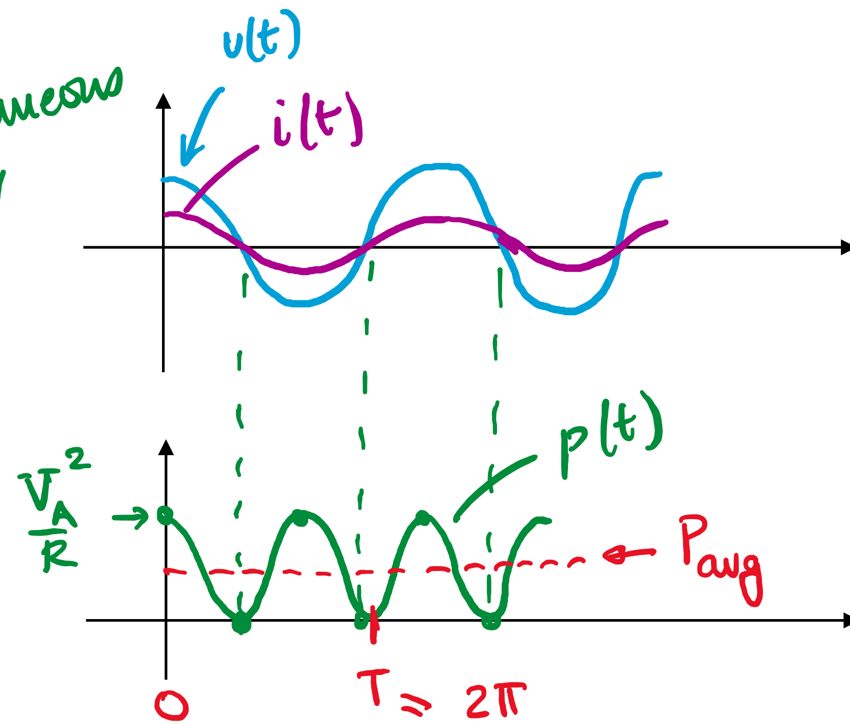
$\Rightarrow \cos^2(\omega t) = \frac{1 + \cos(2\omega t)}{2}$

$p(t) = v(t)\, i(t) = \frac{V_A^2}{R}\cos^2(\omega t)$

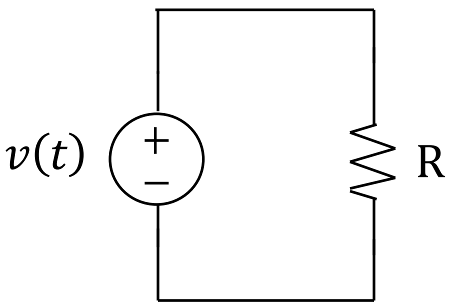
$p(t) = \frac{V_A^2}{2R}\left[1 + \cos(2\omega t)\right]$ ← Instantaneous Power

$P_{avg} = \frac{1}{T}\int_0^T p(t)\, dt = \frac{1}{2} \cdot \frac{V_A^2}{R}$

Average Power (also called "Real Power" & "Active Power")



Root Mean Square (RMS) Value



$$v(t) = V_A \cos(\omega t)$$

The RMS value of a time varying voltage or current is the equivalent constant (i.e., dc) value that delivers the same amount of average power to a resistive load

Delivers same average power to resistive load as 120 V dc

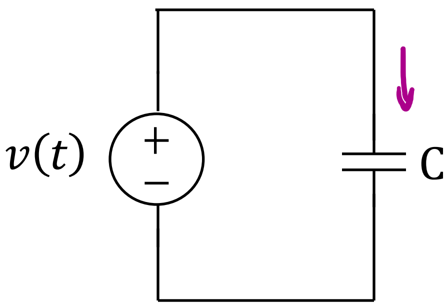
$v(t) = V\cos(\omega t)$

$V_{rms} = 120\,V_{rms} \rightarrow V = \sqrt{2} \cdot 120$

$\approx 170\,V$

$V_{rms} = \dfrac{V}{\sqrt{2}} \Leftarrow$ $\boxed{V_{rms} = \sqrt{\dfrac{1}{T}\displaystyle\int_t^{t+T} v(t)^2\, dt}}$

Power Delivered to a Capacitive Load



$i_c(t) = C\frac{dv_c}{dt}$

$= -V_A \omega C \sin(\omega t)$

$v(t) = V_A \cos(\omega t)$

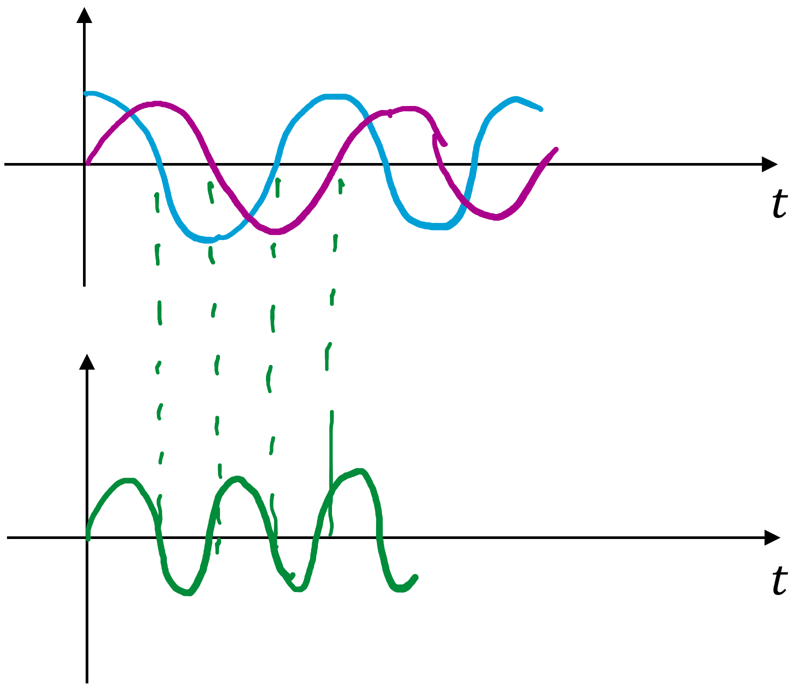
Trigonometric Identity

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

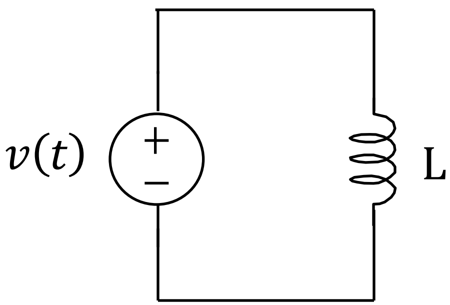
$p(t) = v(t)i(t) = -V_A^2 \omega C \cos(\omega t) \sin(\omega t)$

$p(t) = -\frac{V_A^2 \omega C}{2} \sin(2\omega t)$

$P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt = 0$

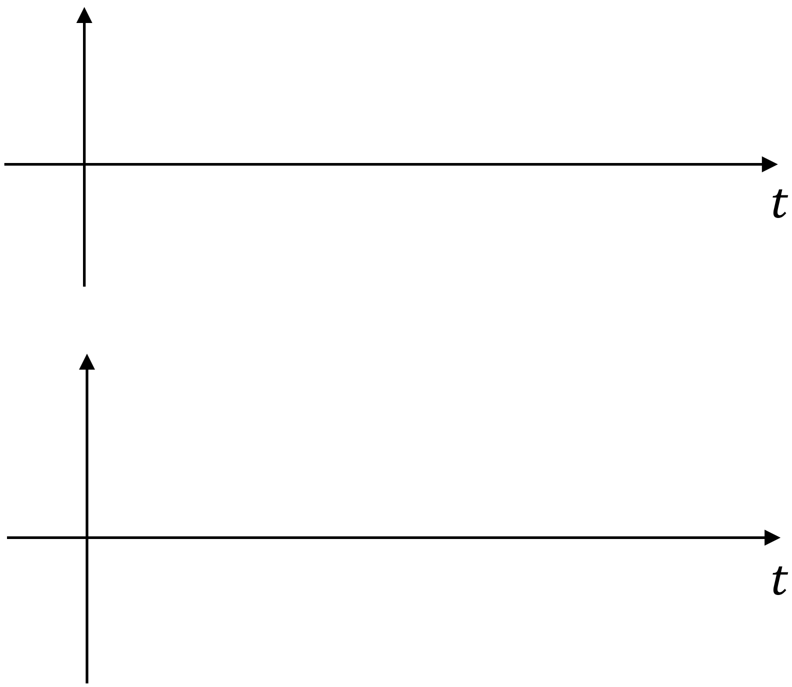


Power Delivered to an Inductive Load

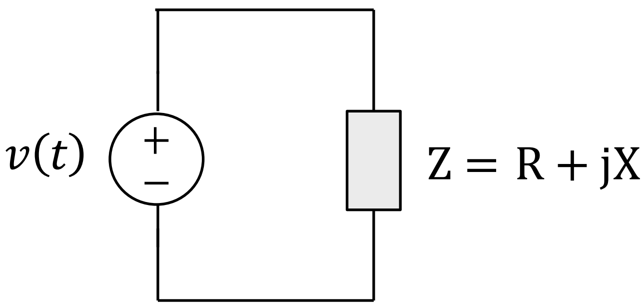


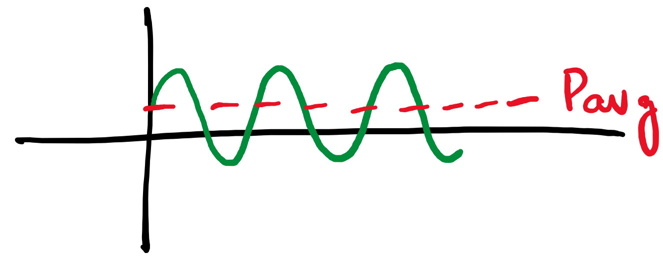
$$v(t) = V_A \cos(\omega t)$$

$$P_{avg} = \frac{1}{T}\int p(t)\,dt = 0$$



Power Delivered to an Arbitrary R-L-C Load





$$v(t) = V_A \cos(\omega t)$$

$v(t) = \bar{V} \cos(\omega t)$

$i(t) = I \cos(\omega t + \phi)$

$P_{avg} = V I \cos(\phi)$

General Method

Find either the magnitude of the voltage across the resistor $\mathrm{V_R}$, or the magnitude of the current through the resistor $\mathrm{I_R}$, and then apply:

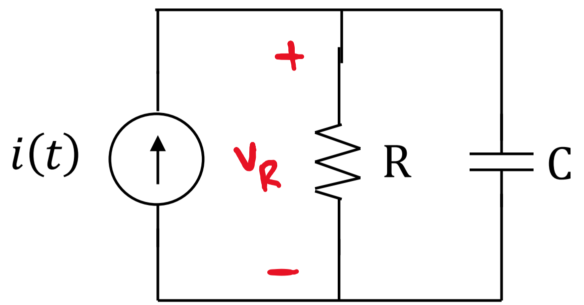
$$\mathrm{P_R} = \frac{1}{2}\frac{\mathrm{V_R^2}}{\mathrm{R}}$$

Reactive Power

$= V I \sin(\phi)$

or $$\mathrm{P_R} = \frac{1}{2}\mathrm{I_R^2 R}$$

Average Power Example

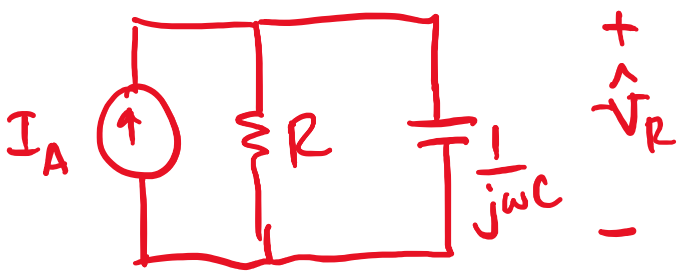


$i(t) = I_A \cos(\omega t)$

Determine average power delivered to the R-C load

$P_{avg} = \frac{1}{2}\frac{V_R^2}{R}$

$v_R = V_R \cos(\omega t + \phi)$

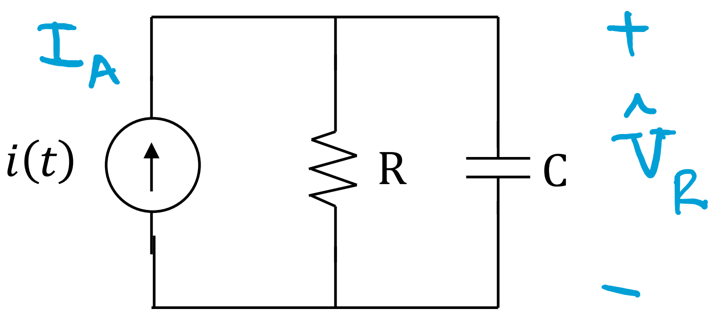


$\hat{V}_R = \left[R \| (1/j\omega C)\right] I_A$

Average Power in Terms of Phasors

$$\mathrm{P_{avg}} = \frac{1}{2}\mathrm{Re}\{\hat{V}\hat{I}^*\} = \frac{1}{2}\mathrm{Re}\{\hat{V}^*\hat{I}\}$$

$$P_{avg} = \frac{1}{2}\mathrm{Re}\{\hat{V}_R e^{-j\phi_R} I_A\}$$



$$\hat{V}_R = V_R e^{j\phi_R}$$

$i(t) = I_A \cos(\omega t)$