ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 38

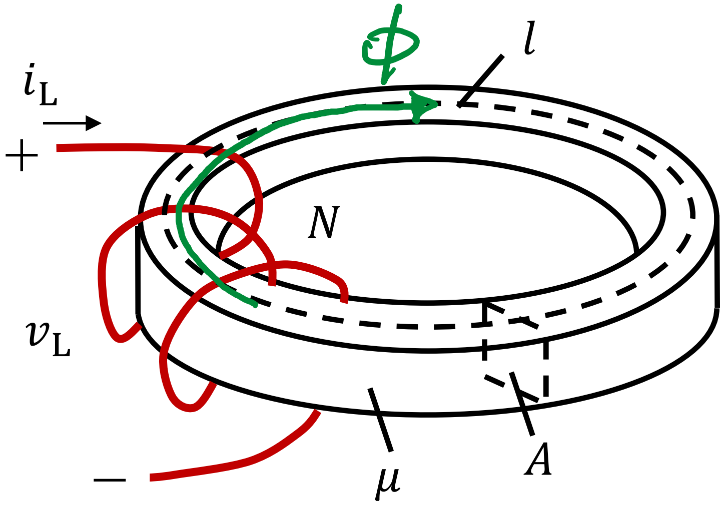
Mutual Inductance and Transformers

Announcements

- Recommended Reading:
  - Textbook Chapter 6.4-6.5 and Chapter 9.10-9.11

- Upcoming due dates:
  - Homework 5 due by 11:59 pm on Monday April 29, 2019
  - Prelab 6 due by 11:59 pm on Monday April 29, 2019
  - Lab report 6 due by 11:59 pm on Friday May 3, 2019
  - Homework 6 due by 11:59 pm on Tuesday May 7, 2019

Inductance (Self-Inductance)



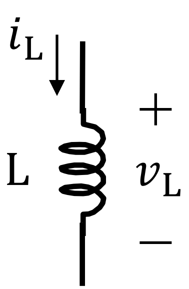
Faraday's Law: $v_L = \dfrac{d\lambda_L}{dt}$

$$\lambda_L = N\Phi = N(BA) = N(\mu H)A = \mu N\left(\frac{N i_L}{l}\right)A$$

$$\lambda_L = \frac{\mu A N^2 i_L}{l} \qquad (\mu_0 = 4\pi\times10^{-7}\ \mathrm{H/m})$$

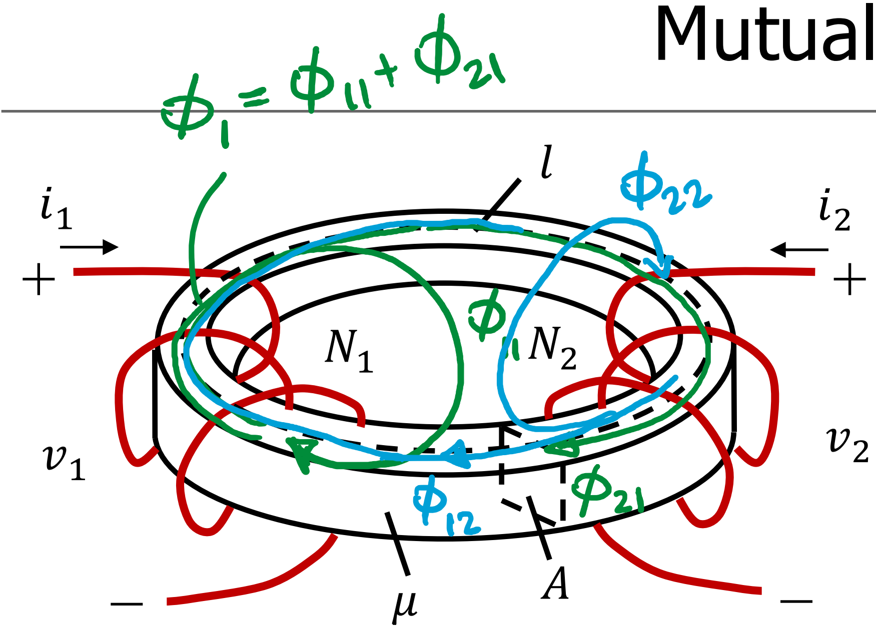
$$\lambda_L = \mathrm{L} i_L$$

$$\mathrm{L} \equiv \frac{d\lambda_L}{di_L} = \frac{\mu A N^2}{l} = \mathcal{P}N^2$$



$$v_L = \mathrm{L}\frac{di_L}{dt}$$

Mutual Inductance



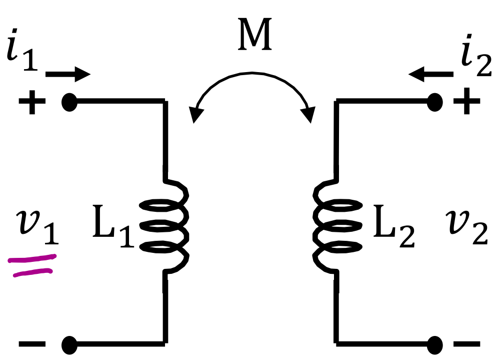
- Use right-hand-rule to determine direction of flux

self-inductance

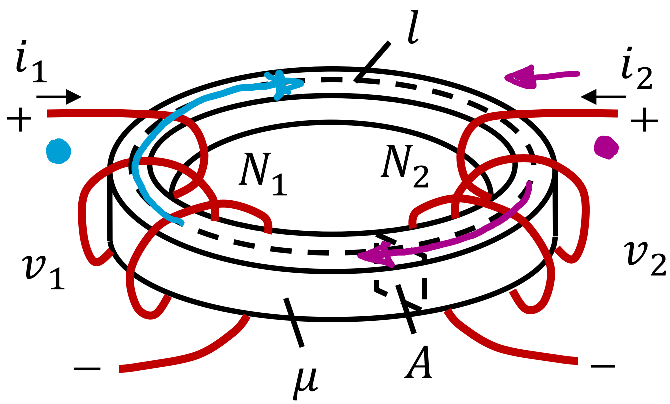
mutual inductance

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

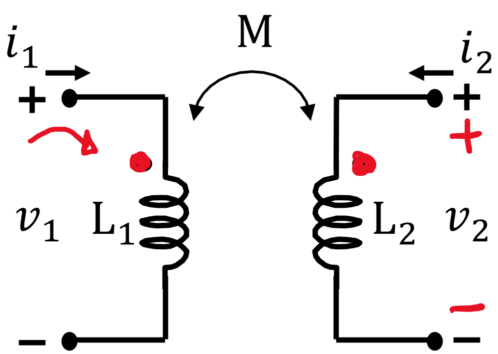
$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$



Dot Convention

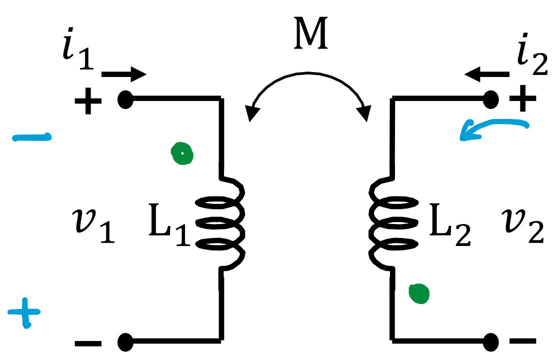


- Currents into the dot generate flux inside the coupled coils in the same direction
- When the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal



$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

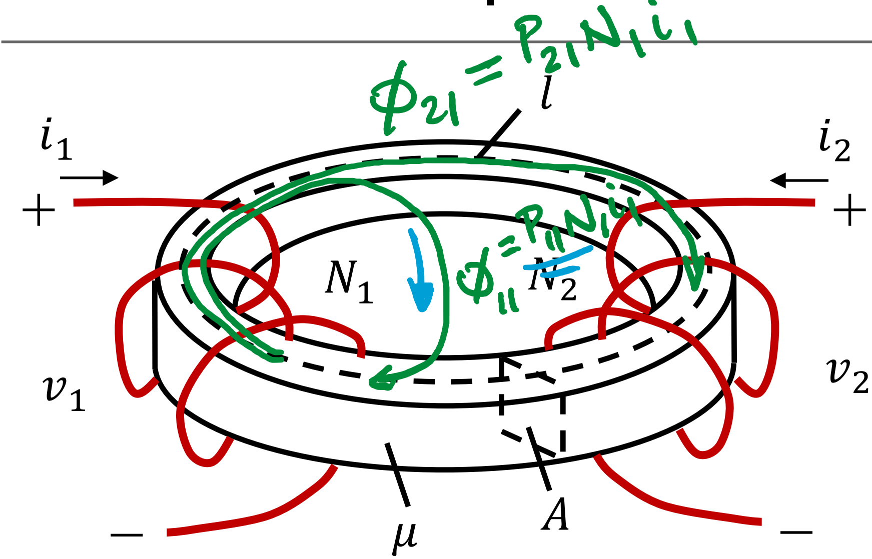
$$v_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$



$$v_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$v_2 = -M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

Relationship Between Mutual and Self Inductances



$$\mathrm{L}_1 = \frac{d\lambda_1}{di_1} = \mathcal{P}_1 N_1^2 = (\mathcal{P}_{11} + \mathcal{P}_{12}) N_1^2$$

$$\mathrm{L}_2 = \frac{d\lambda_2}{di_2} = \mathcal{P}_2 N_2^2 = (\mathcal{P}_{22} + \mathcal{P}_{21}) N_2^2$$

$$\mathrm{M} = \frac{d\lambda_2}{di_1} = \frac{d\lambda_1}{di_2} = \mathcal{P}_{21} N_2 N_1 = \mathcal{P}_{12} N_1 N_2$$

$P_{21} = P_{12}$

$$M_{21} = \frac{\lambda_{21}}{i_1} = \frac{N_2 P_{21} N_1 i_1}{i_1} = P_{21} N_2 N_1 = P_{12} N_1 N_2 = M_{12} \equiv M$$

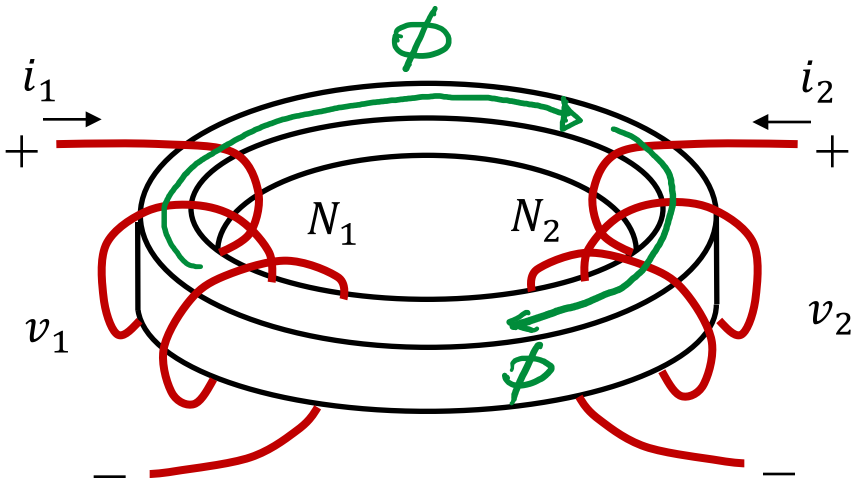
$$\mathrm{L}_1 \mathrm{L}_2 = \mathcal{P}_1 N_1^2 \mathcal{P}_2 N_2^2 = (\mathcal{P}_{11} + \mathcal{P}_{12})(\mathcal{P}_{22} + \mathcal{P}_{21}) N_1^2 N_2^2 = P_{12} P_{21} \left(1 + \frac{P_{11}}{P_{21}}\right)\left(1 + \frac{P_{22}}{P_{12}}\right) N_1^2 N_2^2$$

$$L_1 L_2 = \underbrace{(P_{12} N_1 N_2)}_{M} \underbrace{(P_{21} N_1 N_2)}_{M} \left(1 + \frac{P_{11}}{P_{21}}\right)\left(1 + \frac{P_{22}}{P_{12}}\right)$$

$$M = \frac{1}{\sqrt{\left(1 + \frac{P_{11}}{P_{21}}\right)\left(1 + \frac{P_{22}}{P_{12}}\right)}} \sqrt{L_1 L_2} \equiv k \sqrt{L_1 L_2} \qquad 0 \leq k \leq 1$$

↑ Coupling Coefficient

Ideal Transformer



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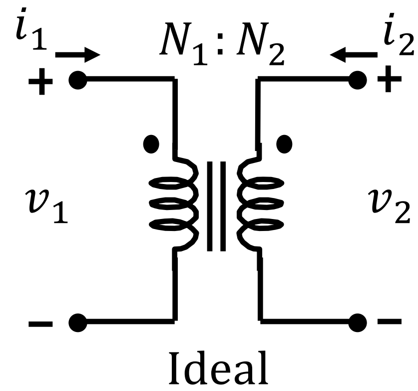


$$v_1 = \frac{d\lambda_1}{dt} = \frac{dN_1\phi}{dt} = N_1\frac{d\phi}{dt}$$

$$v_2 = \frac{d\lambda_2}{dt} = \frac{dN_2\phi}{dt} = N_2\frac{d\phi}{dt}$$

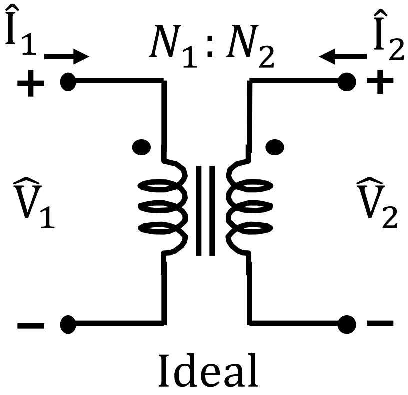
$$\frac{v_2}{v_1} = \frac{N_2\frac{d\phi}{dt}}{N_1\frac{d\phi}{dt}} = \frac{N_2}{N_1}$$

Lossless and stores no energy $\Rightarrow P_1 = P_2 \Rightarrow v_1 i_1 = -v_2 i_2$

$$\Rightarrow \frac{i_2}{i_1} = -\frac{v_1}{v_2} = -\frac{N_1}{N_2}$$

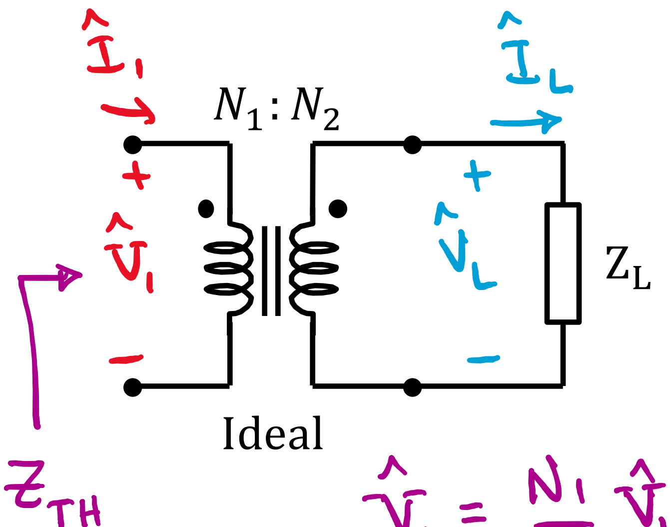
$$\boxed{\frac{v_2}{v_1} = \frac{N_2}{N_1}} \quad \& \quad \boxed{\frac{i_2}{i_1} = -\frac{N_1}{N_2}}$$

Ideal Transformer – Impedance Transformation



$$\frac{\hat{V}_2}{\hat{V}_1} = \frac{N_2}{N_1}$$

$$\frac{\hat{I}_2}{\hat{I}_1} = -\frac{N_1}{N_2}$$



$$Z_L = \frac{\hat{V}_L}{\hat{I}_L}$$

$$Z_{TH} \equiv \frac{\hat{V}_1}{\hat{I}_1} = \frac{N_1}{N_2}\hat{V}_L \cdot \frac{N_1}{N_2}\frac{1}{\hat{I}_L}$$

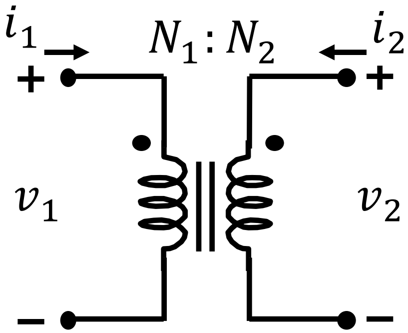
$$\hat{V}_1 = \frac{N_1}{N_2}\hat{V}_L$$

$$\hat{I}_1 = \frac{N_2}{N_1}\hat{I}_L$$

$$\Rightarrow Z_{TH} = \left(\frac{N_1}{N_2}\right)^2 \frac{\hat{V}_L}{\hat{I}_L}$$

$$\Rightarrow \boxed{Z_{TH} = \left(\frac{N_1}{N_2}\right)^2 Z_L}$$

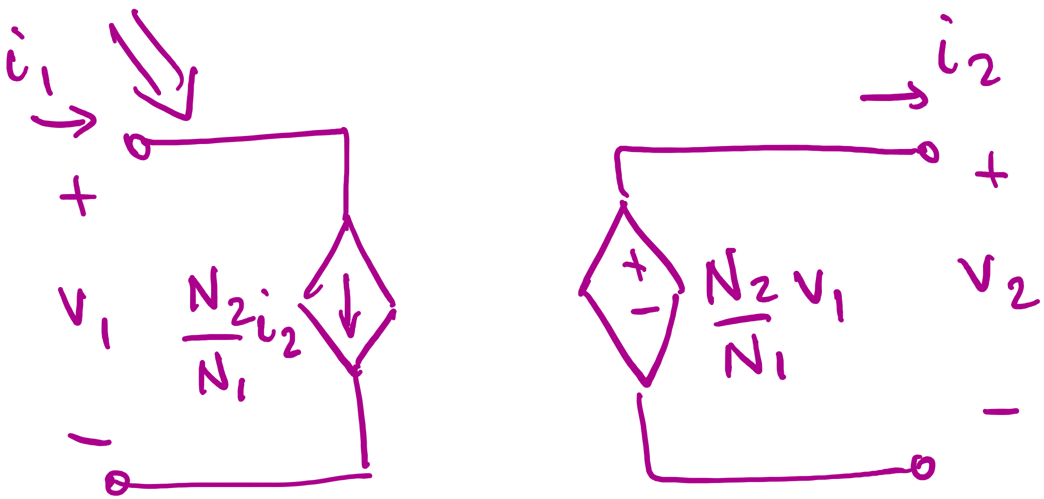
Ideal Transformer – Dependent Source Model



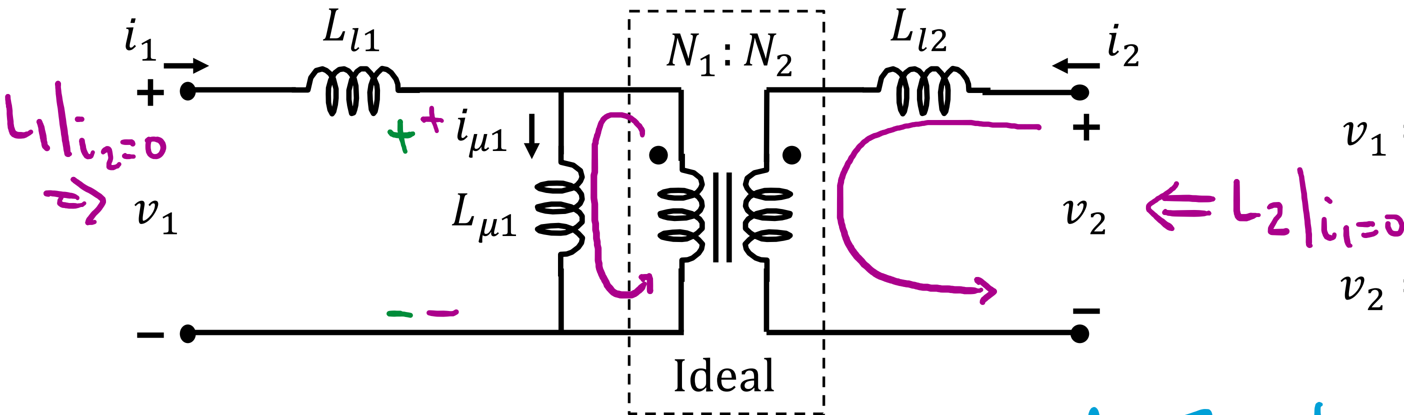
Ideal

$$\frac{v_2}{v_1} = \frac{N_2}{N_1}$$

$$\frac{i_2}{i_1} = -\frac{N_1}{N_2}$$



Non-Ideal Transformer Model



$$v_1 = \mathrm{L}_1 \frac{di_1}{dt} + \mathrm{M} \frac{di_2}{dt}$$

$$v_2 = \mathrm{M} \frac{di_1}{dt} + \mathrm{L}_2 \frac{di_2}{dt}$$

$$L_1 = L_{l_1} + L_{\mu_1}$$

$$L_2 = L_{l_2} + \left(\frac{N_2}{N_1}\right)^2 L_{\mu_1}$$

$$M = \frac{N_2}{N_1} L_{\mu_1}$$

Ideal Transformer

$$\left.\begin{array}{l} L_{l_1} = 0 \\ L_{l_2} = 0 \end{array}\right\} \Rightarrow \begin{array}{l} L_1 = L_{\mu_1} \\ L_2 = \left(\frac{N_2}{N_1}\right)^2 L_{\mu_1} \end{array}$$

$$M = \frac{N_2}{N_1} L_{\mu_1}$$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{\frac{N_2}{N_1} L_{\mu_1}}{\sqrt{L_{\mu_1} \left(\frac{N_2}{N_1}\right)^2 L_{\mu_1}}} = 1$$

$$L_{\mu_1} \to \infty$$