ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 37

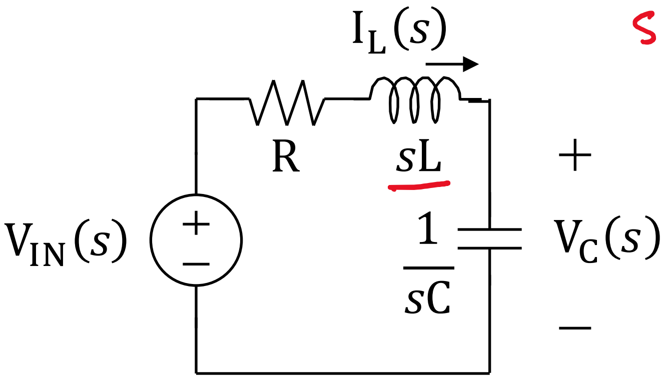
Transfer Functions and Bode Plots Revisited

Announcements

- Recommended Reading:
  - Textbook Chapter 13 and Appendix E

- Upcoming due dates:
  - Homework 5 due by 11:59 pm on Monday April 29, 2019
  - Prelab 6 due by 11:59 pm on Monday April 29, 2019
  - Lab report 6 due by 11:59 pm on Friday May 3, 2019
  - Homework 6 due by 11:59 pm on Tuesday May 7, 2019

Transfer Function in s-Domain



$s = \sigma + j\omega$

$$H(s) \equiv \frac{Y(s)}{X(s)}$$

output (any variable of interest)

input (drive)

with initial conditions = 0

$$H_{IL}(s) \equiv \frac{I_L(s)}{V_{IN}(s)} = \frac{\frac{s}{L}}{\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

$$H_{VC}(s) \equiv \frac{V_C(s)}{V_{IN}(s)} = \frac{1/sC}{sL + R + 1/sC} = \frac{1}{s^2LC + sRC + 1} = \frac{1/LC}{\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

All Transfer Functions of a circuit have the same denominator

The denominator set equal to zero is the characteristic equation of the circuit: $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ ← Solutions of this give information about natural response of circuit

Poles and Zeros of the Transfer Function

$$H(s) \equiv \frac{Y(s)}{X(s)} = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}$$

$$H(s) = \frac{a_n(s+z_1)(s+z_2)\cdots(s+z_n)}{b_m(s+p_1)(s+p_2)\cdots(s+p_m)}$$

Poles are the roots of the denominator polynomial – $D(s)$ $\Rightarrow s = -p_1, -p_2, \cdots, -p_m$

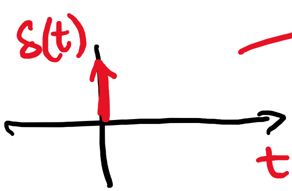
Zeros are the roots of the numerator polynomial – $N(s)$ $\Rightarrow s = -z_1, -z_2, \cdots, -z_n$

By applying an impulse, we are getting the natural response of the circuit

$$Y(s) = H(s)X(s) = \frac{N(s)}{D(s)}X(s)$$

Impulse Response: $X(s) = 1$ $\Rightarrow$ $Y(s) = H(s) = \frac{N(s)}{D(s)}$

The poles of the transfer function determine the natural response of the circuit



$\mathcal{L}\{\delta(t)\} = 1$

$$D(s) = b_m(s+p_1)(s+p_2)\cdots(s+p_m) = 0$$

$$\boxed{s = -\alpha \pm j\sqrt{\alpha^2 - \omega^2}}$$

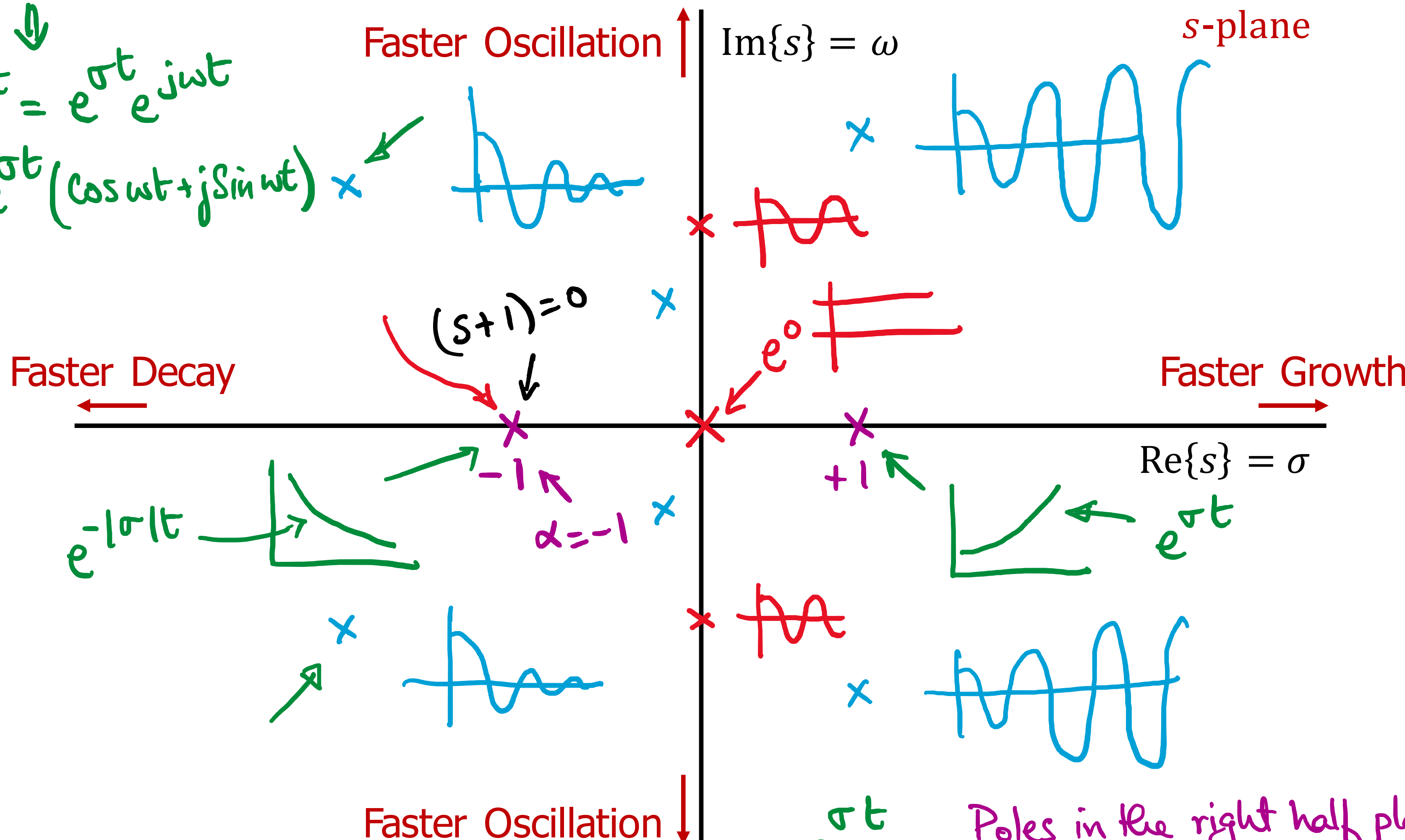
Pole Location and Natural Response

$s = \underline{\underline{\sigma}} + j\omega$

$\Downarrow$

$e^{st} = e^{\sigma t} e^{j\omega t}$

$= e^{\sigma t}(\cos\omega t + j\sin\omega t)$



Poles in the right half plane mean that the circuit is unstable

Sinusoidal Steady-State Frequency Response

s-Domain

$$X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s) = H(s)X(s)$$

$$s = j\omega$$

The s-domain transfer function can also be used to find the sinusoidal steady-state response of the circuit. Simply replace "s" by "jω"

Phasor-Domain

$$\hat{X}(j\omega) \longrightarrow \boxed{H(j\omega)} \longrightarrow \hat{Y}(j\omega) = H(j\omega)\hat{X}(j\omega) = X|H(j\omega)|e^{j(\phi+\angle H)}$$

$$\hat{X}(j\omega) = Xe^{j\phi}$$

$$H(j\omega) = |H(j\omega)|e^{j\angle H(j\omega)}$$

Time-Domain

$$x(t) = X\cos(\omega t + \phi) \longrightarrow \boxed{\text{Circuit in } t\text{-domain}} \longrightarrow y(t) = X|H(j\omega)|\cos(\omega t + \phi + \angle H(j\omega))$$

Bode Plots (Revisited)

$$H(s) = H_1(s)H_2(s)$$

$\Updownarrow$ $s = j\omega$

$$H(j\omega) = H_1(j\omega)H_2(j\omega)$$

$\Rightarrow$ $$H(j\omega) = \overbrace{|H_1(j\omega)|e^{j\angle H_1(j\omega)}}^{H_1(j\omega)}\overbrace{|H_2(j\omega)|e^{j\angle H_2(j\omega)}}^{H_2(j\omega)}$$

$\Rightarrow$ $$|H(j\omega)|e^{j\angle H(j\omega)} = |H_1(j\omega)||H_2(j\omega)|e^{j(\angle H_2(j\omega)+\angle H_1(j\omega))}$$

$$|H(j\omega)| = |H_1(j\omega)||H_2(j\omega)|$$ $\Rightarrow$ $$|H(j\omega)|_{\text{dB}} = 20\log_{10}(|H_1(j\omega)||H_2(j\omega)|)$$

$$|H(j\omega)|_{dB} = 20\log_{10}|H_1(j\omega)| + 20\log_{10}|H_2(j\omega)|$$

$$\angle H(j\omega) = \angle H_2(j\omega) + \angle H_1(j\omega)$$

Bode Plot Example 1

$$H(s) = \frac{10^5(s + 10^3)}{(s + 10^1)(s + 10^5)}$$

$s = j\omega$

$$H(j\omega) = \frac{10^5(j\omega + 10^3)}{(j\omega + 10^1)(j\omega + 10^5)} \Rightarrow H(j\omega) = \frac{10^2(1 + j\omega/10^3)}{(1 + j\omega/10^1)(1 + j\omega/10^5)}$$

$$H(j\omega) = H_1(j\omega)H_2(j\omega)H_3(j\omega)H_4(j\omega)$$

$$H_1(j\omega) = 10^2$$

$$H_2(j\omega) = (1 + j\omega/10^3)$$

$$H_3(j\omega) = \frac{1}{(1 + j\omega/10^1)}$$

$$H_4(j\omega) = \frac{1}{(1 + j\omega/10^5)}$$

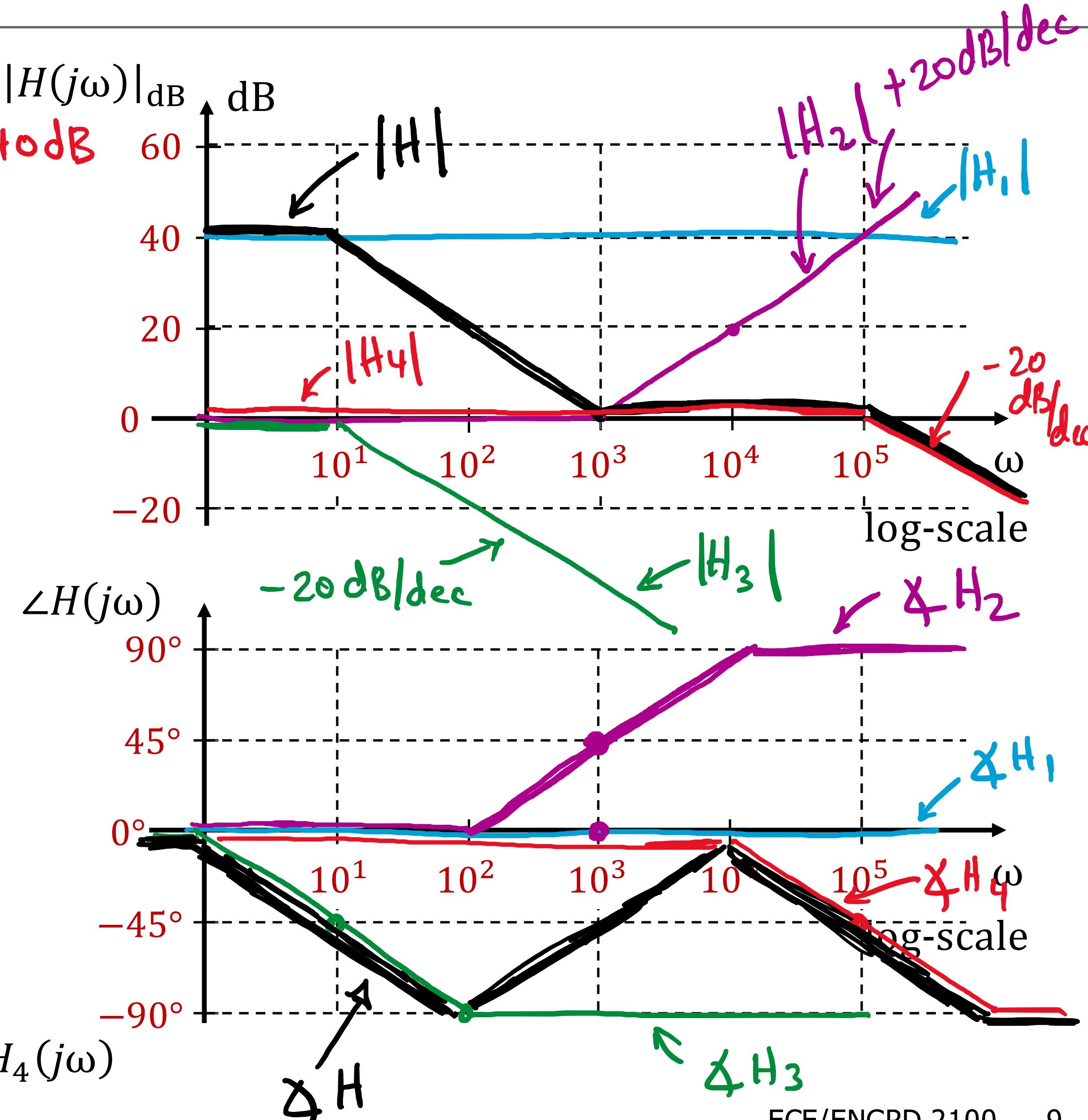
Bode Plot Example 1 (Cont.)

$H_1(j\omega) = 10^2$ → $20\log_{10} 10^2 = 40\text{dB}$

$H_2(j\omega) = (1 + j\omega/10^3)$

$$H_3(j\omega) = \frac{1}{(1 + j\omega/10^1)}$$

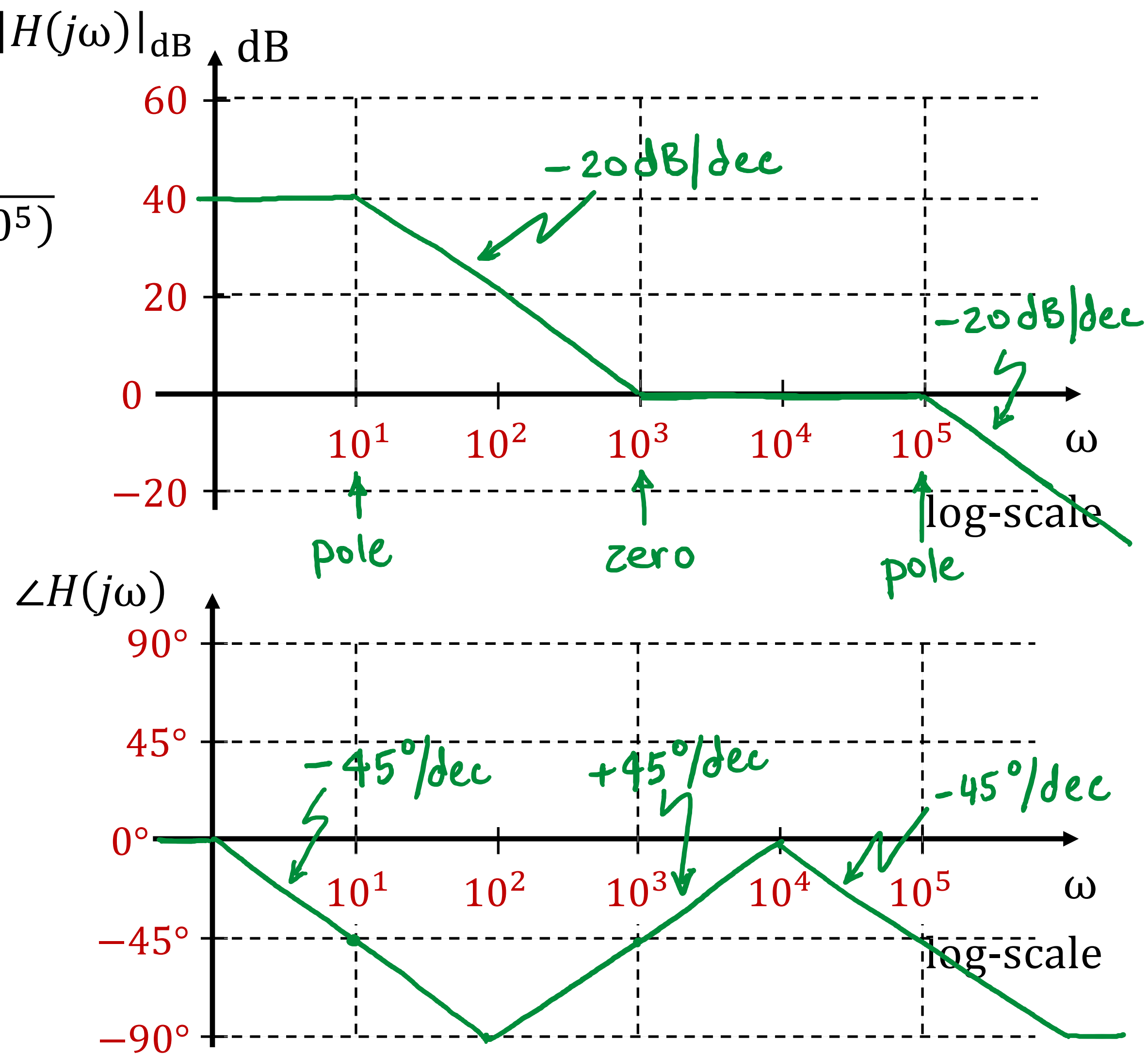
$$H_4(j\omega) = \frac{1}{(1 + j\omega/10^5)}$$



$H(j\omega) = H_1(j\omega)H_2(j\omega)H_3(j\omega)H_4(j\omega)$

Bode Plot Example 1 – Using Poles and Zeros

$$H(j\omega) = \frac{10^2(1 + j\omega/10^3)}{(1 + j\omega/10^1)(1 + j\omega/10^5)}$$



Bode Plot Example 2

$$H(s) = \frac{10^7}{s^2 + 10s + 10^6}$$

$$s = j\omega$$

$$H(j\omega) = \frac{10^7}{-\omega^2 + 10j\omega + 10^6} \leftarrow \omega_o^2 = 10^6$$

$$2\alpha = 10$$

$$= \frac{10}{1 + j\omega/10^5 - (\omega/10^3)^2}$$

$\omega_o = 10^3$ rad/s

$\alpha = 5$ rad/s

Since $\omega_o > \alpha \Rightarrow$ underdamped

$Q = \frac{\omega_o}{2\alpha} = 10^2$

$\Rightarrow Q_{dB} = 40$ dB