ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 32

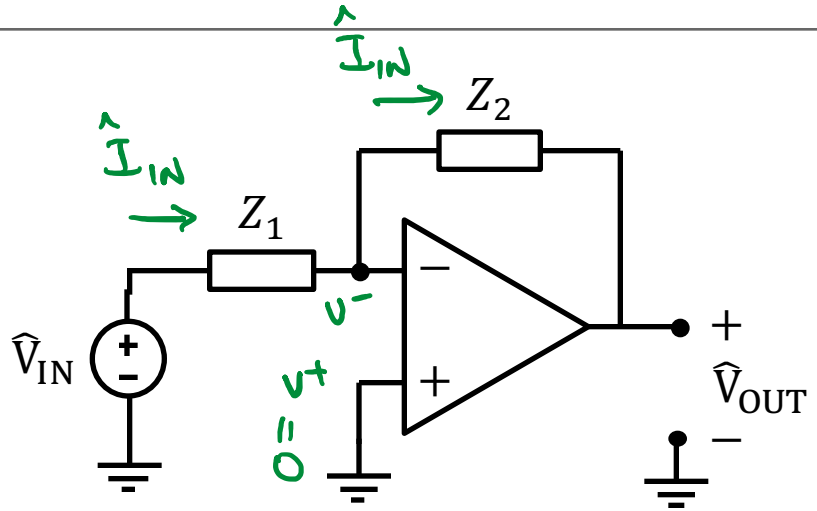
Active Filters

Announcements

- Recommended Reading:
  - Textbook Chapter 15

- Upcoming due dates:
  - Prelab 5 due by 11:59 pm on Monday April 15, 2019
  - Homework 5 due by 11:59 pm on Monday April 22, 2019
  - Lab report 5 due by 11:59 pm on Friday April 19, 2019

Active Filters – Elegant Approach



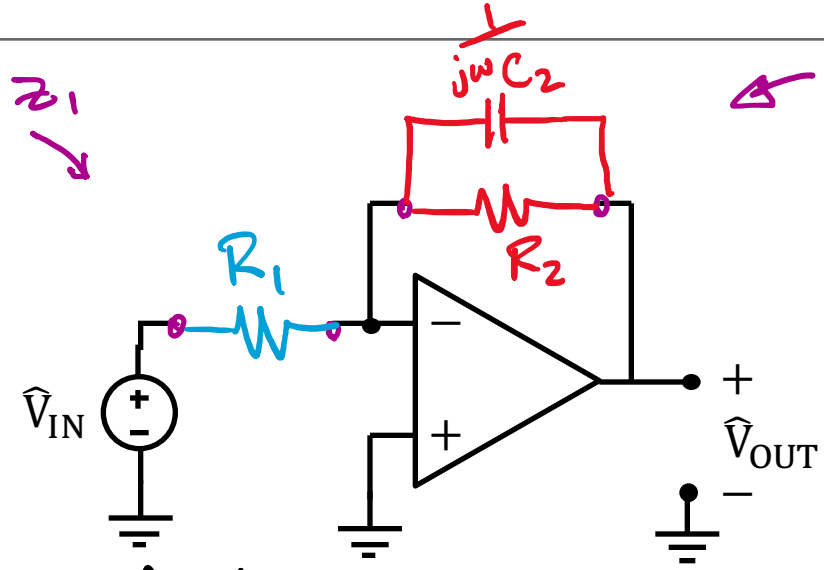
$v^+ \approx v^-$

$\hat{I}_{IN} = \dfrac{\hat{V}_{IN}}{Z_1}$

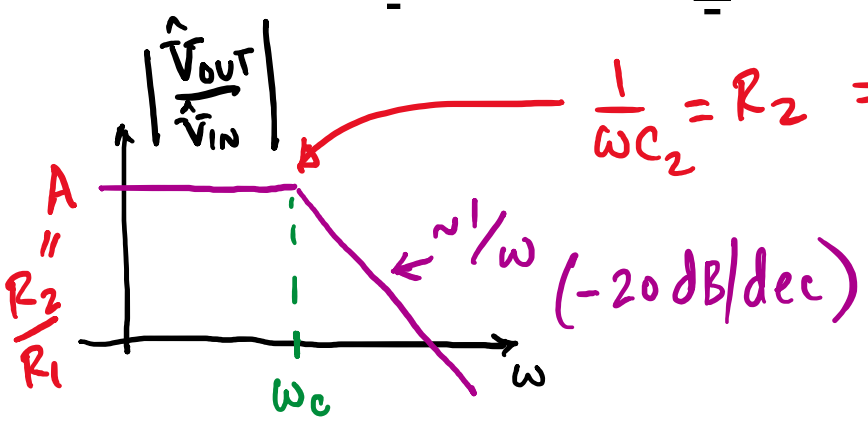
$\hat{V}_{OUT} = 0 - \hat{I}_{IN} Z_2$

$$\hat{V}_{OUT} = -\frac{Z_2}{Z_1}\hat{V}_{IN}$$

Low Pass Active Filter Synthesis

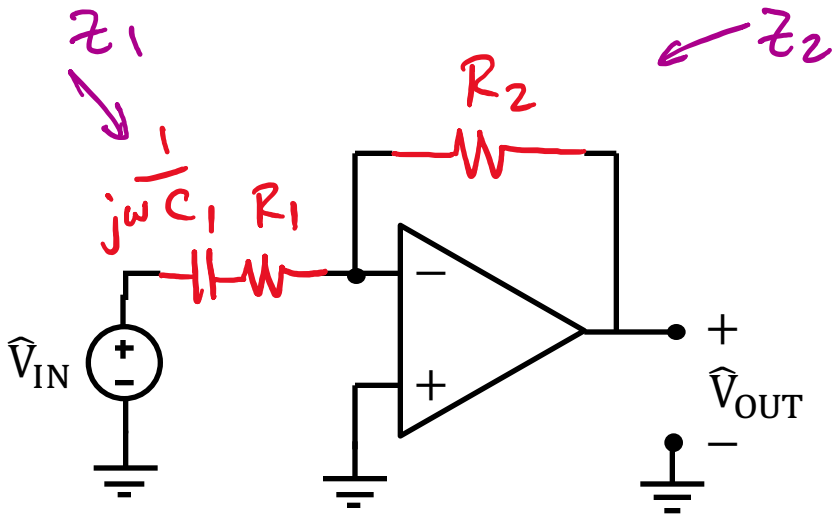


$$\frac{\hat{V}_{OUT}}{\hat{V}_{IN}} = -\frac{Z_2}{Z_1}$$

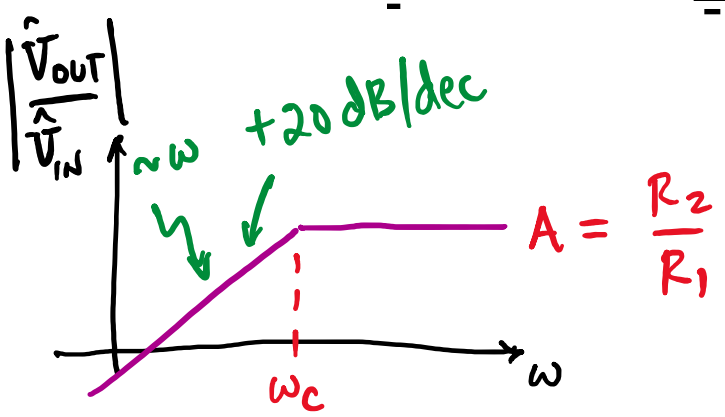


$$\frac{1}{\omega C_2} = R_2 \Rightarrow \omega_c = \frac{1}{R_2 C_2}$$

High Pass Active Filter Synthesis



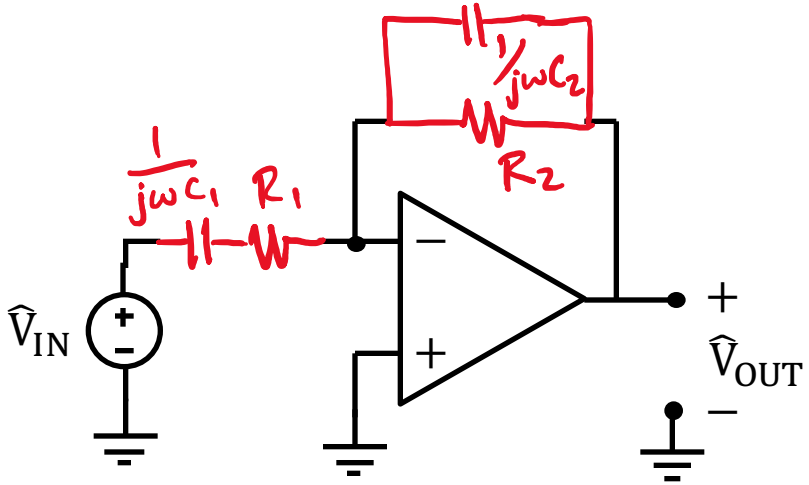
$$\frac{\hat{V}_{OUT}}{\hat{V}_{IN}} = -\frac{Z_2}{Z_1}$$



$$\omega_c: \frac{1}{\omega C_1} = R_1 \Rightarrow \omega_c = \frac{1}{R_1 C_1}$$

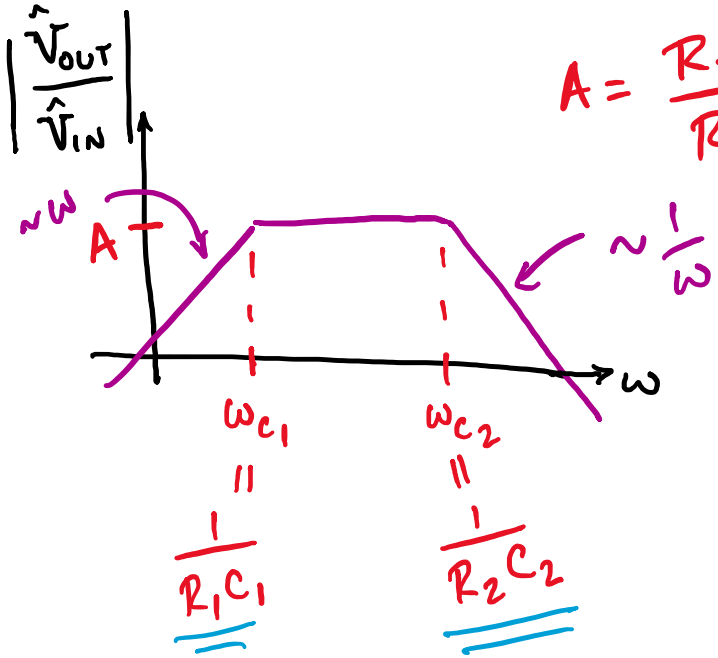
$$\frac{\hat{V}_{OUT}}{\hat{V}_{IN}} = -\frac{R_2}{\frac{1}{j\omega C_1} + R_1} = \frac{-j\omega R_2 C_1}{1 + j\omega R_1 C_1}$$

Band Pass Active Filter Synthesis



$$\frac{\hat{V}_{OUT}}{\hat{V}_{IN}} = -\frac{Z_2}{Z_1}$$

Need: $\omega_{c_1} < \omega_{c_2}$



$$A = \frac{R_2}{R_1}$$

$$\frac{\hat{V}_{OUT}}{\hat{V}_{IN}} = -\frac{R_2/j\omega C_2}{R_2 + 1/j\omega C_2}$$

$$\frac{1}{j\omega C_1} + R_1$$

$$= -\frac{\frac{R_2}{1+j\omega R_2 C_2}}{\frac{1+j\omega R_1 C_1}{j\omega C_1}}$$

Band Pass Active Filter – Quality Factor

$$\frac{\hat{V}_{OUT}}{\hat{V}_{IN}} = \frac{-j\omega R_2 C_1}{(1+j\omega R_2 C_2)(1+j\omega R_1 C_1)} = \frac{-j\omega R_2 C_1}{(1+j\omega/\omega_{c_2})(1+j\omega/\omega_{c_1})}$$

$$\left[\frac{(j\omega)^2}{\omega_{c_1}\omega_{c_2}} + j\omega\left(\frac{1}{\omega_{c_1}}+\frac{1}{\omega_{c_2}}\right) + 1\right]\hat{V}_{OUT} = -j\omega R_2 C_1 \hat{V}_{IN}$$

$$\frac{1}{\omega_{c_1}\omega_{c_2}}\frac{d^2 v_{out}}{dt^2} + \left(\frac{1}{\omega_{c_1}}+\frac{1}{\omega_{c_2}}\right)\frac{dv_{out}}{dt} + v_{out} = -R_2 C_1 \frac{dv_{IN}}{dt}$$

0

homogeneous diff. eqn

$$v_{out} \sim e^{st} \Rightarrow \frac{s^2}{\omega_{c_1}\omega_{c_2}} + \left(\frac{1}{\omega_{c_1}}+\frac{1}{\omega_{c_2}}\right)s + 1 = 0$$

$$s^2 + \overbrace{(\omega_{c_1}+\omega_{c_2})}^{2\alpha}s + \overbrace{\omega_{c_1}\omega_{c_2}}^{\omega_0^2} = 0 \quad \leftarrow \text{Characteristic Eqn}$$

$$Q \equiv \frac{\omega_0}{2\alpha} = \frac{\sqrt{\omega_{c_1}\omega_{c_2}}}{\omega_{c_1}+\omega_{c_2}}$$

Band Pass Active Filter – Quality Factor (Cont.)

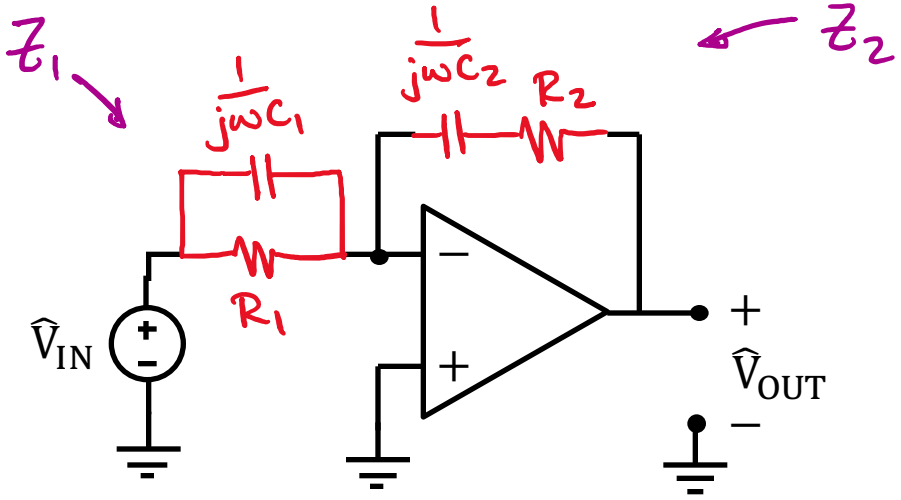
$$Q = \frac{\sqrt{\omega_{c_1}\omega_{c_2}}}{\omega_{c_1}+\omega_{c_2}} \Rightarrow \frac{dQ}{d\omega_{c_1}} = 0$$

maximum at

$$\Rightarrow \omega_{c_1} = \omega_{c_2} \Rightarrow Q_{max} = \frac{\sqrt{\omega_{c_2}\omega_{c_2}}}{\omega_{c_2}+\omega_{c_2}} = \frac{\omega_{c_2}}{2\omega_{c_2}} = \frac{1}{2}$$

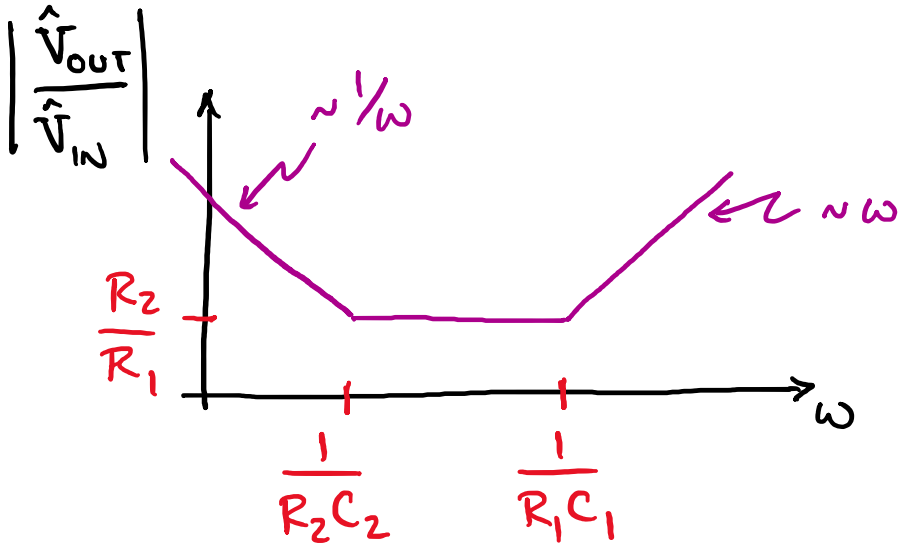
$$\boxed{Q_{max} = \frac{1}{2}}$$

Band Stop Active Filter Synthesis

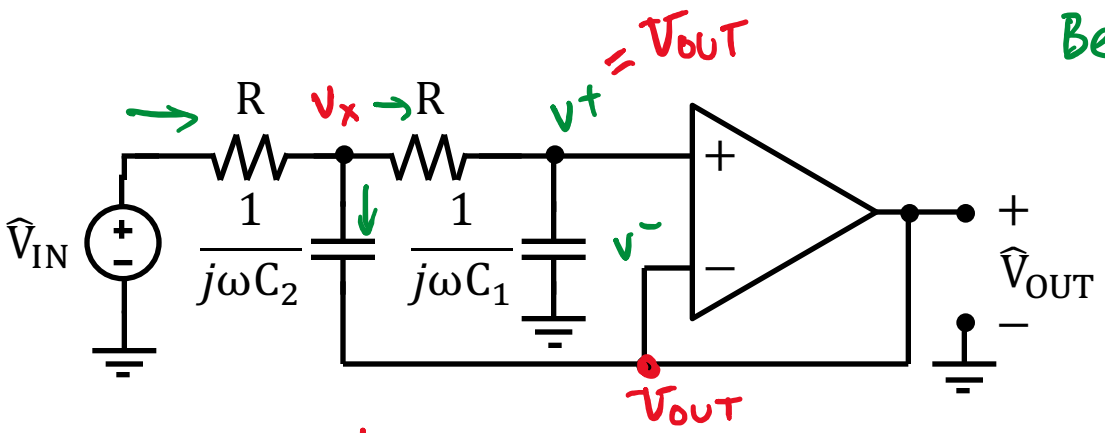


$$\frac{\hat{V}_{OUT}}{\hat{V}_{IN}} = -\frac{Z_2}{Z_1}$$

swap Z_1 & Z_2 of band pass filter to create band stop filter



Sallen-Key Low Pass Active Filter



Because $\hat{V}_{OUT}$ is finite (system stable)

$v^+ \approx v^- = \hat{V}_{OUT}$

$$v^+ = \frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + R} v_x = \frac{1}{1 + j\omega RC_1} v_x \Rightarrow v_x = (1 + j\omega RC_1)\hat{V}_{OUT}$$

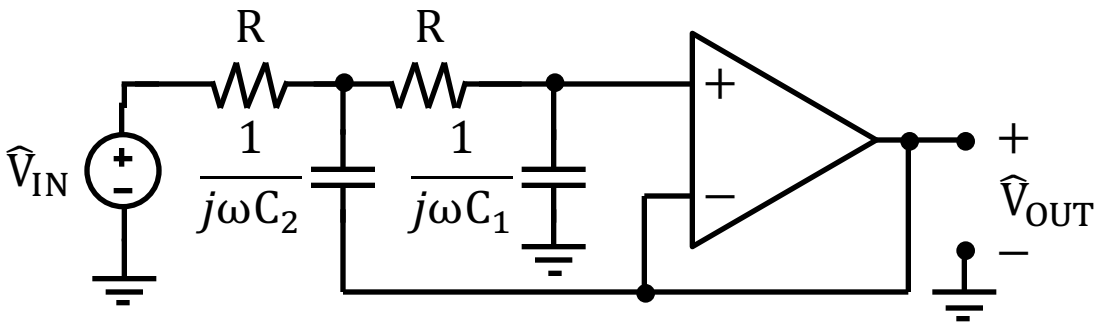
$v^+ = \hat{V}_{OUT}$

$$\frac{\hat{V}_{IN} - v_x}{R} = \frac{v_x - \hat{V}_{OUT}}{\frac{1}{j\omega C_2}} + \frac{v_x - \hat{V}_{OUT}}{R}$$

Do algebra

$$\hat{V}_{IN} = \left(j\omega 2RC_1 + 2 + (j\omega)^2 R^2 C_1 C_2 - 1\right)\hat{V}_{OUT}$$

Sallen-Key Low Pass Active Filter (Cont.)



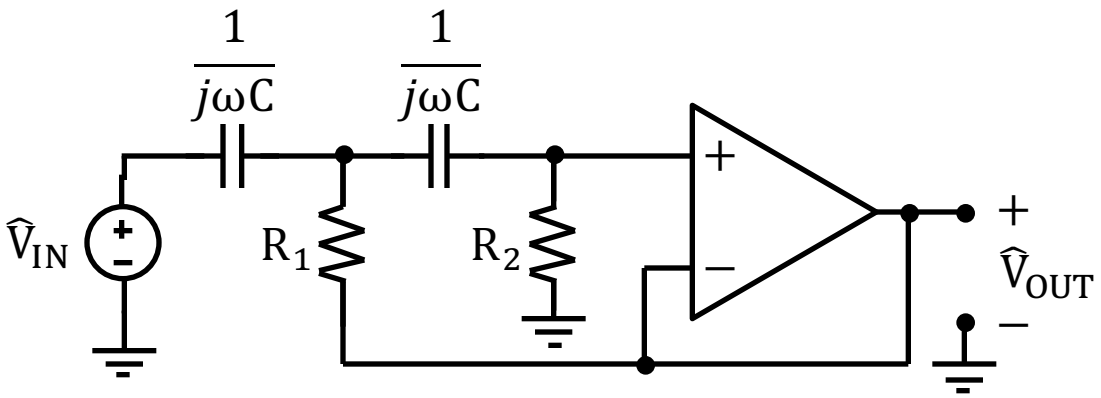
$$\frac{\hat{V}_{OUT}}{\hat{V}_{IN}} = \frac{1}{(j\omega)^2(R^2C_1C_2) + (j\omega)(2RC_1) + 1}$$

Char Eqn: $s^2R^2C_1C_2 + s\,2RC_1 + 1 = 0$

$\Rightarrow$ $s^2 + \overbrace{\frac{2s}{RC_2}}^{2\alpha} + \overbrace{\frac{1}{R^2C_1C_2}}^{\omega_0^2} = 0$

$$Q = \frac{\omega_0}{2\alpha} = \frac{1}{2}\sqrt{\frac{C_2}{C_1}}$$ ← Can get Q values greater than 1/2 by making C_2 greater than C_1

Sallen-Key High Pass Active Filter



Swap R's and C's to go from Low Pass Filter to High Pass Filter