ECE/ENGRD 2100

Introduction to Circuits for ECE

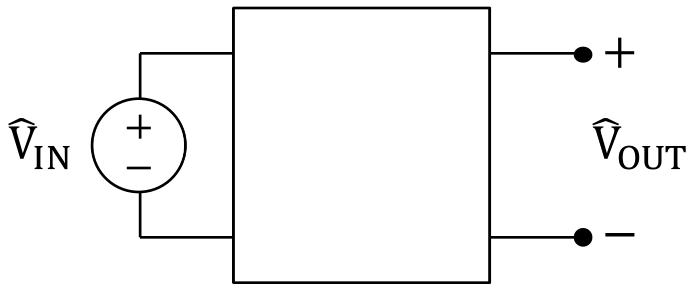
Lecture 31

Passive and Active Filters

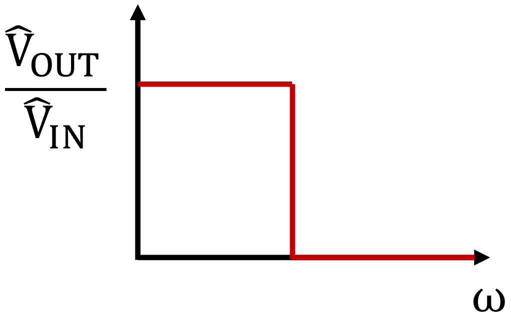
Announcements

- Recommended Reading:
  - Textbook Chapter 14 and Chapter 15
- Upcoming due dates:
  - Prelab 5 due by 11:59 pm on Monday April 15, 2019
  - Homework 5 due by 11:59 pm on Friday April 19, 2019
  - Lab report 5 due by 11:59 pm on Friday April 19, 2019

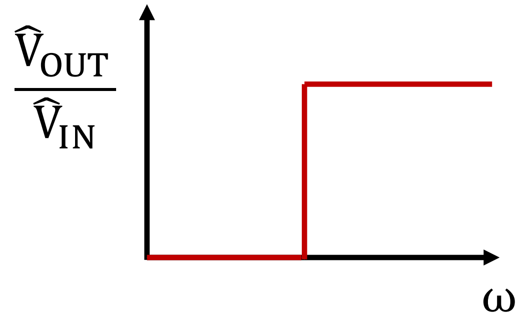
Filters



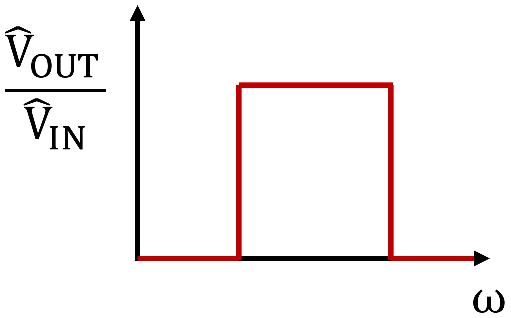
Low Pass Filter



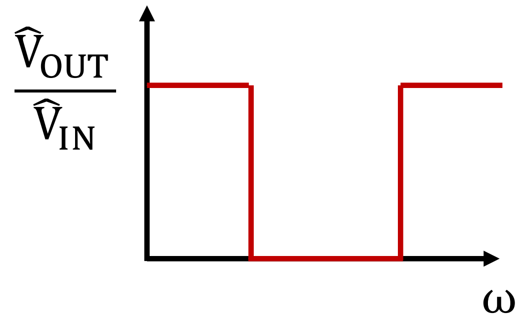
High Pass Filter



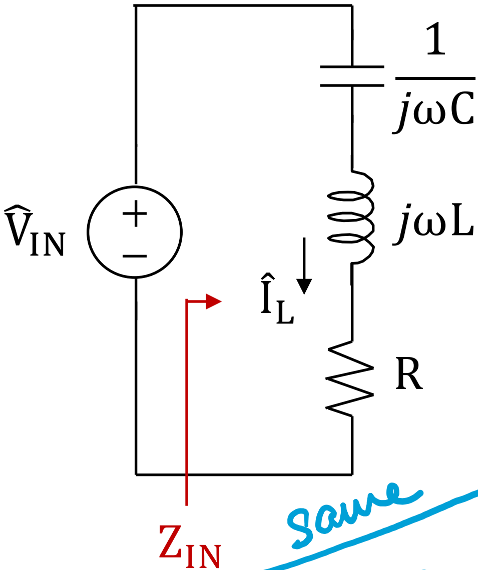
Band Pass Filter



Band Stop Filter



Connection Between Frequency & Time Domain



$$\hat{I}_L = \frac{\hat{V}_{IN}}{Z_{IN}} \Rightarrow \frac{\hat{V}_{IN}}{\hat{I}_L} = Z_{IN} = \frac{1}{j\omega C} + R + j\omega L$$

$$\Rightarrow \frac{\hat{V}_{IN}}{\hat{I}_L} = \frac{1 + j\omega RC + (j\omega)^2 LC}{j\omega C} \leftarrow$$

$$(j\omega)^2 LC\,\hat{I}_L + j\omega RC\,\hat{I}_L + \hat{I}_L = j\omega C\,\hat{V}_{IN}$$

same

Diff. Eq.

$$LC\frac{d^2 i_L}{dt^2} + \overbrace{RC}^{2\alpha}\frac{di_L}{dt} + i_L = C\frac{dv_{IN}}{dt}$$

$v_{IN} \rightarrow \hat{V}_{IN} e^{j\omega t}$

$i_L \rightarrow \hat{I}_L e^{j\omega t}$

Plug in Diff Eqn

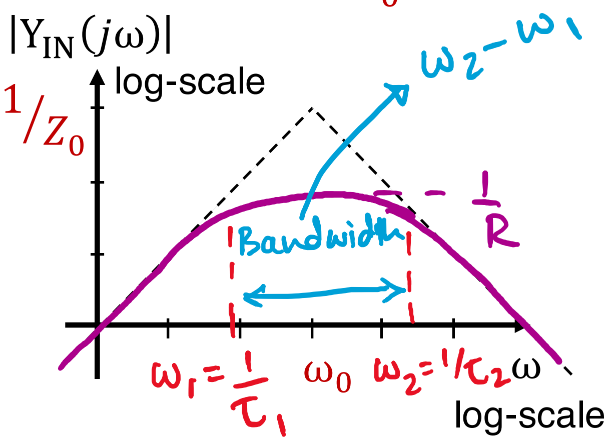
$$(j\omega)^2 LC\,\hat{I}_L \cancel{e^{j\omega t}} + (j\omega) RC\,\hat{I}_L \cancel{e^{j\omega t}} + \hat{I}_L \cancel{e^{j\omega t}} = (j\omega) C\,\hat{V}_{IN} \cancel{e^{j\omega t}}$$

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad Q \equiv \frac{\omega_0}{2\alpha} \longrightarrow Q = \frac{Z_0}{R}$$

Frequency Response and Natural Response

Overdamped

$\mathrm{R} > 2\mathrm{Z}_0$



$$Q \equiv \frac{\omega_0}{2\alpha} = \frac{Z_0}{R} < \frac{1}{2}$$

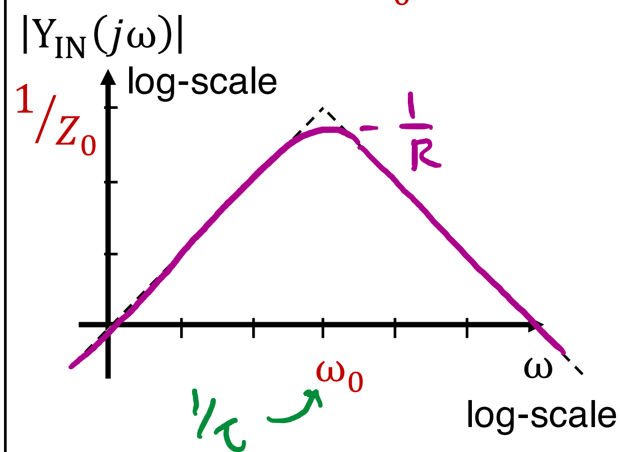
$$\Rightarrow \quad \alpha > \omega_0$$

$$A_1 e^{-\left(\alpha - \sqrt{\alpha^2 - \omega_0^2}\right)t} + A_2 e^{-\left(\alpha + \sqrt{\alpha^2 - \omega_0^2}\right)t}$$

$$= A_1 e^{-t/\tau_1} + A_2 e^{-t/\tau_2}$$

Critically damped

$\mathrm{R} = 2\mathrm{Z}_0$



$$Q \equiv \frac{\omega_0}{2\alpha} = \frac{Z_0}{R} = \frac{1}{2}$$

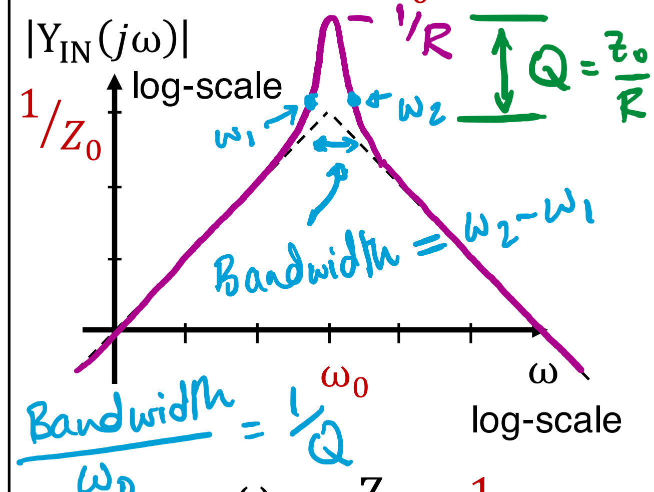
$$\Rightarrow \quad \alpha = \omega_0$$

$$A_1 e^{-\alpha t} + A_2 t e^{-\alpha t}$$

$$= A_1 e^{-t/\tau} + A_2 t\, e^{-t/\tau}$$

Underdamped

$\mathrm{R} < 2\mathrm{Z}_0$



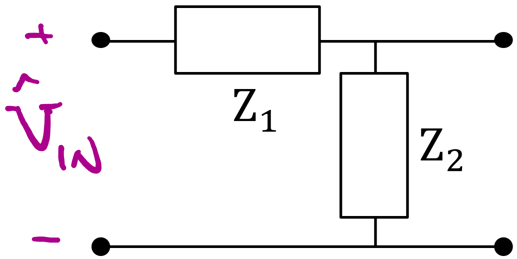
$$Q \equiv \frac{\omega_0}{2\alpha} = \frac{Z_0}{R} > \frac{1}{2}$$

$$\Rightarrow \quad \alpha < \omega_0$$

$$A_1 e^{-\alpha t}\cos(\omega_d t) + A_2 e^{-\alpha t}\sin(\omega_d t)$$

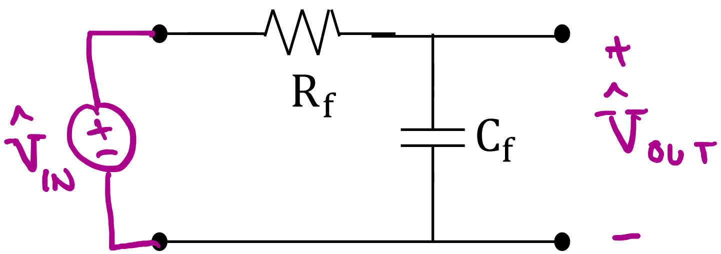
$$\omega_d \equiv \sqrt{\omega_0^2 - \alpha^2}$$

Passive Filter Synthesis

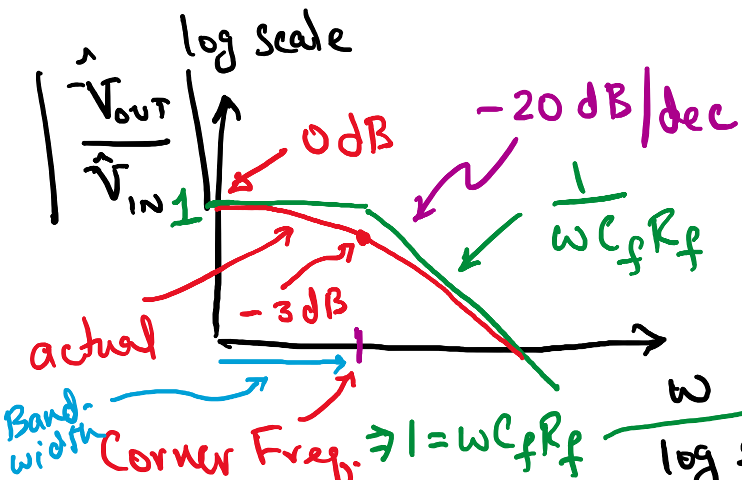


$$\hat{V}_{OUT} = \frac{Z_2}{Z_1 + Z_2}$$

Low Pass Filter



$$\frac{\hat{V}_{OUT}}{\hat{V}_{IN}} = \frac{\frac{1}{j\omega C_f}}{\frac{1}{j\omega C_f} + R_f} = \frac{1}{1 + j\omega C_f R_f}$$



$\omega = \frac{1}{R_f C_f} = \frac{1}{\tau}$

$$\text{Bel} = \log_{10}\left(\frac{P_{OUT}}{P_{IN}}\right)$$

$$\text{Decibel} = 10\log_{10}\left(\frac{P_{OUT}}{P_{IN}}\right)$$

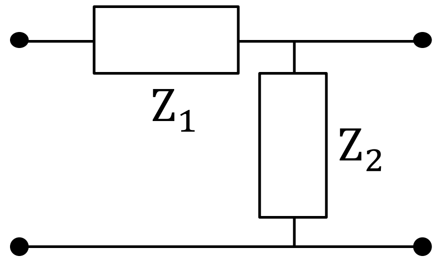
$\parallel$

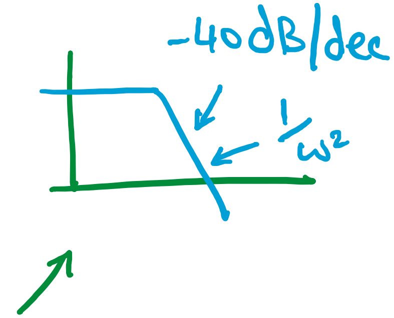
dB

$$= 10\log_{10}\left(\frac{V_{OUT}^2/\cancel{R}}{V_{IN}^2/\cancel{R}}\right)$$

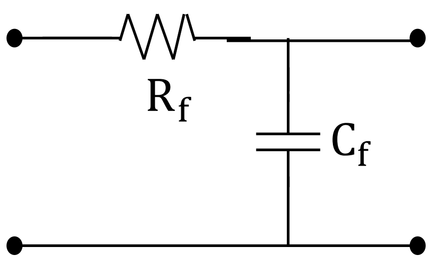
$$= 20\log_{10}\left(\frac{V_{OUT}}{V_{IN}}\right)$$

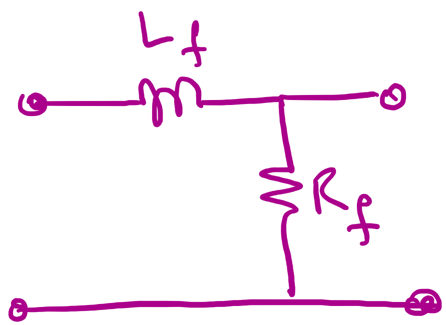
Passive Filter Synthesis (Cont.)

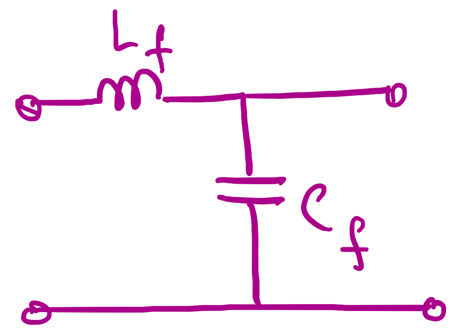




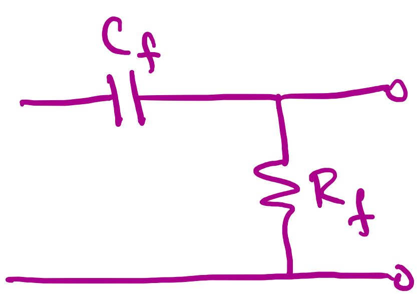
Low Pass Filters

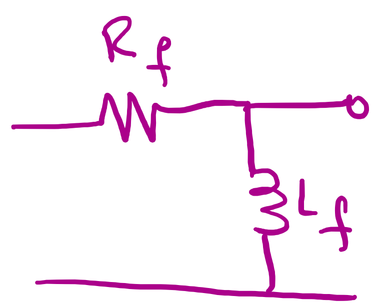


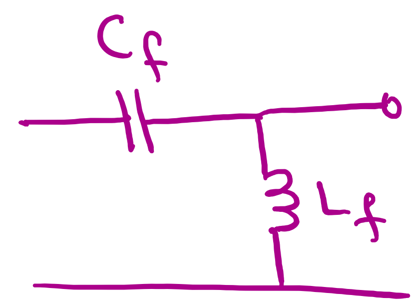




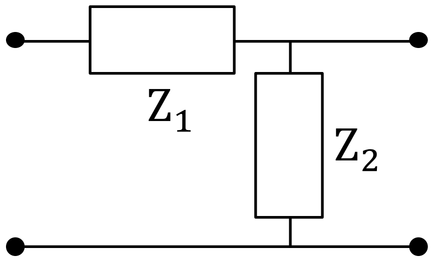
High Pass Filters



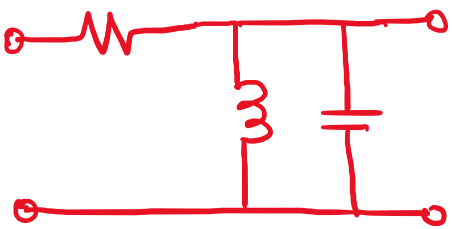


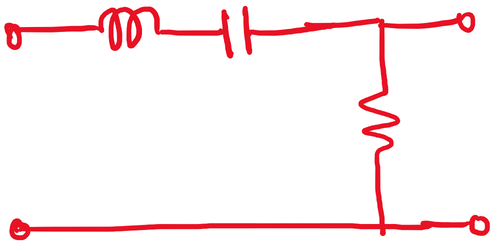


Passive Filter Synthesis (Cont.)

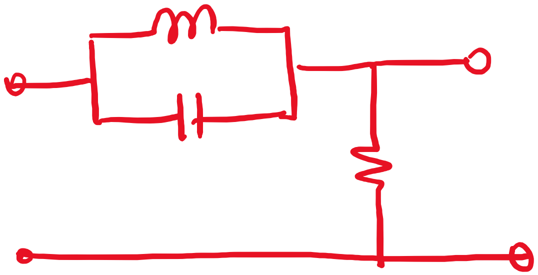


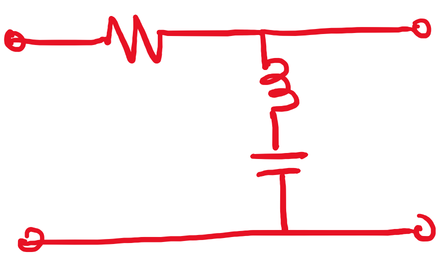
Band Pass Filters



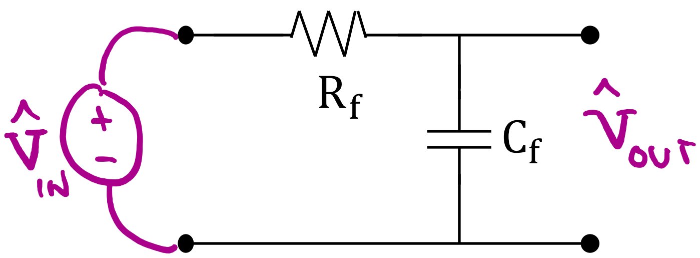


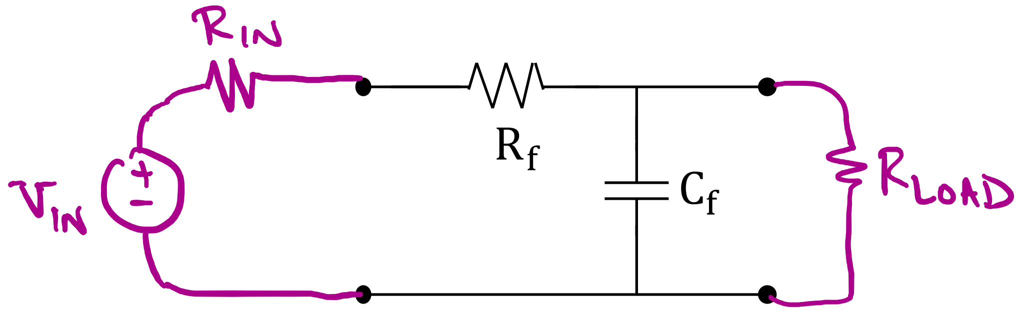
Band Stop Filters





Limitations of Passive Filters



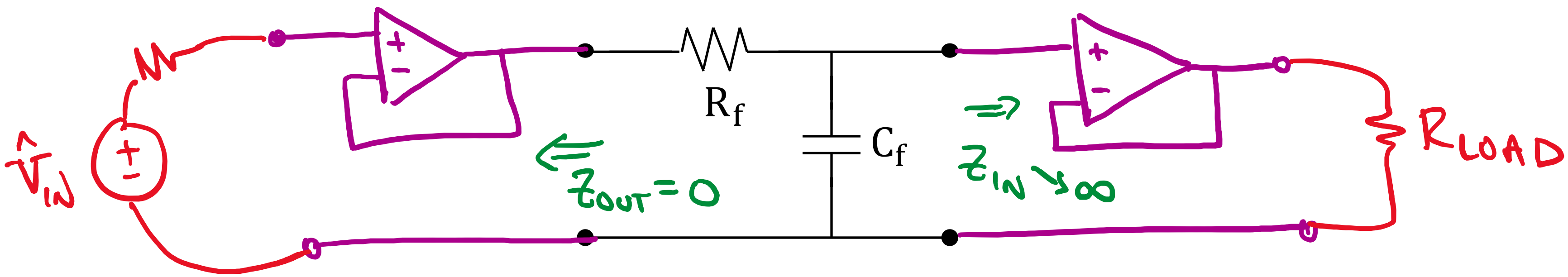


Corner Freq $= \frac{1}{R_f C_f}$

Corner Freq $= \frac{1}{R_{TH} C_f}$

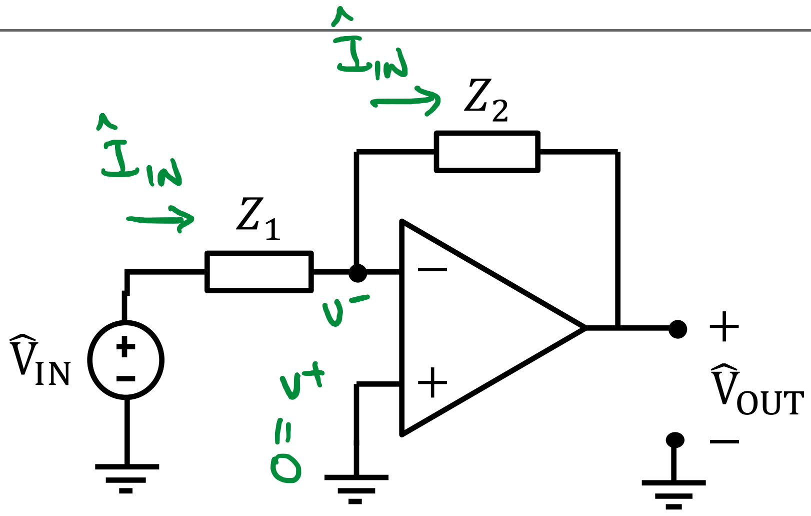
$R_{TH} = (R_{IN} + R_f) \| R_{LOAD}$

Active Op Amp Filters – Brute Force Approach



Corner Freq = $\frac{1}{R_f C_f}$

Active Filters – Elegant Approach



$$v^+ \approx v^-$$

$$\hat{I}_{IN} = \frac{\hat{V}_{IN}}{Z_1}$$

$$\hat{V}_{OUT} = 0 - \hat{I}_{IN} Z_2$$

$$\boxed{\hat{V}_{OUT} = -\frac{Z_2}{Z_1}\hat{V}_{IN}}$$