ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 30

Second Order Systems in Frequency Domain

Announcements

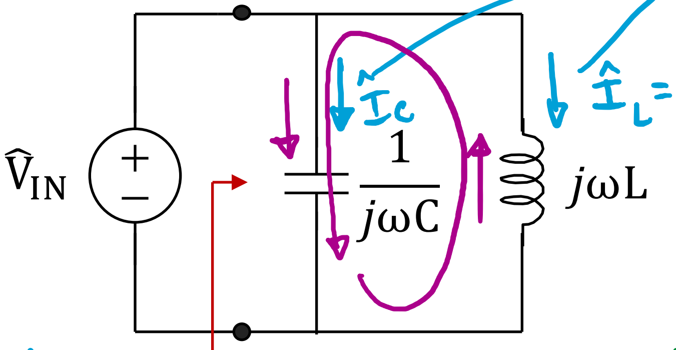
- Recommended Reading:
  - Textbook Chapter 14

- Upcoming due dates:
  - Prelab 5 due by 11:59 pm on Monday April 15, 2019
  - Homework 5 due by 11:59 pm on Wednesday April 17, 2019
  - Lab report 5 due by 11:59 pm on Friday April 19, 2019

Impedance of Parallel L-C Circuit

zero current

Currents are equal in magnitude but opposite in phase



$$\hat{I}_L = \frac{\hat{V}_{IN}}{Z_o e^{j90°}} = \frac{\hat{V}_{IN}}{Z_o} e^{-j90°}$$

$$Z_{IN} = \frac{1}{j\omega C} \Big\| \, j\omega L$$

$$\Rightarrow \quad Z_{IN} = \frac{j\omega L}{1-\omega^2 LC}$$

$$\hat{I}_C = \frac{\hat{V}_{IN}}{Z_o e^{-j90°}} = \frac{\hat{V}_{IN}}{Z_o} e^{j90°}$$

$$1 = \omega^2 LC \Rightarrow \omega = \frac{1}{\sqrt{LC}} = \omega_0$$

$$Z_{IN}(j\omega) = \begin{cases} j\omega L & \omega \text{ low} \quad \omega \ll 1/\sqrt{LC} = \omega_0 \\ \dfrac{1}{j\omega C} & \omega \text{ high} \quad \omega \gg 1/\sqrt{LC} = \omega_0 \end{cases}$$

$\omega L e^{j90°}$

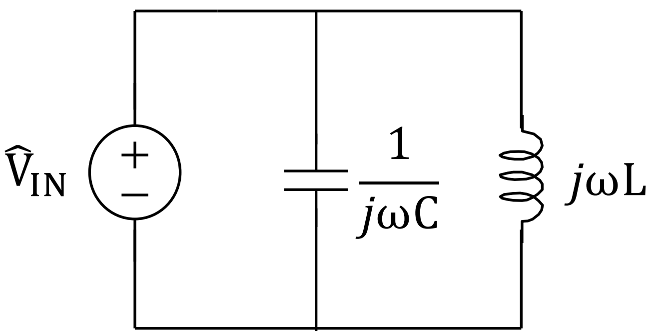
$\frac{1}{\omega C} e^{-j90°}$

$$@\ \omega = \omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow Z_{IN}(j\omega) = \frac{j\omega L}{0} \rightarrow \infty$$

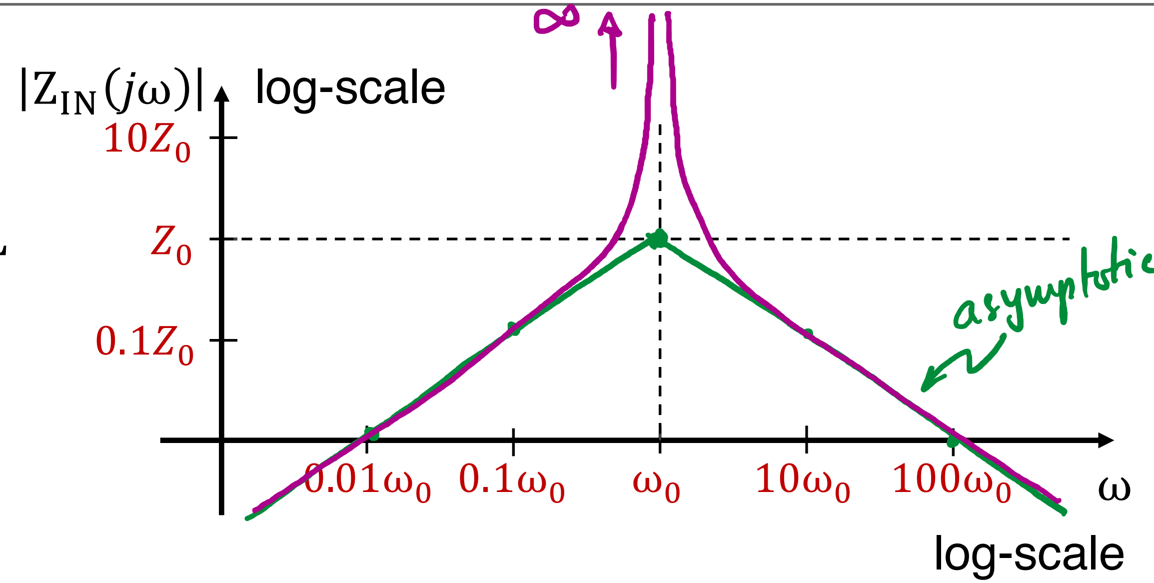
$$@\ \omega = \omega_0 \Rightarrow \omega L = \omega_0 L = \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}} = Z_0$$

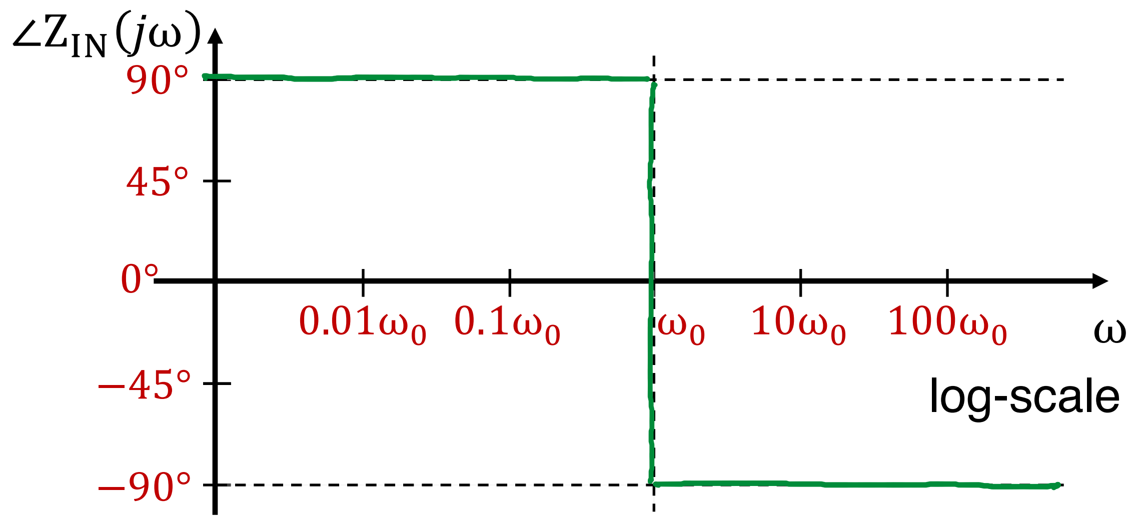
$$@\ \omega = \omega_0 \Rightarrow \frac{1}{\omega C} = \frac{1}{\omega_0 C} = \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}} = Z_0$$

Impedance of Parallel L-C Circuit (Cont.)



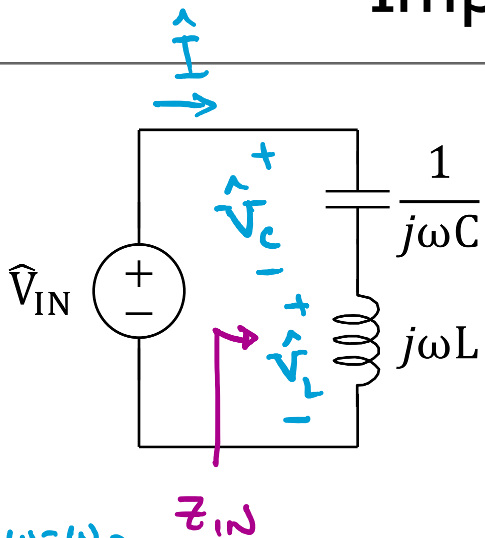
$$\omega_0 = \frac{1}{\sqrt{LC}}$$





$$Z_{IN}(j\omega) = \begin{cases} \omega L e^{j90^\circ} & \omega \ll \omega_0 \\ \infty & \omega = \omega_0 \\ \dfrac{1}{\omega C} e^{-j90^\circ} & \omega \gg \omega_0 \end{cases}$$

Impedance of Series L-C Circuit



$$Z_{IN} = \frac{1}{j\omega C} + j\omega L = \frac{1-\omega^2 LC}{j\omega C}$$

$$Z_{IN} = \begin{cases} \frac{1}{j\omega C} = \frac{1}{\omega C} e^{-j90^\circ} & \omega \ll \omega_0 = 1/\sqrt{LC} \\ 0 & \omega = \omega_0 \\ j\omega L = \omega L e^{+j90^\circ} & \omega \gg \omega_0 = 1/\sqrt{LC} \end{cases}$$

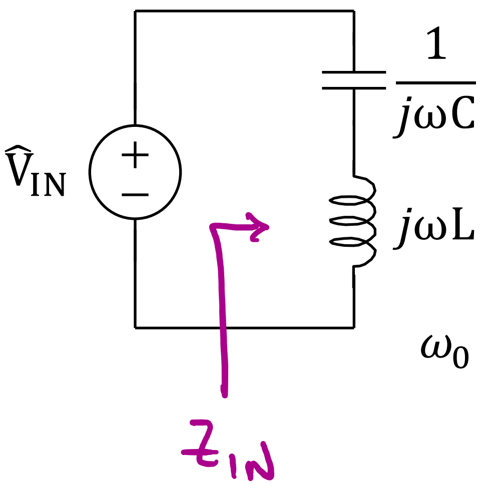
@ $\omega = \omega_0$

$$\hat{V}_c = \frac{1}{j\omega_0 C}\hat{I} = Z_0 \hat{I} e^{-j90^\circ}$$

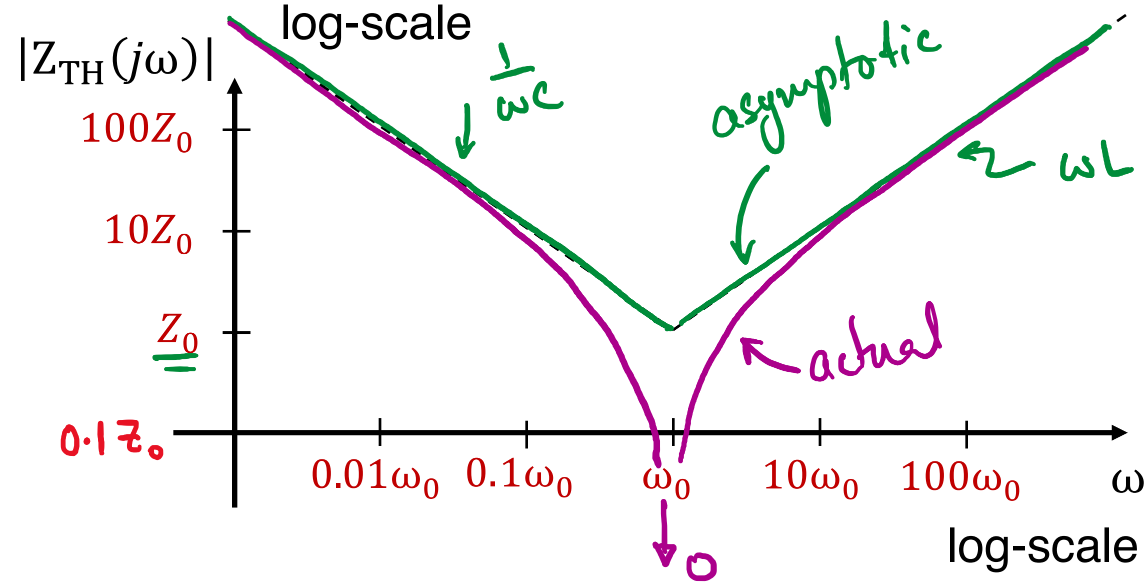
$$\hat{V}_L = j\omega_0 L \hat{I} = Z_0 \hat{I} e^{+j90^\circ}$$

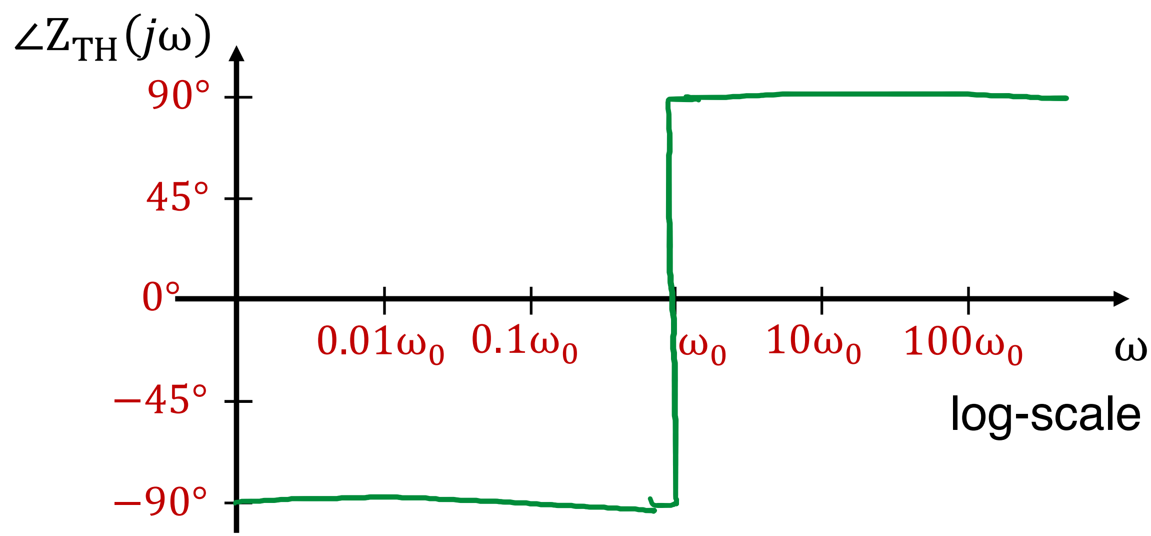
Same magnitude but opposite phase } so cancel

Impedance of Series L-C Circuit (Cont.)

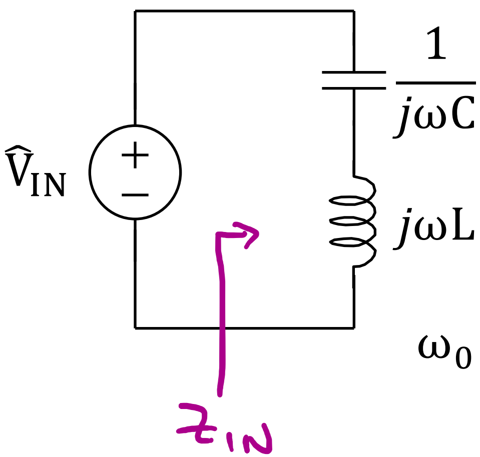


$$\omega_0 = \frac{1}{\sqrt{LC}}$$



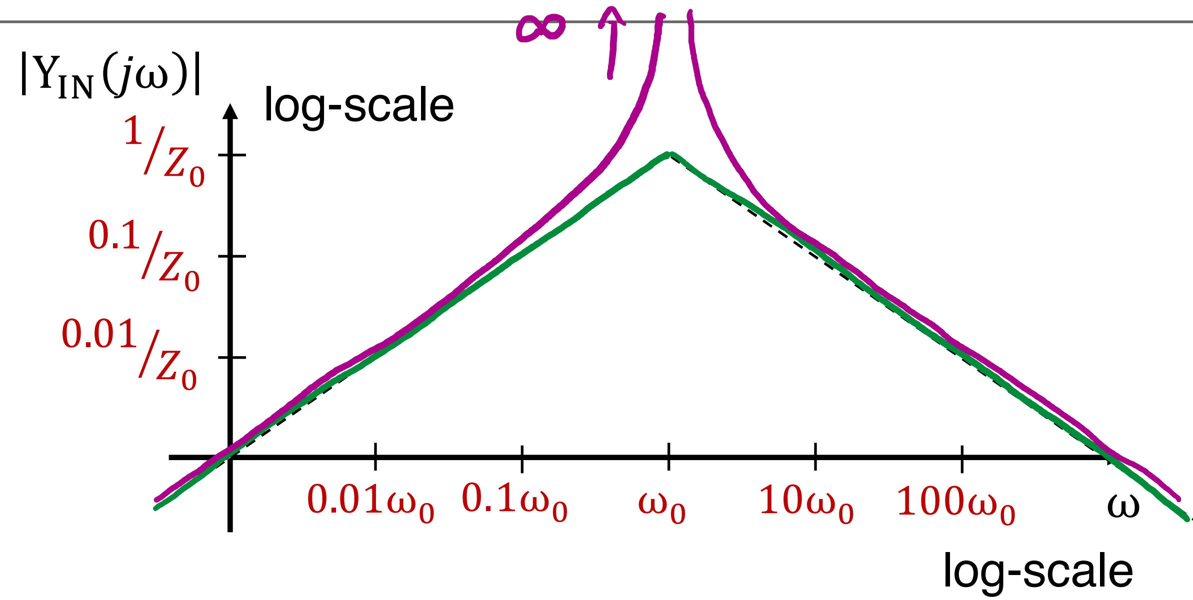


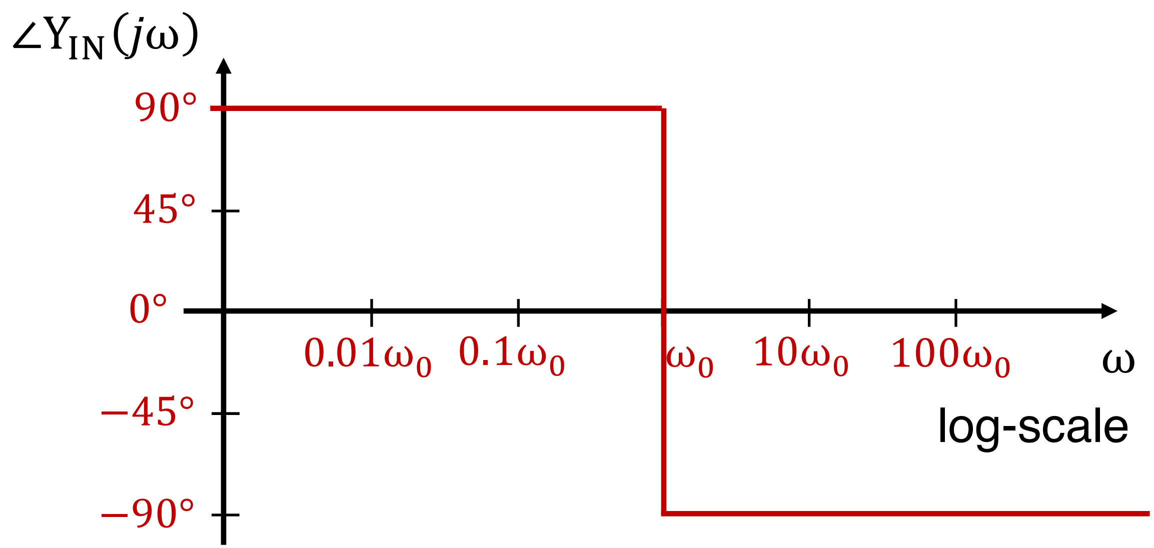
Admittance of Series L-C Circuit



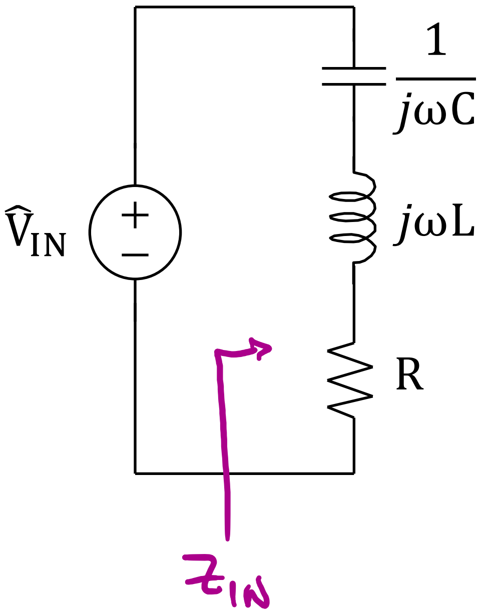
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Y_{IN} = \frac{1}{Z_{IN}}$$





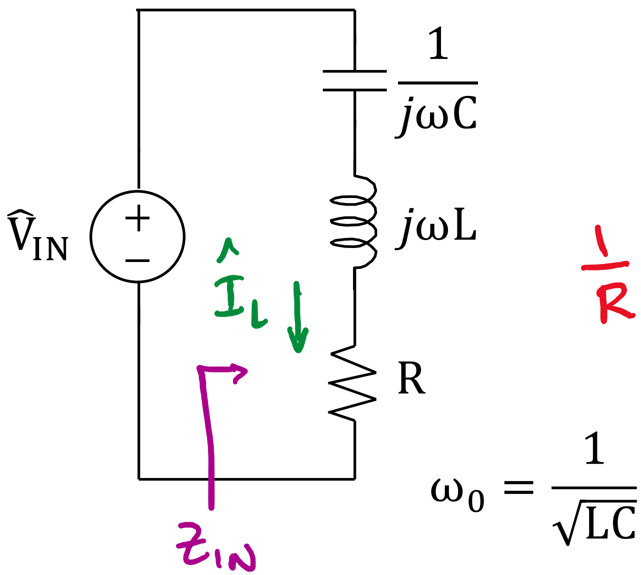
Series L-C Circuit with Damping

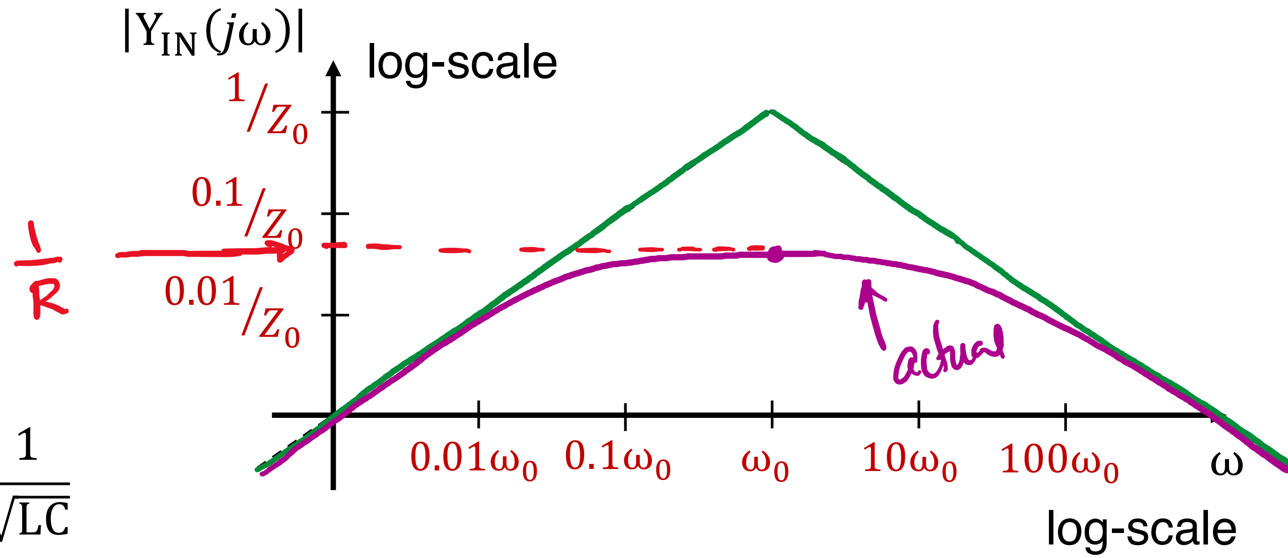


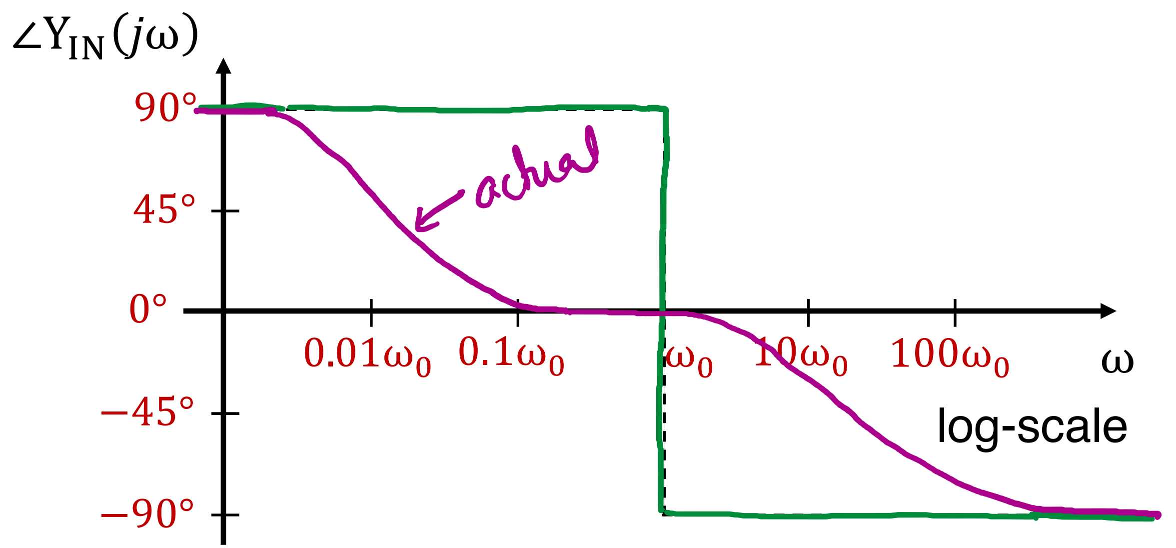
$$Z_{IN} = \frac{1}{j\omega C} + R + j\omega L = \frac{1 + j\omega RC - \omega^2 LC}{j\omega C}$$

$$Z_{IN} = \begin{cases} \frac{1}{j\omega C} = \frac{1}{\omega C} e^{-j90^\circ} & \omega \ll \omega_0 = 1/\sqrt{LC} \\ \frac{\cancel{1} + j\cancel{\omega_0} R\cancel{C} - \cancel{1}}{\cancel{j\omega_0 C}} = R & \omega = \omega_0 \\ \frac{-\omega^{\cancel{2}} L\cancel{C}}{j\cancel{\omega}\cancel{C}} = j\omega L = \omega L e^{+j90^\circ} & \omega \gg \omega_0 = 1/\sqrt{LC} \end{cases}$$

Case 1 – Overdamped





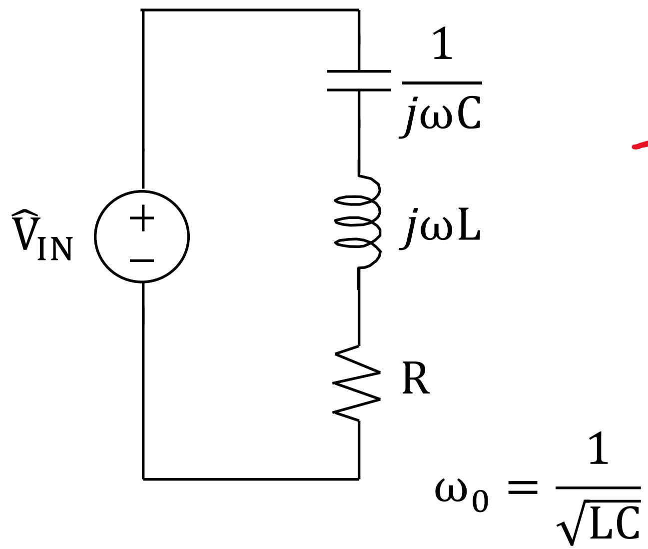


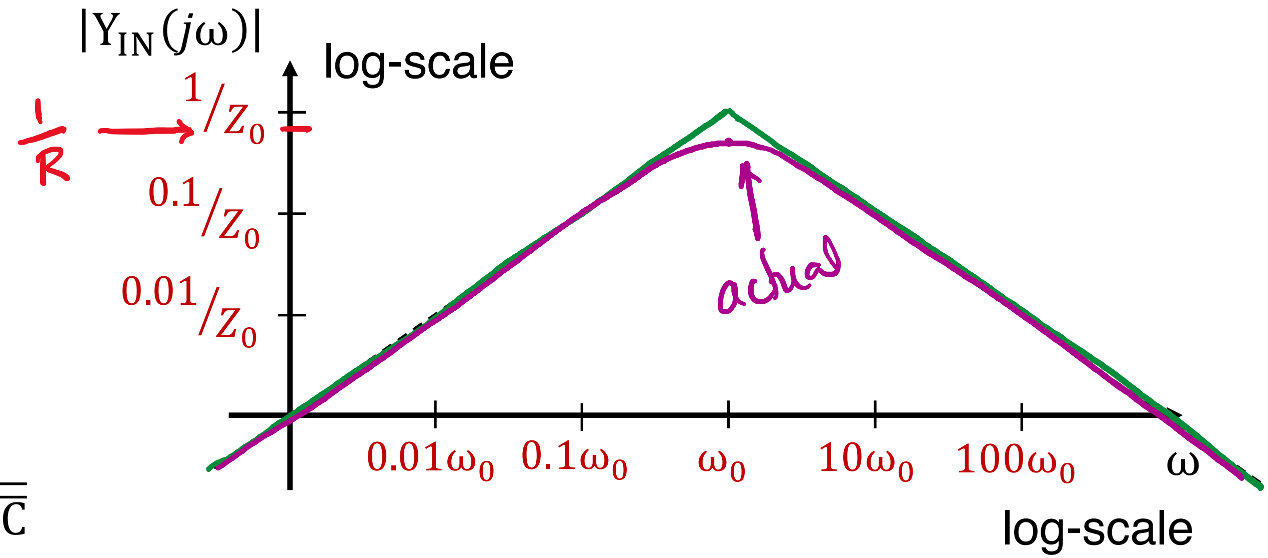
$$Y_{IN} \equiv \frac{1}{Z_{IN}}$$

$$R_0 > 2Z_0$$

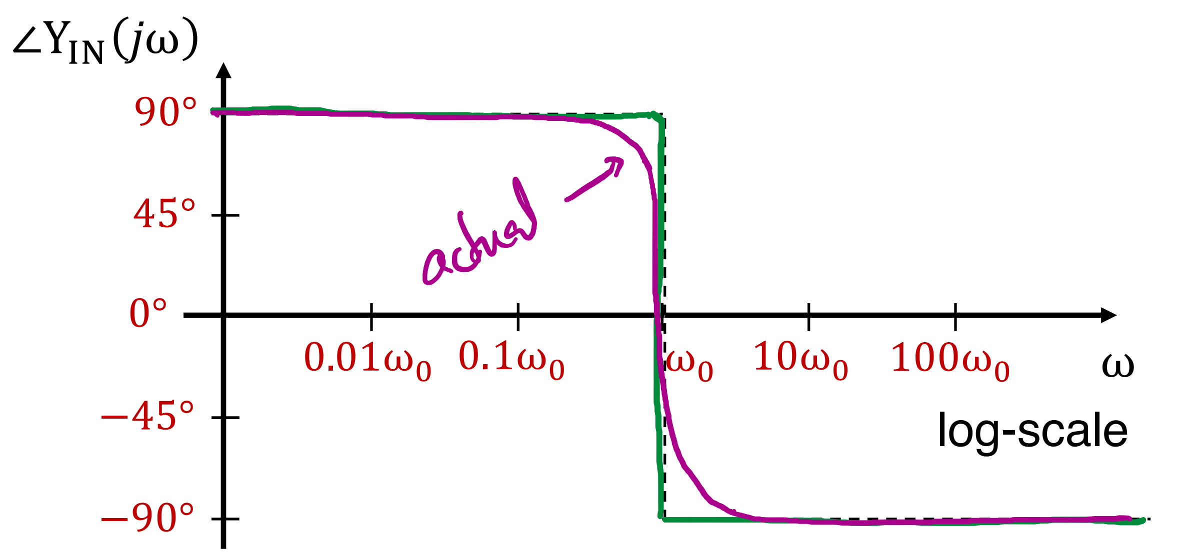
$$\hat{I}_L = \frac{\hat{V}_{IN}}{Z_{IN}} = Y_{IN}\hat{V}_{IN}$$

Case 2 – Critically Damped

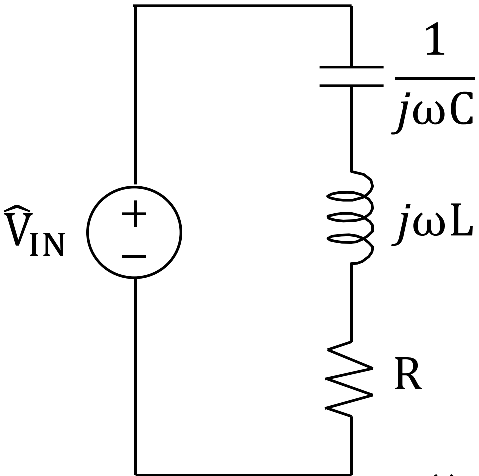




$R = 2Z_0$

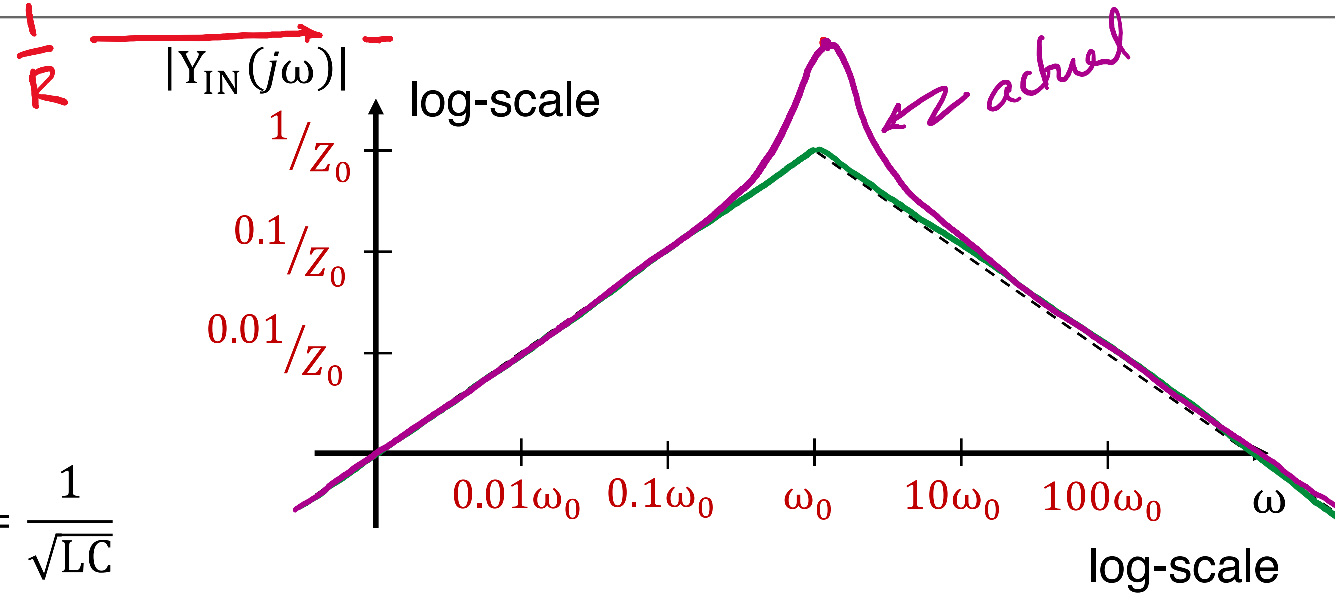


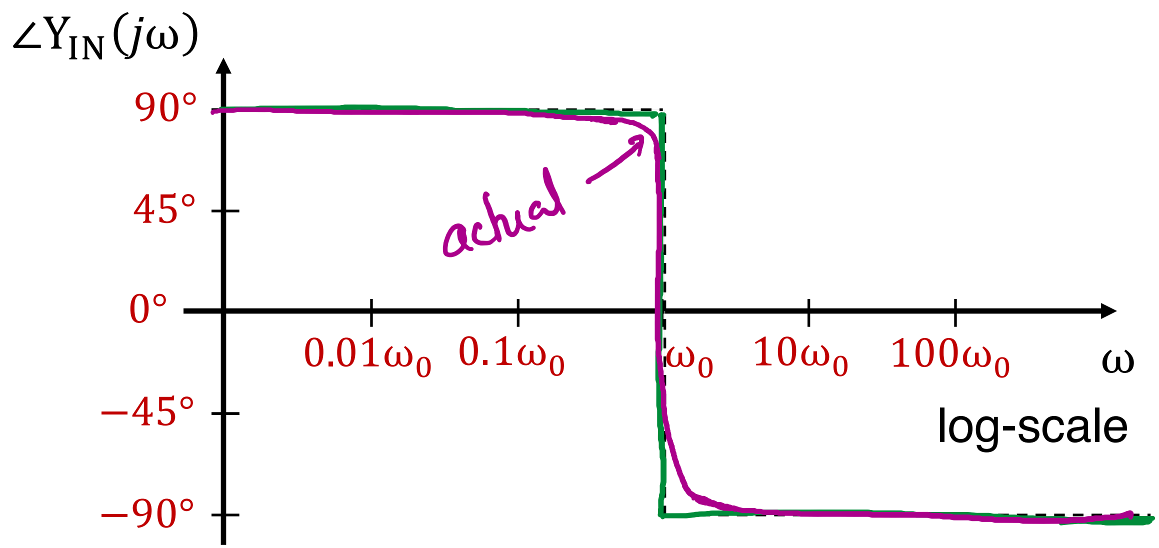
Case 3 – Underdamped



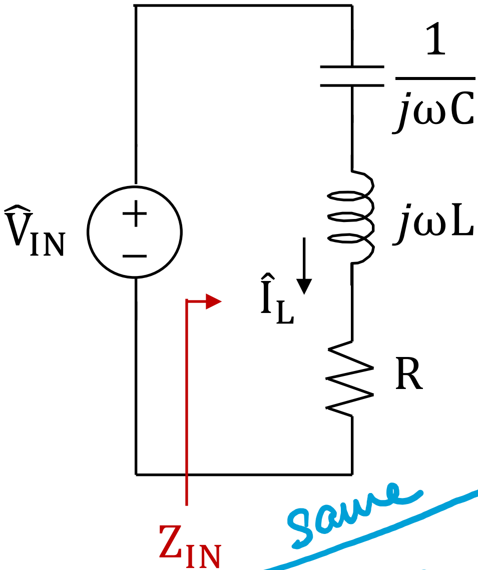
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$R < 2Z_0$





Connection Between Frequency & Time Domain



$$\hat{I}_L = \frac{\hat{V}_{IN}}{Z_{IN}} \Rightarrow \frac{\hat{V}_{IN}}{\hat{I}_L} = Z_{IN} = \frac{1}{j\omega C} + R + j\omega L$$

$$\Rightarrow \frac{\hat{V}_{IN}}{\hat{I}_L} = \frac{1 + j\omega RC + (j\omega)^2 LC}{j\omega C} \leftarrow$$

$$(j\omega)^2 LC \hat{I}_L + j\omega RC \hat{I}_L + \hat{I}_L = j\omega C \hat{V}_{IN}$$

same

Diff. Eq.

$$LC \frac{d^2 i_L}{dt^2} + \overbrace{RC}^{2\alpha} \frac{di_L}{dt} + i_L = C \frac{dv_{IN}}{dt}$$

$$v_{IN} \rightarrow \hat{V}_{IN} e^{j\omega t}$$

$$i_L \rightarrow \hat{I}_L e^{j\omega t}$$

Plug in Diff Eqn

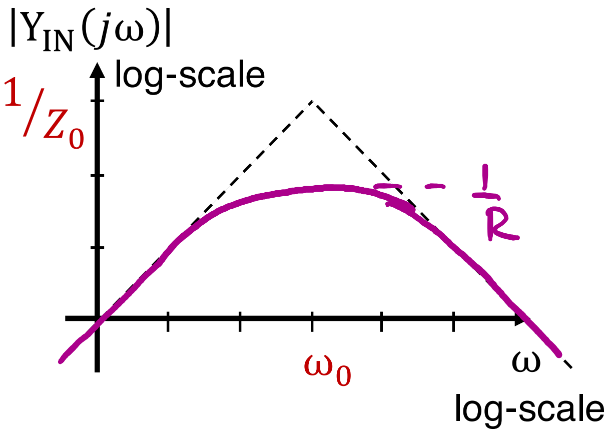
$$(j\omega)^2 LC \hat{I}_L \cancel{e^{j\omega t}} + (j\omega) RC \hat{I}_L \cancel{e^{j\omega t}} + \hat{I}_L \cancel{e^{j\omega t}} = (j\omega) C \hat{V}_{IN} \cancel{e^{j\omega t}}$$

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad Q \equiv \frac{\omega_0}{2\alpha} \longrightarrow Q = \frac{Z_0}{R}$$

Frequency Response and Natural Response

Overdamped

$R > 2Z_0$



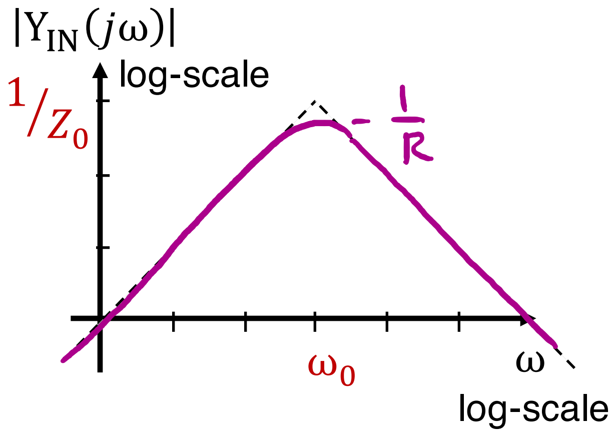
$$Q \equiv \frac{\omega_0}{2\alpha} = \frac{Z_0}{R} < \frac{1}{2}$$

$\Rightarrow$ $\alpha > \omega_0$

$$A_1 e^{-\left(\alpha - \sqrt{\alpha^2 - \omega_0^2}\right)t} + A_2 e^{-\left(\alpha + \sqrt{\alpha^2 - \omega_0^2}\right)t}$$

Critically damped

$R = 2Z_0$



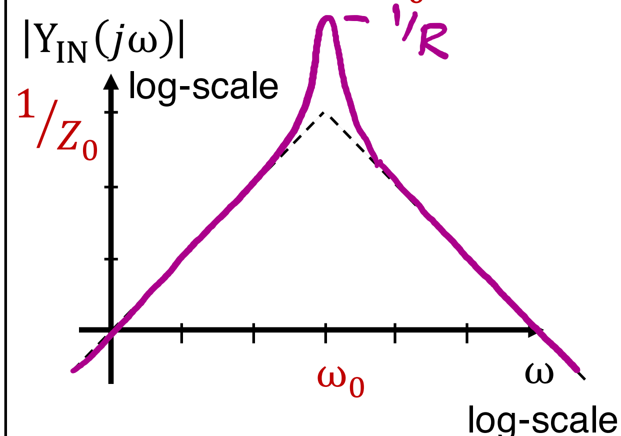
$$Q \equiv \frac{\omega_0}{2\alpha} = \frac{Z_0}{R} = \frac{1}{2}$$

$\Rightarrow$ $\alpha = \omega_0$

$$A_1 e^{-\alpha t} + A_2 t e^{-\alpha t}$$

Underdamped

$R < 2Z_0$



$$Q \equiv \frac{\omega_0}{2\alpha} = \frac{Z_0}{R} > \frac{1}{2}$$

$\Rightarrow$ $\alpha < \omega_0$

$$A_1 e^{-\alpha t} \cos(\omega_d t) + A_2 e^{-\alpha t} \sin(\omega_d t)$$

$$\omega_d \equiv \sqrt{\omega_0^2 - \alpha^2}$$