ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 29

Magnitude and Phase versus Frequency Plots

Announcements

- Recommended Reading:
  - Textbook Chapter 9 and Chapter 14

- Upcoming due dates:
  - Lab report 4 due by 11:59 pm on Monday April 8, 2019
  - Prelab 5 due by 11:59 pm on Monday April 15, 2019
  - Homework 5 due by 11:59 pm on Wednesday April 17, 2019
  - Lab report 5 due by 11:59 pm on Friday April 19, 2019

Sinusoidal Steady State Analysis using Impedances

1. Create impedance (frequency domain) model of the circuit
   - Replace sinusoidal sources by the equivalent phasor

$$v_{\mathrm{IN}}(t) = \mathrm{V_I}\cos(\omega t + \phi_{\mathrm{I}}) \rightarrow \mathrm{V_I} e^{j\phi_{\mathrm{I}}}$$

   - Replace circuit elements by their impedances models

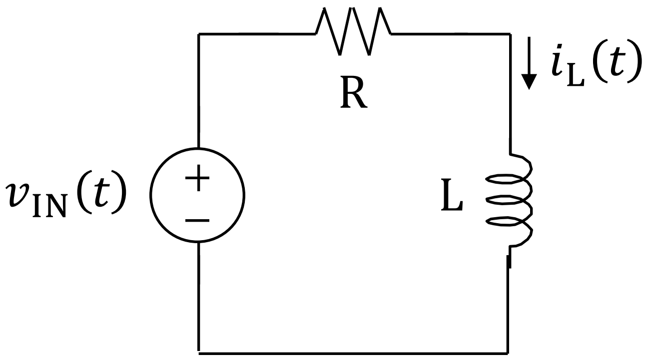
$$\mathrm{R} \rightarrow \mathrm{R} \qquad \mathrm{L} \rightarrow j\omega\mathrm{L} \qquad \mathrm{C} \rightarrow \frac{1}{j\omega\mathrm{C}}$$

2. Solve frequency domain circuit (with algebraic constitutive relationships) for phasors of interest
   - Solve frequency domain circuit like a resistive circuit $\rightarrow \hat{V}_C$

3. Convert phasors of interest into time domain by multiplying by $e^{j\omega t}$ and taking the real part

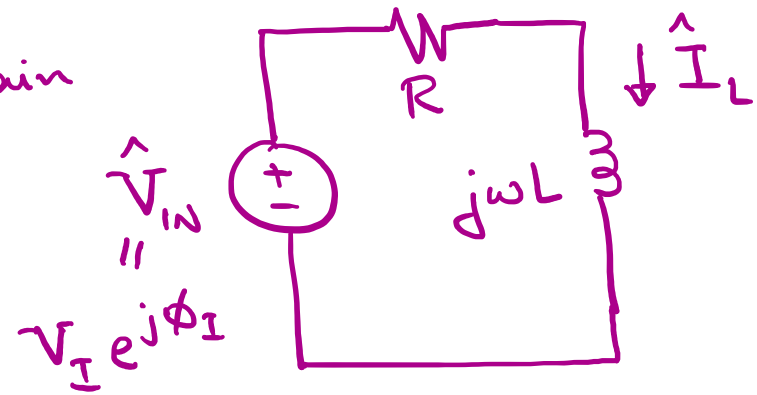
$$v_{\mathrm{C}}(t) = \mathrm{Re}\left\{\hat{\mathrm{V}}_{\mathrm{C}} e^{j\omega t}\right\} = \mathrm{Re}\left\{\mathrm{V_C} e^{j\phi_{\mathrm{C}}} e^{j\omega t}\right\} = \mathrm{V_C}\cos(\omega t + \phi_{\mathrm{C}})$$

Circuit Analysis using Impedances – Example 2



$$v_{IN}(t) = V_I \cos(\omega t + \phi_I)$$

Freq. Domain

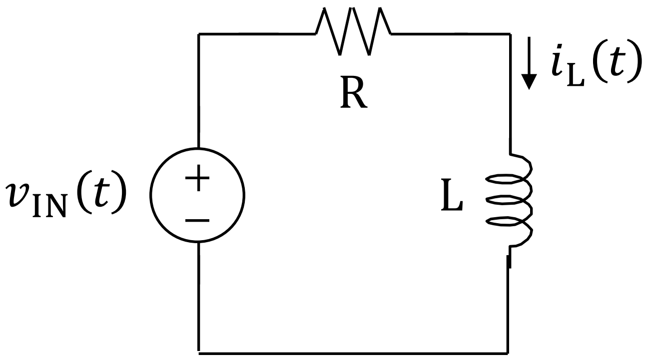


$$\hat{I}_L = \frac{\hat{V}_{IN}}{R + j\omega L} = \frac{V_I e^{j\phi_I}}{\sqrt{R^2 + (\omega L)^2}} e^{-j\tan^{-1}\left(\frac{\omega L}{R}\right)}$$

$$i_L(t) = Re\left\{\hat{I}_L e^{j\omega t}\right\} = Re\left\{\frac{V_I e^{j(\omega t + \phi_I - \tan^{-1}(\omega L/R))}}{\sqrt{R^2 + (\omega L)^2}}\right\}$$

$$i_L(t) = \frac{V_I}{\sqrt{R^2 + (\omega L)^2}} \cos\left(\omega t + \phi_I - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

Circuit Behavior in Frequency Domain

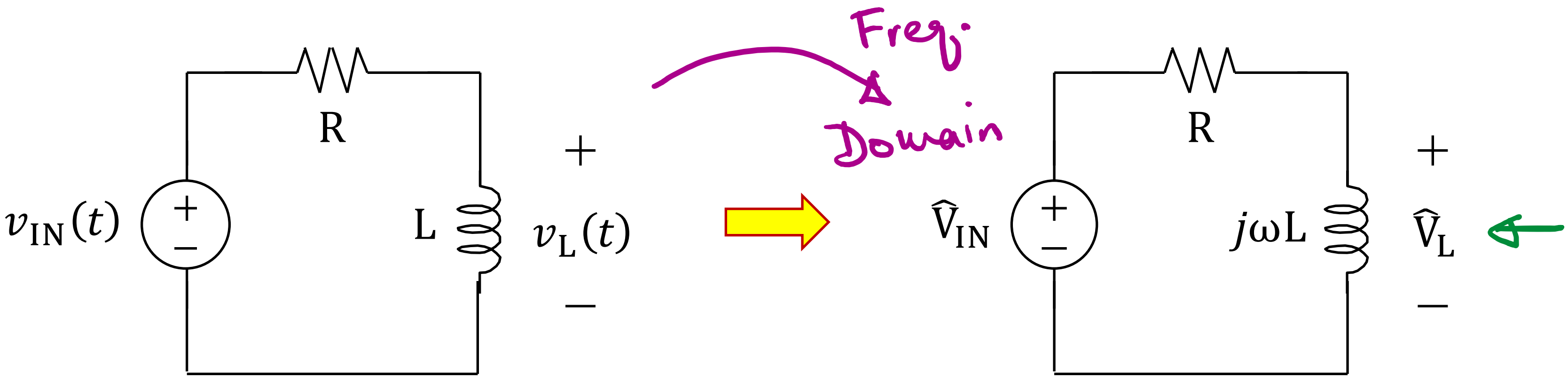


$$v_{IN}(t) = V_I \cos(\omega t + \phi_I)$$

$$\hat{I}_L = \frac{\hat{V}_{IN} e^{-j \tan^{-1}(\omega L/R)}}{\sqrt{R^2 + (\omega L)^2}}$$

Quite often we are not interested in going back into the time domain and simply interested in the behavior of the output as a function of the frequency of the driving sinusoid (e.g., when designing filters)

Frequency Domain Circuit Analysis



$$v_{IN}(t) = V_I \cos(\omega t + \phi_I)$$

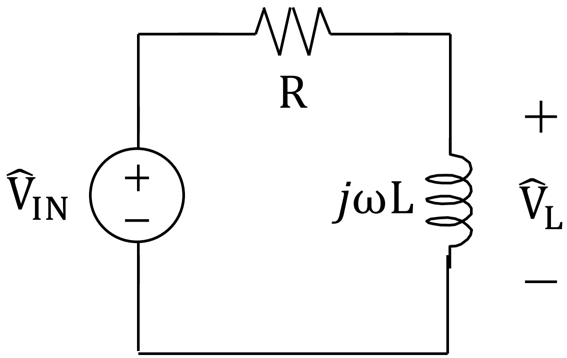
$$+ V_2 \cos(\omega_2 t + \phi_2)$$

$$+ V_3 \cos(\omega_3 t + \phi_3)$$

$$\hat{V}_L = \frac{j\omega L}{R + j\omega L} \hat{V}_{IN}$$

Transfer Function $\quad H(j\omega) \equiv \dfrac{\hat{V}_L}{\hat{V}_{IN}} = \dfrac{j\omega L}{R + j\omega L}$

Transfer Function



$$\hat{V}_L = \frac{j\omega L}{R + j\omega L}\hat{V}_{IN}$$

$$H(j\omega) \equiv \frac{\hat{V}_L}{\hat{V}_{IN}} = \frac{j\omega L}{R + j\omega L}$$

Low Freq: $H(j\omega) \approx \frac{j\omega L}{R}$ if $R \gg \omega L \Rightarrow \omega \ll \frac{R}{L}$

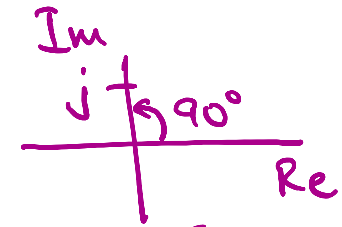
Corner Freq: $\omega = R/L$

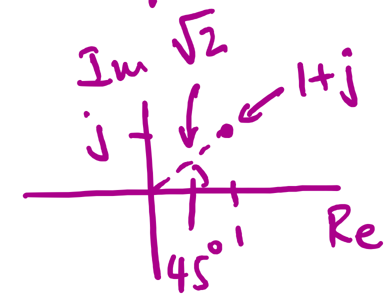
$H(j\omega) = \frac{j\cancel{\omega L}}{\cancel{\omega L} + j\cancel{\omega L}} = \frac{j}{1+j}$ if $R = \omega L \Rightarrow \boxed{\omega = \frac{R}{L}}$

High Freq: $H(j\omega) \approx \frac{j\omega L}{j\omega L} = 1$ if $R \ll \omega L \Rightarrow \omega \gg \frac{R}{L}$

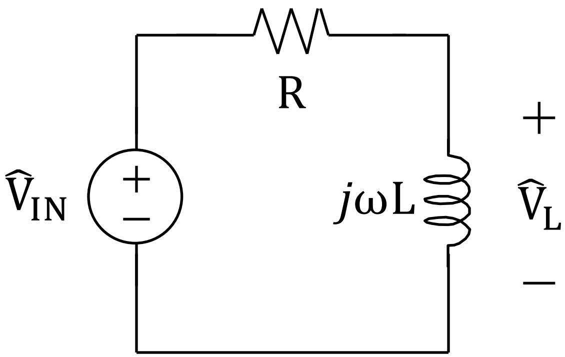
Transfer Function Magnitude and Phase

$$H(j\omega) = \begin{cases} \dfrac{j\omega L}{R} = \dfrac{\omega L}{R} e^{j90^\circ} & \omega \ll \dfrac{R}{L} \\[2ex] \dfrac{j}{1+j} = \dfrac{e^{j90^\circ}}{\sqrt{2}\, e^{j45^\circ}} = \dfrac{1}{\sqrt{2}} e^{j45^\circ} & \omega = \dfrac{R}{L} \\[2ex] 1 = 1\, e^{j0^\circ} & \omega \gg \dfrac{R}{L} \end{cases}$$



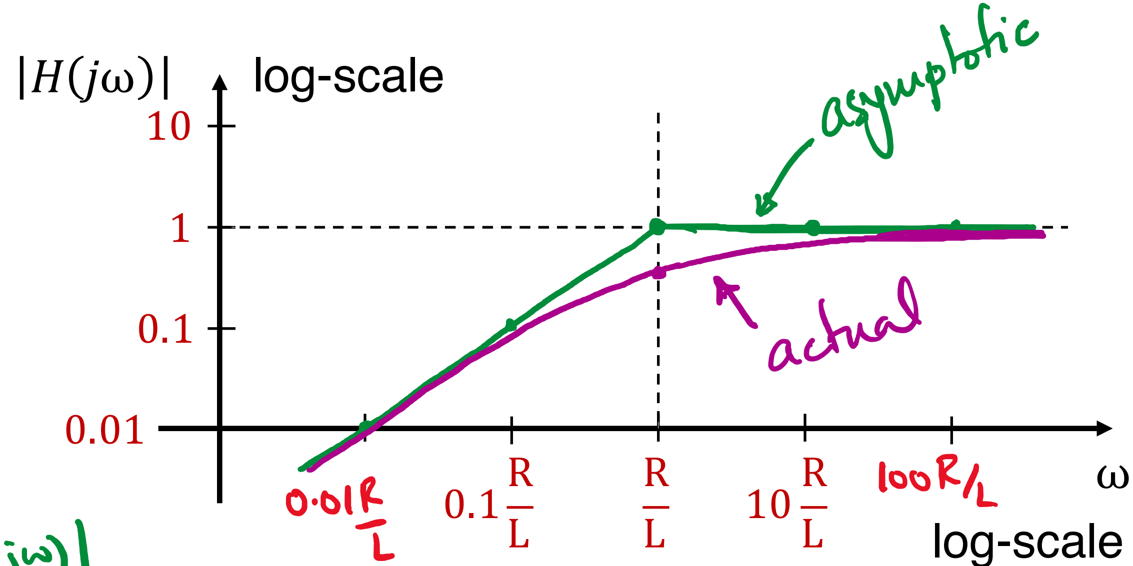


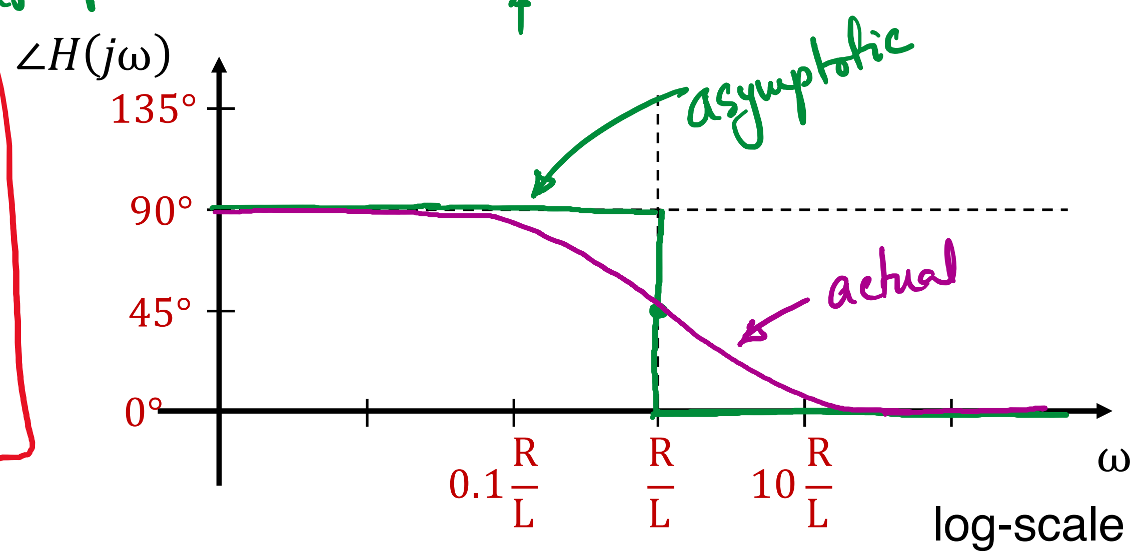
Magnitude and Phase versus Frequency Plots



$$H(j\omega) \equiv \frac{\hat{V}_L}{\hat{V}_{IN}} = \frac{j\omega L}{R + j\omega L}$$

$$H(j\omega) = \begin{cases} \overbrace{\frac{\omega L}{R}}^{|H(j\omega)|} e^{j90^\circ} & \omega \ll R/L \\ \frac{1}{\sqrt{2}} e^{j45^\circ} & \omega = R/L \\ 1 e^{j0^\circ} & \omega \gg R/L \end{cases}$$



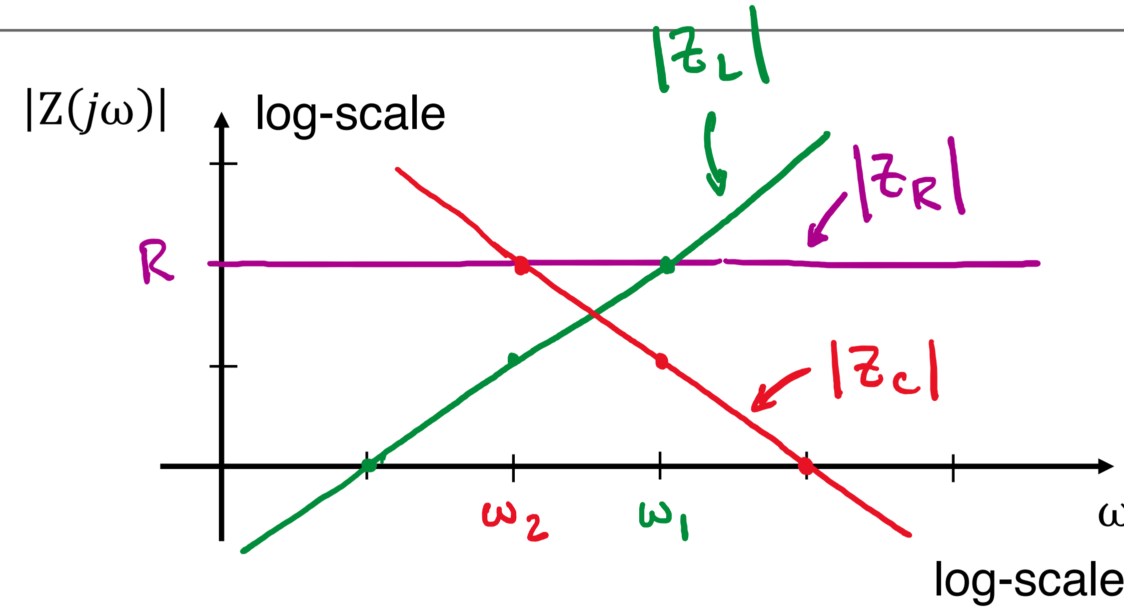


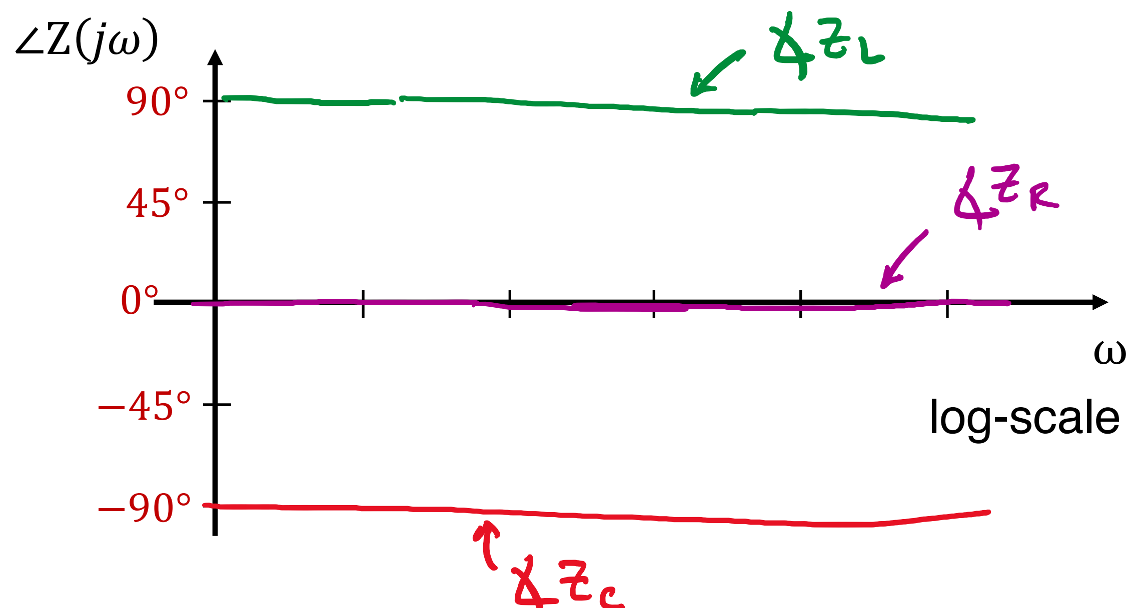
Impedance Plots

$$Z_R = R = Re^{j0^\circ}$$

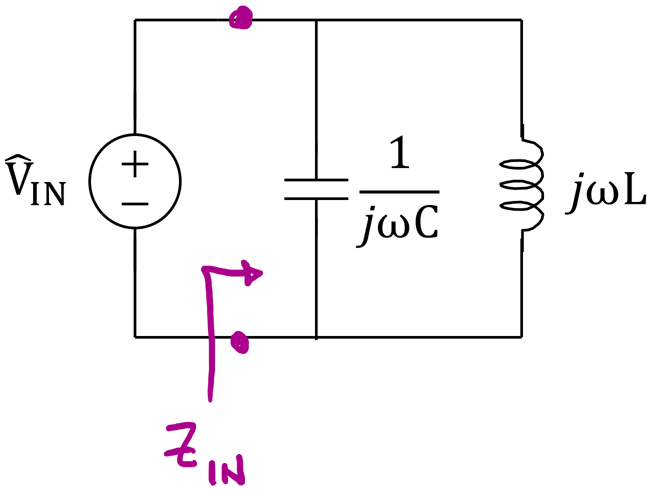
$$Z_L = j\omega L = \omega L e^{j90^\circ}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{\omega C} e^{-j90^\circ}$$





Impedance of Parallel L-C Circuit



$$Z_{IN} = \frac{1}{j\omega C} \,\|\, j\omega L$$

$$Z_{IN} = \frac{\frac{1}{j\omega C}\cdot j\omega L}{\frac{1}{j\omega C} + j\omega L} = \frac{j\omega L}{1-\omega^2 LC}$$

$$Z_{IN} = \begin{cases} \dfrac{j\omega L}{1} = j\omega L & \text{low Freq (looks like L)} \\[2ex] \dfrac{j\cancel{\omega}\cancel{L}}{-\omega^{\cancel{2}}\cancel{L}C} = \dfrac{1}{j\omega C} & \text{Hig Freq (looks like C)} \end{cases}$$

at ω where $\frac{1}{j\omega C} = j\omega L$