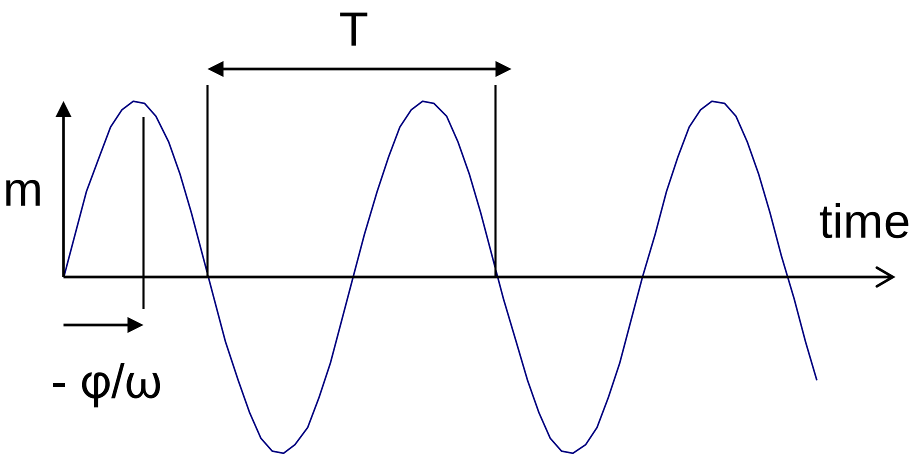
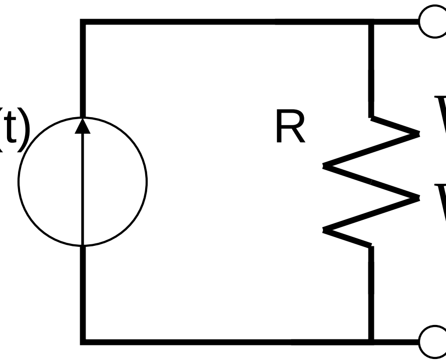
Basic Sinusoid terms

- $I(t)=m\cos(\omega t+\varphi)$
- m: amplitude
  - Peak = m
  - Peak-to-peak =2m
  - $RMS=[mean(I(t))^2]^{1/2}= 2^{-1/2}m$
- ω: frequency
  - ω is in radians/s
  - $\omega = 2\pi f$, f is frequency in Hertz (Hz)=cycles/second
  - Period, $T = 1/f = 2\pi/\omega$
- φ : phase
  - Usually in radians
  - Sometimes in degrees
  - Time delay = $-\varphi/\omega$

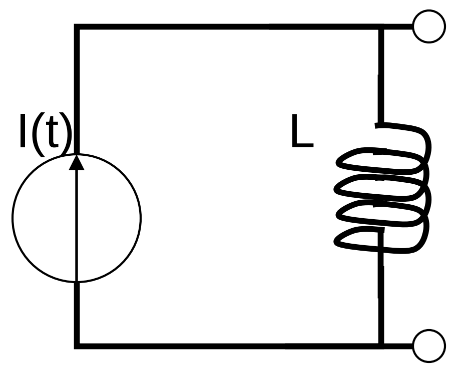


Responses to sinusoids

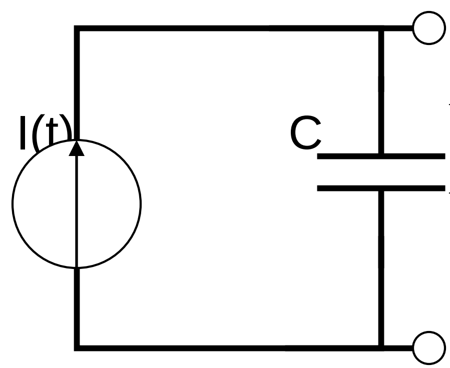
- responses of basic components to sinusoidal currents
- In each case, assume
- I(t)=mcos(ωt)



$$V_R = IR$$
$$V_R(t) = Rm\cos(\omega t)$$



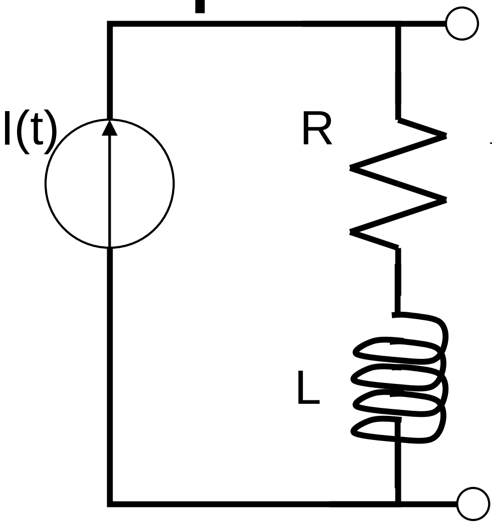
$$V_L = L\frac{dI}{dt} = L\frac{d}{dt}\left(m\cos(\omega t)\right)$$
$$V_L(t) = -L\omega m\sin(\omega t)$$



$$V_C = \frac{1}{C}\int I(t)dt = \frac{1}{C}\int m\cos(\omega t)dt$$
$$V_R(t) = \frac{1}{\omega C}m\sin(\omega t)$$

Responses to sinusoids add in series:

- $I(t)=m\cos(\omega t)$

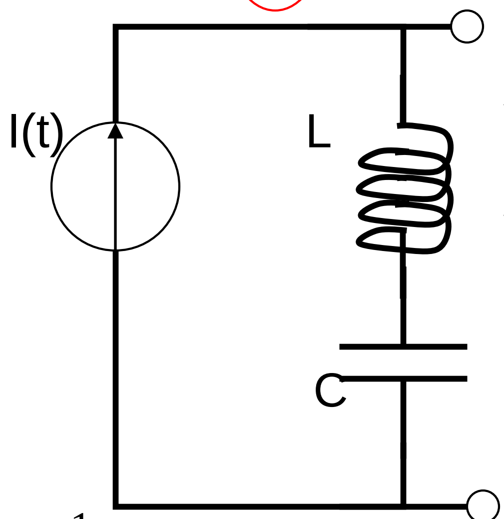


$$V_{out} = IR + L\frac{dI}{dt}$$

$$V_{out}(t) = Rm\cos(\omega t) - L\omega m\sin(\omega t)$$

$$= Rm\left(\cos(\omega t) - \frac{L}{R}\omega\sin(\omega t)\right)$$

τω

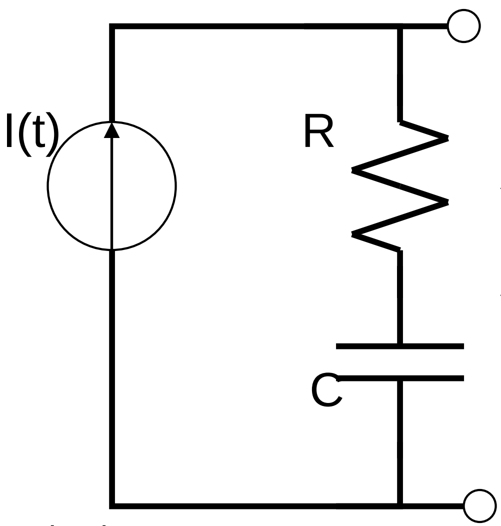


$$V_{out} = L\frac{dI}{dt} + \frac{1}{C}\int m\cos(\omega t)dt$$

$$V_{out}(t) = \left(\frac{1}{\omega C} - L\omega\right)m\sin(\omega t)$$

$$= L\omega\left(\frac{1}{\omega^2 LC} - 1\right)m\sin(\omega t)$$

(ω_o / ω) Opposite sign: can cancel!



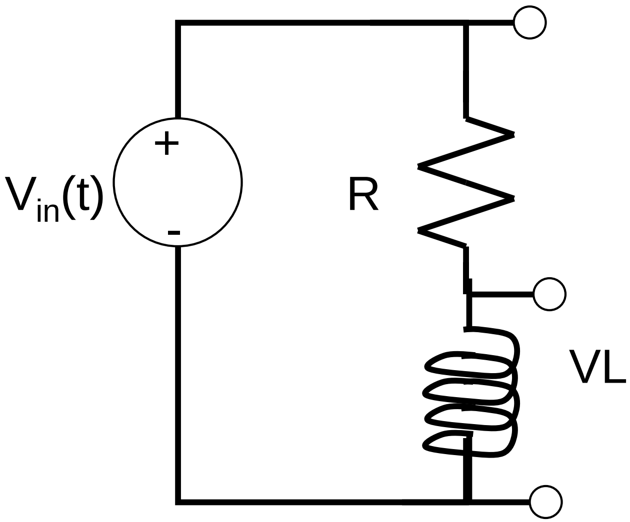
$$V_{out} = IR + \frac{1}{C}\int Idt$$

$$V_{out}(t) = Rm\cos(\omega t) + \frac{1}{\omega C}m\sin(\omega t)$$

$$= Rm\left(\cos(\omega t) + \frac{1}{\omega RC}\sin(\omega t)\right)$$

1/(τω)

Harder:



$V_{in}(t)$=mcos(ωt)

- Want to find V_L.
  - Know that :

$$V_L = L\frac{dI}{dt}$$

  - But what is I(t)?
- Guess:

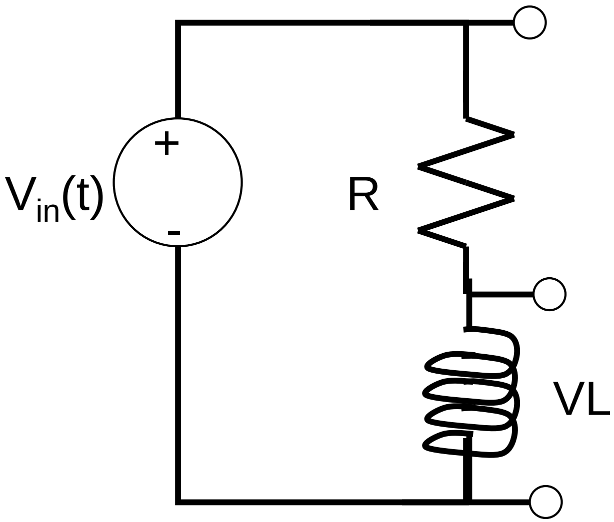
$$I(t) = A\cos(\omega t) + B\sin(\omega t)$$

  - Now plug this in, solve for A,B:

$$Vin(t) = RI(t) + L\frac{dI}{dt}$$

$$m\cos(\omega t) = RA\cos(\omega t) + RB\sin(\omega t) - \omega LA\sin(\omega t) + \omega LB\cos(\omega t)$$

Harder, part 2:



$V_{in}(t)=m\cos(\omega t)$

- Separate out cos, sin
  - In general, there is no real-value constant k for which

$$k\sin(\omega t)=\cos(\omega t)$$

  - So get two equations

$$m\cos(\omega t)=RA\cos(\omega t)+\omega LB\cos(\omega t)$$

$$0=RB\sin(\omega t)-\omega LA\sin(\omega t)\rightarrow\frac{\omega LA}{R}=B$$

  - Which solve to: $A=\frac{mR}{R^2+(\omega L)^2}, B=\frac{m(\omega L)}{R^2+(\omega L)^2}$

- Now substitute back:

$$V_L=L\frac{dI}{dt}=-A\omega L\sin(\omega t)+B\omega L\cos(\omega t)=\frac{-m\omega RL}{R^2+(\omega L)^2}\sin(\omega t)+\frac{m(\omega L)^2}{R^2+(\omega L)^2}\cos(\omega t)$$

There has to be an easier way!