ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 2

Circuit Laws and Circuit Analysis

Announcements

- Recommended Reading:
  - Textbook Chapter 3

- Lab 1 is next week (starting Tuesday January 29, 2019)

- Upcoming due dates:
  - Prelab 1 due by 12:20 pm on Tuesday January 29, 2019
  - Homework 1 due by 11:59 pm on Friday February 1, 2019
  - Lab report 1 due by 11:59 pm on Friday February 8, 2019

Discussion Sections

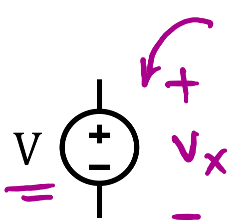
- Option A: Wed 1:25-2:15 pm and Fri 1:25-2:15 pm
- Option B: Wed 12:20-1:10 pm and Fri 1:25-2:15 pm
- Option C: Wed 1:25-2:15 pm and Thu 1:25-2:15 pm
- Option D: Thu 1:25-2:15 pm and Fri 1:25-2:15 pm

Option E: Tue 1:25-2:15pm and Fri 1:25-2:15pm

Both Discussions Sections cover the same material

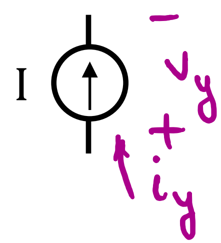
Constitutive Relationships of Basic Circuit Elements

Voltage Source



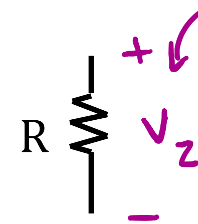
$\boxed{v_x = V}$

Current Source



$\boxed{i_y = I}$

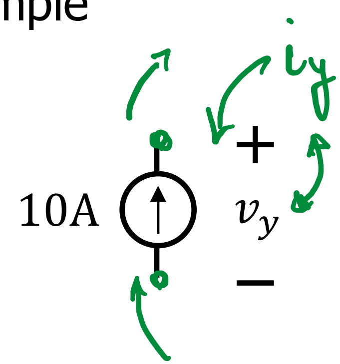
Resistor



$\boxed{v_z = R i_z}$

We could have defined the circuit variables with the opposite polarity as long as we follow the associated variables convention

Example



$\boxed{i_y = -10A}$

$P = v \cdot i$

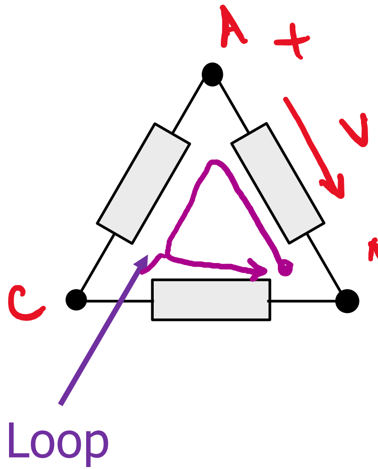
if $P = +ve$ Power going in element

if $P = -ve$ Power coming out of element

Kirchhoff's Circuit Law

- To analyze circuits we need additional relations between the terminal variables of the circuit elements
- These additional relationships come from Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL) that account for how the elements are interconnected

Kirchhoff's Voltage Law (KVL)



$$v_{AB} = -\int_B^A \vec{E}\cdot\vec{dl}$$

$\uparrow \vec{dl}$

$$\oint \vec{E}\cdot\vec{dl} = -\frac{\partial}{\partial t}\iint \vec{B}\cdot\vec{ds} = -\frac{\partial \Phi}{\partial t} \qquad \text{Faraday's Law}$$

$$\frac{\partial \phi}{\partial t} = 0 \quad \text{through loops (\& outside \& elements)}$$

$$\oint \vec{E}\cdot\vec{dl} = 0$$

$$\Rightarrow \underbrace{\int_B^A \vec{E}\cdot\vec{dl}}_{-v_{AB}} + \underbrace{\int_A^C \vec{E}\cdot\vec{dl}}_{-v_{CA}} + \underbrace{\int_C^B \vec{E}\cdot\vec{dl}}_{-v_{BC}} = 0$$

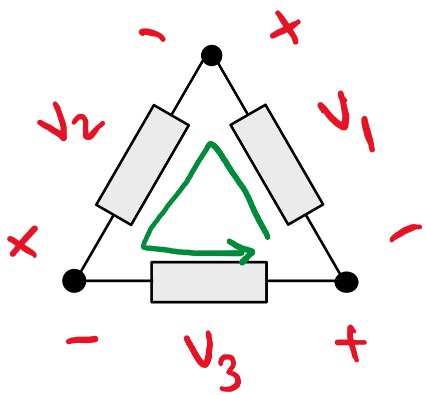
$$\Rightarrow \quad v_{AB} + v_{CA} + v_{BC} = 0 \quad \underset{\text{in general}}{\Longrightarrow} \quad \sum_{k=1}^{N} v_k = 0$$

Sum of voltages around a loop is zero

KVL (Cont.)

- In applying KVL, be careful about signs
- Good approach to remember: While traversing a loop, add with sign that one hits first (or add with sign that one hits second)

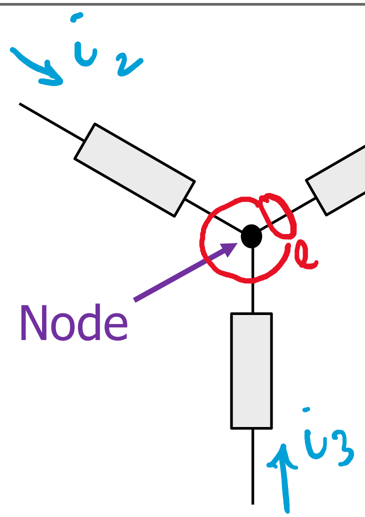
$$-v_1 - v_2 - v_3 = 0$$



$$v_1 + v_2 + v_3 = 0$$

- You can go around the loop in either direction

Kirchhoff's Current Law (KCL)



$$\oint_l \vec{H}\cdot\vec{dl} = \iint_S \vec{J}\cdot\vec{ds} + \frac{\partial}{\partial t}\iint_S \vec{D}\cdot\vec{ds}$$ Ampere's Law

$$\oiint_S \vec{D}\cdot\vec{ds} = \iiint_V \rho\, dv = q$$ Gauss' Law

$$\oint_{l\to 0} \vec{H}\cdot\vec{dl}$$

$$\frac{dq}{dt} = 0$$ inside elements & at nodes

$$\underbrace{\oint_{l\to 0} H\cdot\vec{dl}}_{=0} = \oiint_S \vec{J}\cdot\vec{ds} + \underbrace{\frac{\partial}{\partial t}\oiint_S \vec{D}\cdot\vec{ds}}_{=0}$$

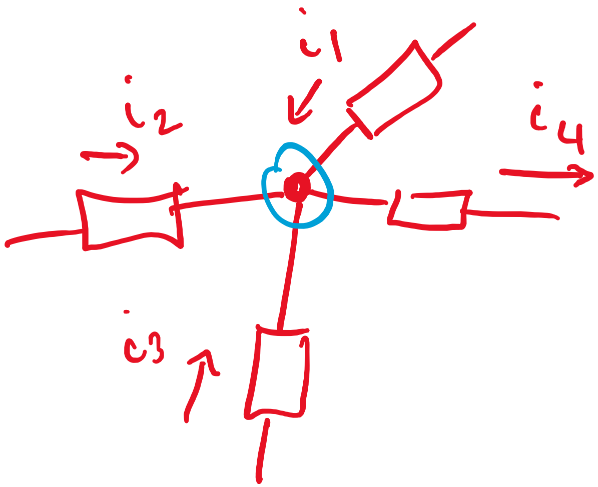
$$\Rightarrow \oiint_S \vec{J}\cdot\vec{ds} = 0 \Rightarrow i_1 + i_2 + i_3 = 0$$

$$\sum_{k=1}^{M} i_k = 0$$ KCL

Sum of currents going into a node is zero

KCL (Cont.)

- In applying KCL, be careful about signs
- Good approach to remember: Either use all currents going out of the node, or all currents going into the node

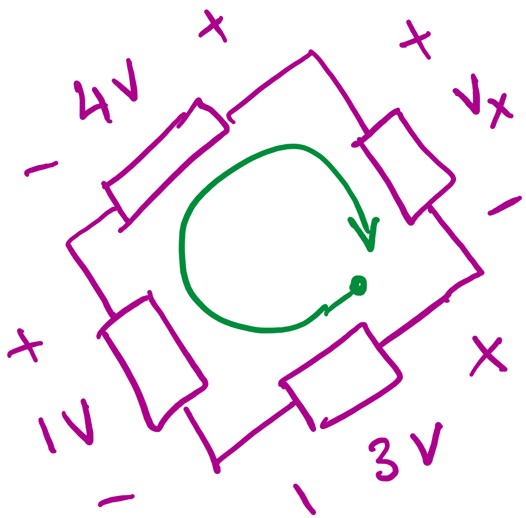


$i_1 + i_2 + i_3 - i_4 = 0$

- KCL also holds for any closed region of a circuit

KVL and KCL Examples

KVL: $\sum_{k=1}^{N} v_k = 0$

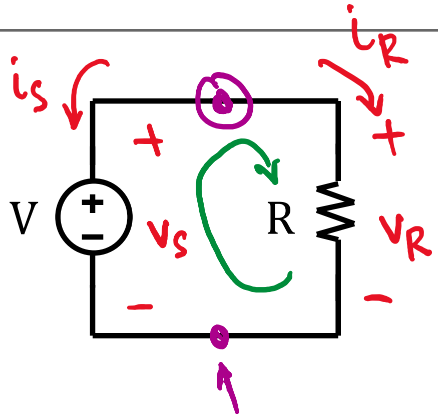


$+3-1-4+v_x=0$

$\underline{v_x = 2\,V}$

KCL: $\sum_{k=1}^{M} i_k = 0$

Our First Circuit Analysis



Analyzing (or "solving") a circuit means finding the voltage across and the current through each element ("branch")

Unknowns: v_s, i_s, v_R, i_R

Constitutive Relationships:

$v_s = V$ — ①

$v_R = R\, i_R$ — ②

Circuit Relationships:

$-v_s + v_R = 0 \Rightarrow v_s = v_R$ — ③ KVL

$-i_s - i_R = 0 \Rightarrow i_s = -i_R$ — ④ KCL

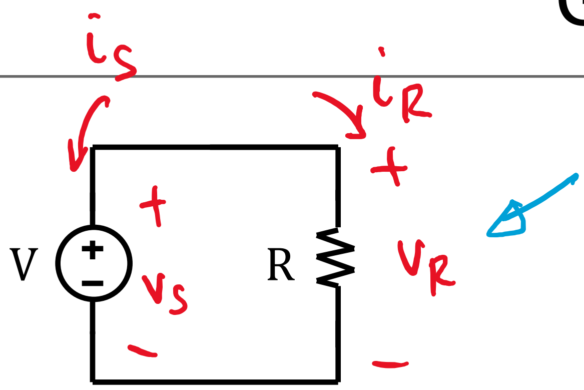
$v_s = V$

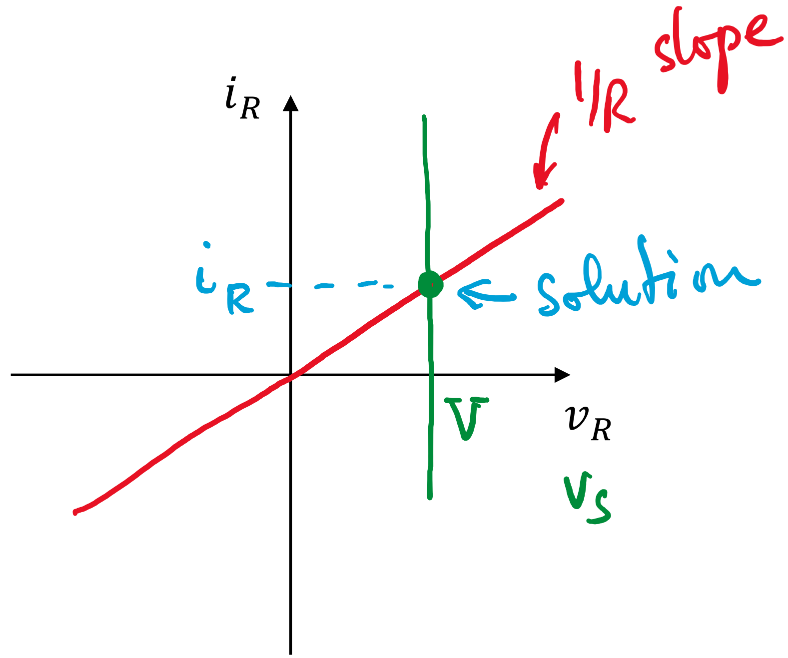
$v_R = v_s = V$

$i_R = \dfrac{v_R}{R} = \dfrac{V}{R}$

$i_s = -i_R = -\dfrac{V}{R}$

Graphical Solution





Constitutive Relationships:

$$v_S = \mathrm{V}$$

$$v_R = \mathrm{R}i_R$$

Circuit Relationships:

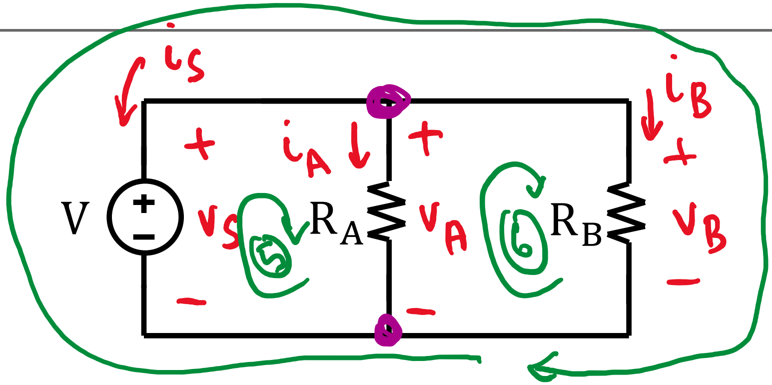
$$v_R = v_S$$

$$i_R = -i_S$$

$$i_S = -i_R$$

Power Calculation

Another Circuit Analysis Example



Unknowns: 6

Constitutive Relationships:

$v_S = V$ — 1

$v_A = R_A i_A$ — 2

$v_B = R_B i_B$ — 3

Circuit Relationships:

KCL: $-i_S - i_A - i_B = 0$ — 4

KVL: $-v_S + v_A = 0$ — 5

$-v_A + v_B = 0$ — 6

Wait to learn easier ways to solve circuits

Another Circuit Analysis Example - Discussion

In general:

- For B branches (elements), there are 2B unknowns
- For B branches, there are B independent element equations
- For N nodes, there are (N-1) independent KCL equations
- For N nodes and B branches, there are (B-N+1) independent KVL equations

$2B-(B+N-1)$

- In our example, we have six unknowns and six equations, so we can find a solution
- While this approach is good enough for a computer, it is too laborious for humans, especially when circuits become large
- So we need techniques and tricks to make solving circuits easier