ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 19

Step Response and Impulse Response

Announcements

- Recommended Reading:
  - Textbook Chapter 7

- Upcoming due dates:
  - Homework 3 due by 11:59 pm on Monday March 11, 2019
  - Lab report 3 due by 11:59 pm on Friday March 15, 2019

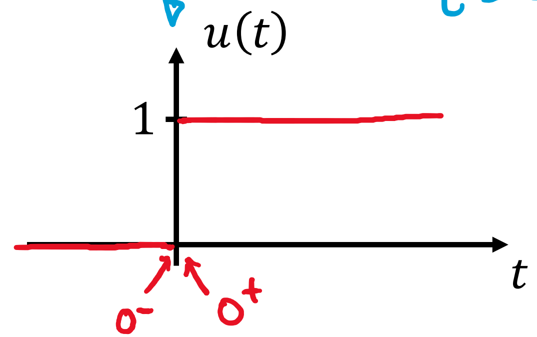
- Prelim 2 on Thursday March 28, 2019 from 7:30 – 9 pm in 203 Phillips
  - Email afridi@cornell.edu if have conflict
  - Will cover material through Lecture 24
  - Prelim is closed-book and closed-notes
  - Two double-sided page formula sheet is allowed
  - Bring a calculator

Unit Step Function

transition occurs at $t = 0$

unit step $\quad t=0$

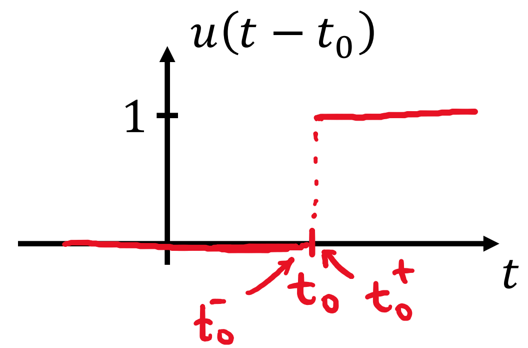
$$u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t > 0 \end{cases}$$



transition

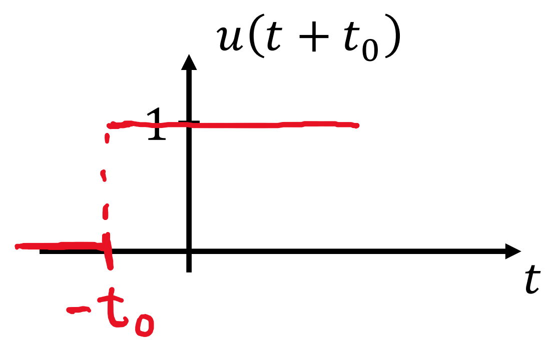
$t - t_o = 0 \Rightarrow t = t_o$

$$u(t - t_0) = \begin{cases} 0 & \text{for } t < t_0 \\ 1 & \text{for } t > t_0 \end{cases}$$



$t + t_o = 0 \Rightarrow t = -t_o$

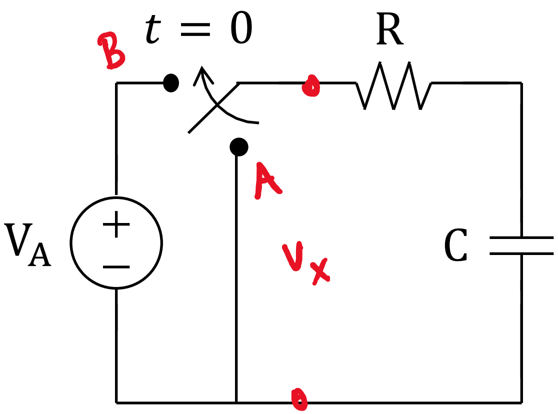
$$u(t + t_0) = \begin{cases} 0 & \text{for } t < -t_0 \\ 1 & \text{for } t > -t_0 \end{cases}$$

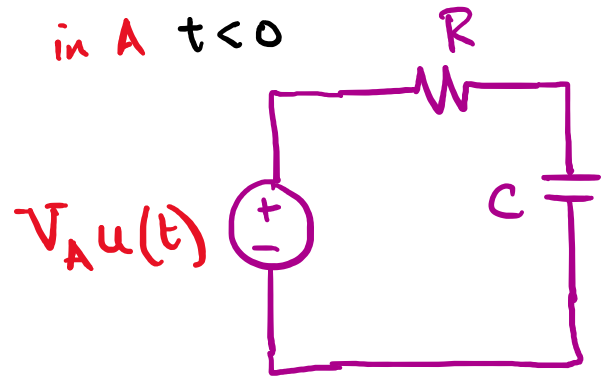


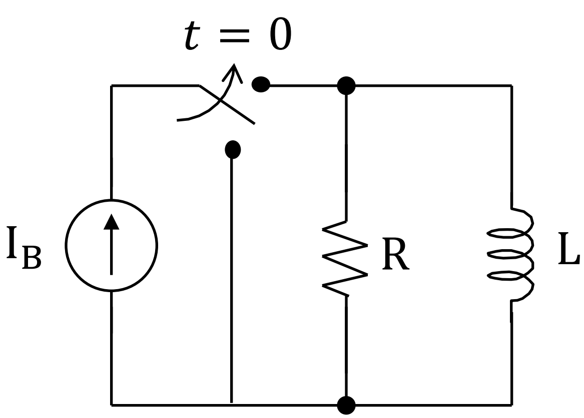
$u(t)$ is unit-less

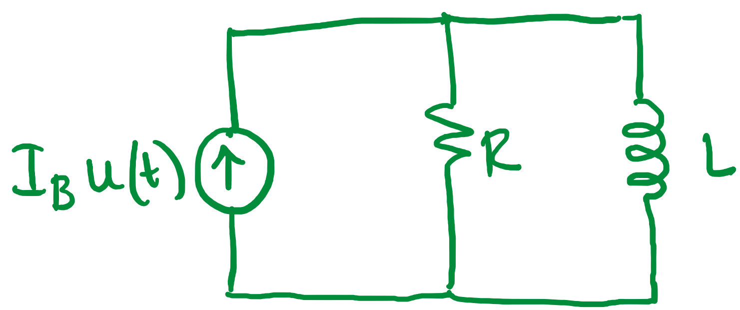
Modeling Switch Action using Unit Step

$$v_x = \begin{cases} V_A & \text{in } B \quad t>0 \\ 0 & \text{in } A \quad t<0 \end{cases}$$



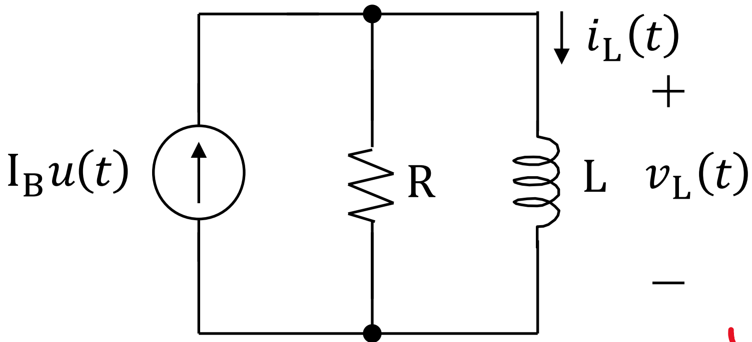






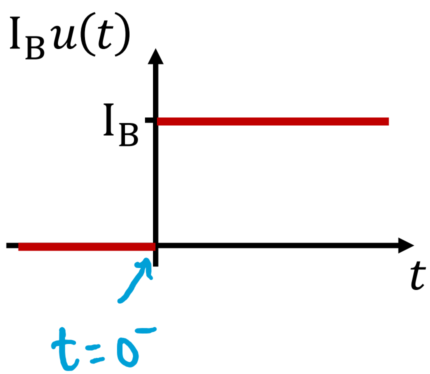
Step Response Example

Find $v_L(t)$ for all t



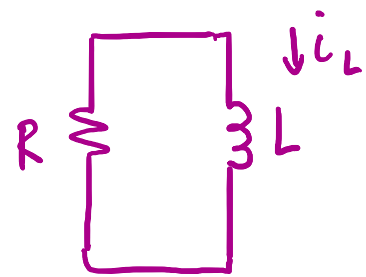
Find $i_L(t)$ first since can apply continuity condition to it

$v_L = L\frac{di_L}{dt}$



$\boxed{\begin{matrix} i_L(0^-) \\ \parallel \\ 0 \end{matrix}}$

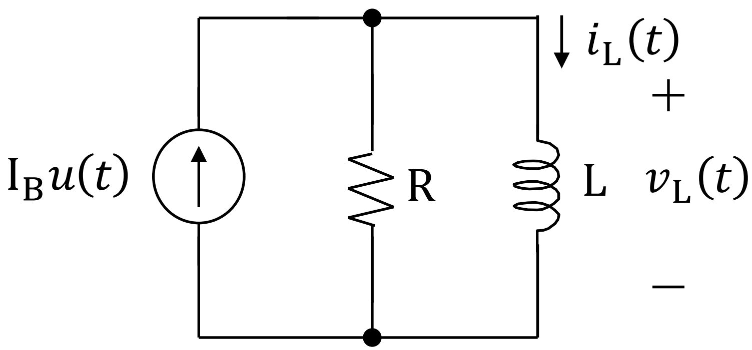
$t<0$



$i_L(t<0) = 0$

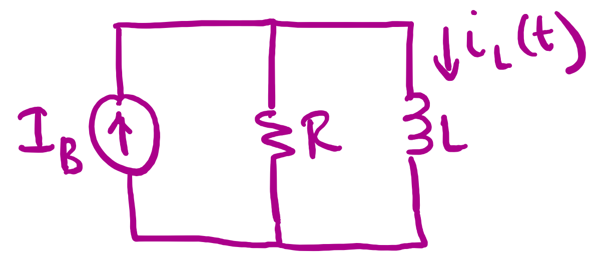
$\underline{\underline{i_L(0^+)}} = i_L(0^-) = 0 \leftarrow$ Initial Condition for $t>0$

Step Response Example (Cont.)

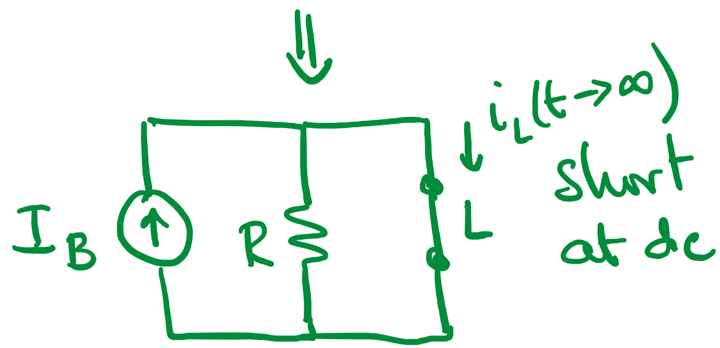


$i_L(0^+) = 0$

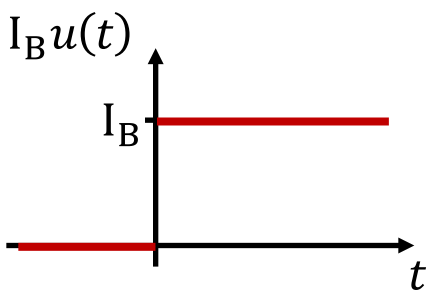
t > 0



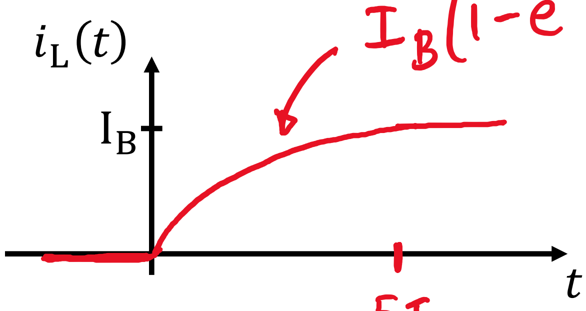
t → ∞



$i_L(t\to\infty) = I_B$

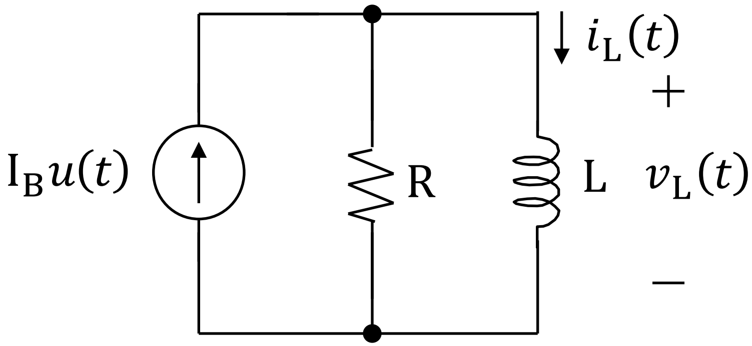


$I_B(1-e^{-t/\tau})$



$\tau = \frac{L}{R}$

Step Response Example (Cont.)



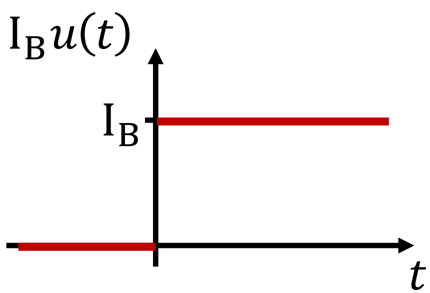
$$i_L(t) = \begin{cases} 0 & \text{for } t < 0 \\ \underline{\mathrm{I_B}\left(1 - e^{-t/\tau}\right)} & \text{for } t > 0 \end{cases}$$

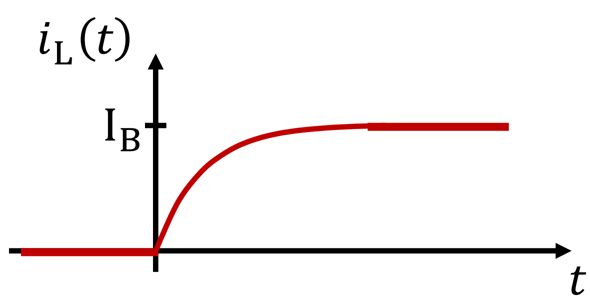
$$v_L = L\frac{di_L}{dt} \Rightarrow v_L = 0 \quad t < 0$$

$\underline{t > 0}$

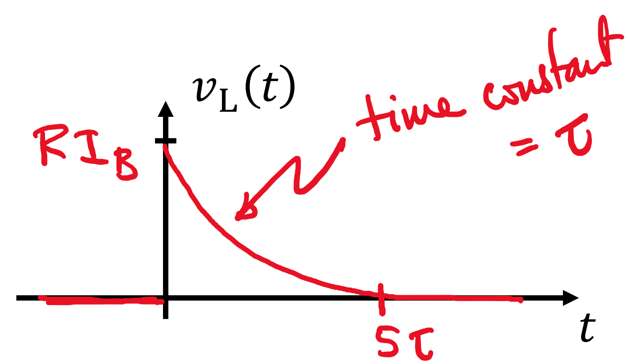
$$v_L = L\mathrm{I_B}\left(-e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right) = \cancel{L}\mathrm{I_B}e^{-t/\tau}\frac{R}{\cancel{L}}$$

$$\underline{v_L = R\mathrm{I_B}e^{-t/\tau}} \quad t > 0$$





Inductor looks like an open circuit under fast transients

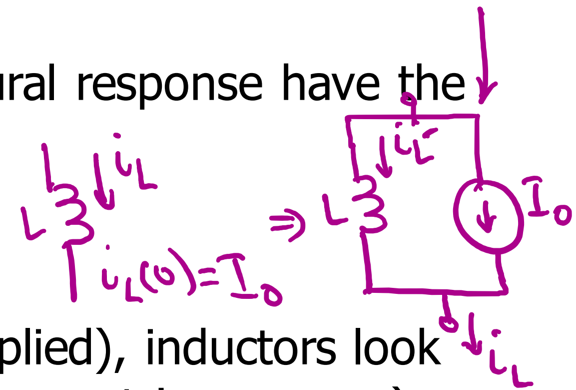


Key Lessons

- Continuity applies to state variables: $i_L(t)$ and $v_C(t)$
  - But not to other currents and voltages in the circuit
- Best to first solve for state variables
  - Other currents and voltages can then be determined

$i_L'(0)=0$

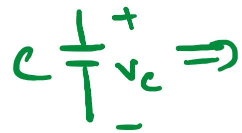
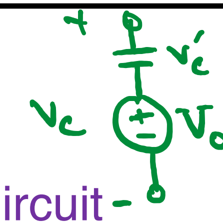
- All voltages and currents in a circuit that undergo natural response have the same time constant
- In dc steady state, inductors look like short circuit
  - But during abrupt transitions (e.g., when step is applied), inductors look like an open circuit (or a current source if inductor current is non-zero)



- In dc steady state, capacitors look like open circuit
  - But during abrupt transitions (e.g., when step is applied), capacitors look like a short circuit (or a voltage source if capacitor voltage is non-zero)

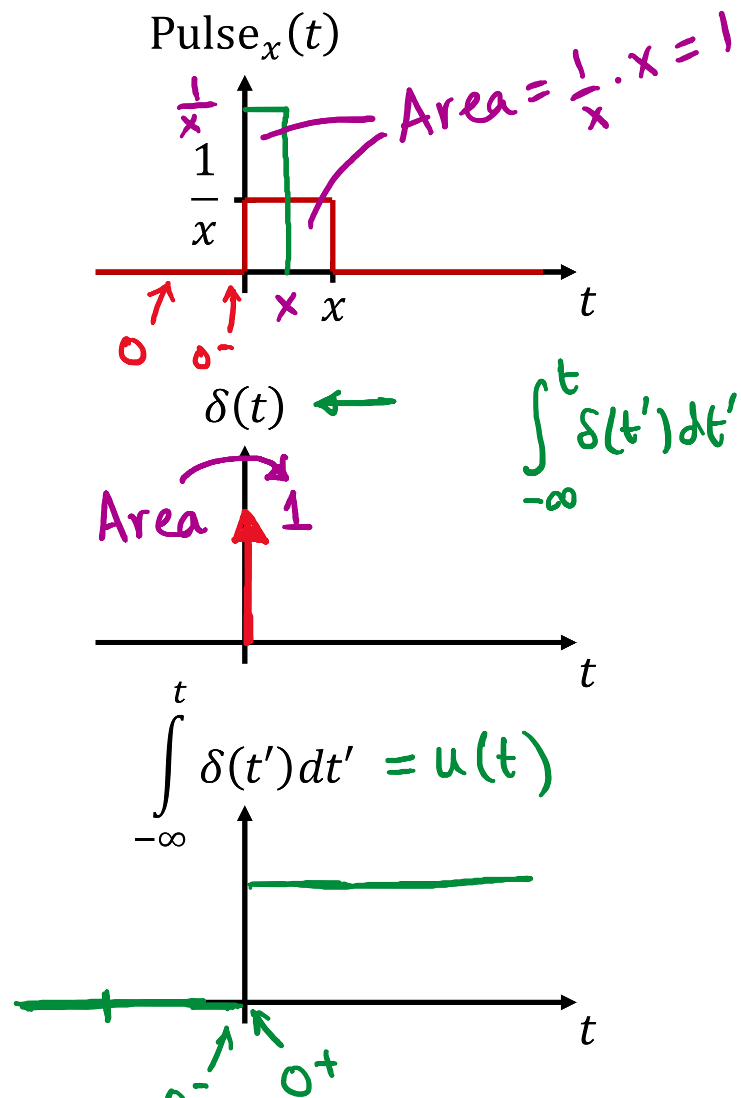
$i_C = C\frac{dv_C}{dt}$

DC Steady State versus Fast Transient Behavior

	DC Steady Steady	Fast Transient
Capacitor	Open Circuit	  $v_c'(0)=0$ $v_c(0)=V_0$ Short Circuit or voltage source of voltage equal to its initial voltage
Inductor	Short Circuit	Open Circuit or current source of current equal to its initial current

Impulse Function

$$\text{Pulse}_x(t) \xrightarrow[x \to 0]{} \delta(t)$$



$$\int_{-\infty}^{t} \delta(t')\,dt' = u(t)$$

$$\delta(t) = \frac{du(t)}{dt}$$

$$\boxed{\delta(t) \text{ has units of } s^{-1}}$$

Current Impulse and Voltage Impulse

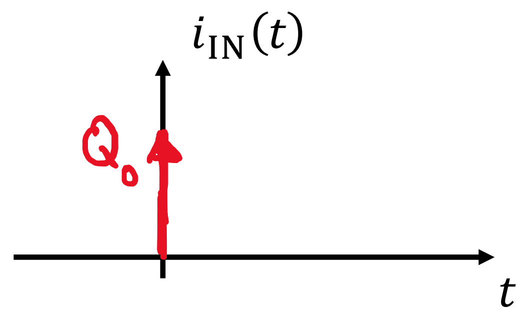
Current Impulse

charge

$$i_{\mathrm{IN}}(t) = Q_0 \delta(t) \quad [\mathrm{A}]$$

C = A·S

s^{-1}



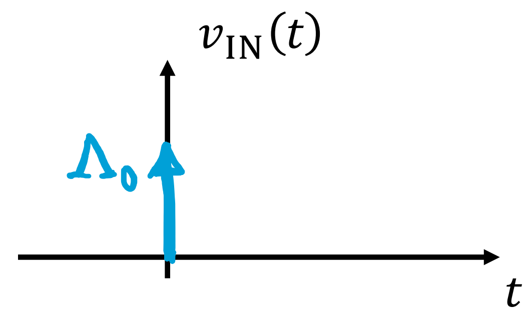
Voltage Impulse

Flux Linkage

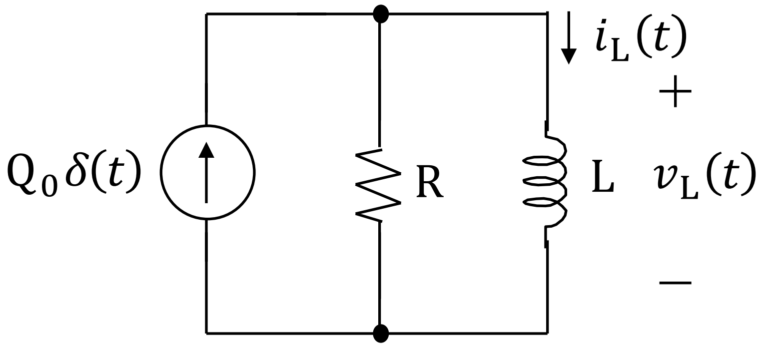
$$v_{\mathrm{IN}}(t) = \Lambda_0 \delta(t) \quad [\mathrm{V}]$$

V·s

s^{-}



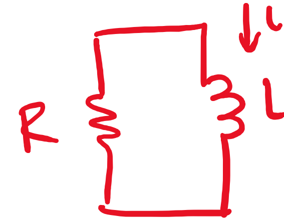
Impulse Response Example



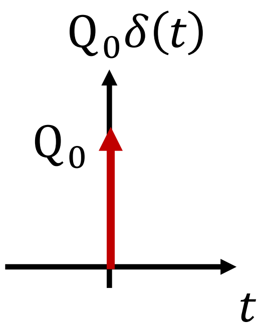
Find $i_L(t)$ for all t

$\underline{t<0}$

$i_L(t<0)=0$



$i_L(0^-)=0$



$v_L(0) = Q_0\delta(t)R$

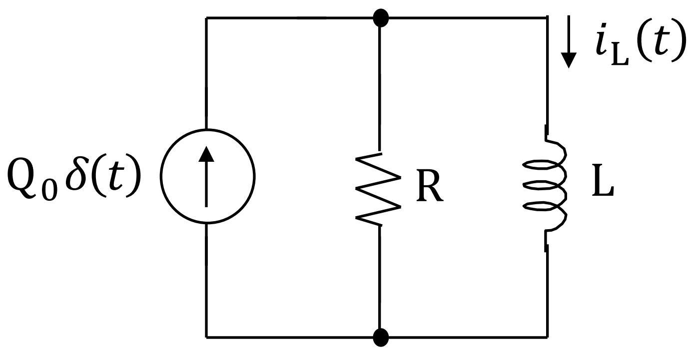
$i_L(0^+) = ?$

$$v_L = L\frac{di_L}{dt} \Rightarrow i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t')\,dt' = \frac{1}{L}\int_{0^-}^{t} v_L(t')\,dt'$$

$$i_L(0^+) = \frac{1}{L}\int_{0^-}^{0^+} v_L(t')\,dt' = \frac{1}{L}\int_{0^-}^{0^+} Q_0R\,\delta(t')\,dt' = \frac{Q_0R}{L}\underbrace{\int_{0^-}^{0^+}\delta(t')\,dt'}_{=1}$$

$$i_L(0^+) = \frac{Q_0R}{L}$$

Impulse Response Example (cont.)



$i_L(t) = Ke^{-t/\tau}$ for $t > 0$

$i_L(0^+) = \frac{Q_0 R}{L}$

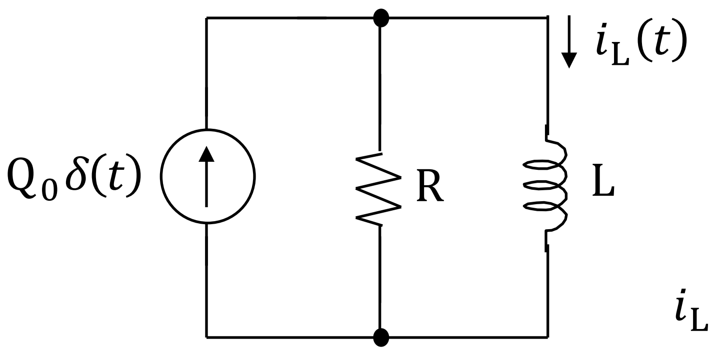
$i_L(0^+) = K$

$\Rightarrow K = \frac{Q_0 R}{L}$

$\therefore \boxed{i_L(t) = \frac{Q_0 R}{L} e^{-t/\tau}}$

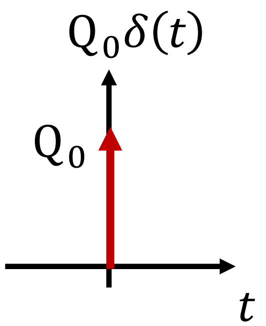
$\tau = \frac{L}{R}$

Impulse Response Example (cont.)



$$i_L(t) = Ke^{-t/\tau} \qquad \text{for } t > 0$$

$$i_L(0^+) = \frac{Q_0 R}{L}$$



$$i_L(t) = \frac{Q_0 R}{L} e^{-t/\tau} \qquad \text{for } t > 0$$