ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 18

First Order RC and RL Circuits – Solution using Intuitive Approach and ZIR/ZSR Method

Announcements

- Recommended Reading:
  - Textbook Chapter 7

- Upcoming due dates:
  - Prelab 3 due by 12:20 pm on Tuesday March 5, 2019
  - Homework 3 due by 11:59 pm on Monday March 11, 2019
  - Lab report 3 due by 11:59 pm on Friday March 15, 2019

Intuitive Approach

Total Solution: $y(t) = y_h(t) + y_p(t)$

$y_h(t)$ - Natural Response (Homogeneous Solution)

- Typically, dies with time
- Form depends on order of circuit
  - First-order circuits (RC, RL): exponential decay

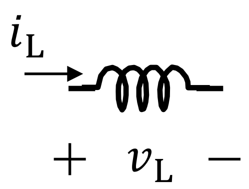
$$Ke^{-t/\tau} \quad \text{where} \quad \tau = \begin{cases} \mathrm{RC} \\ \mathrm{L/R} \end{cases}$$

$y_p(t)$ - Driven Response (Particular Solution)

- Continues in steady state
- Same form as the drive - $f(t)$

Determining DC Steady State

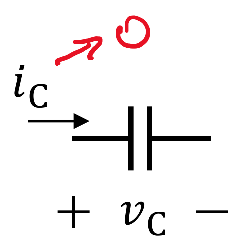
Inductor $v_L = \mathrm{L}\dfrac{di_L}{dt}$ $i_\mathrm{L} = $ constant $= dc$ ⇨ $v_\mathrm{L} = 0$



⇨

short circuit

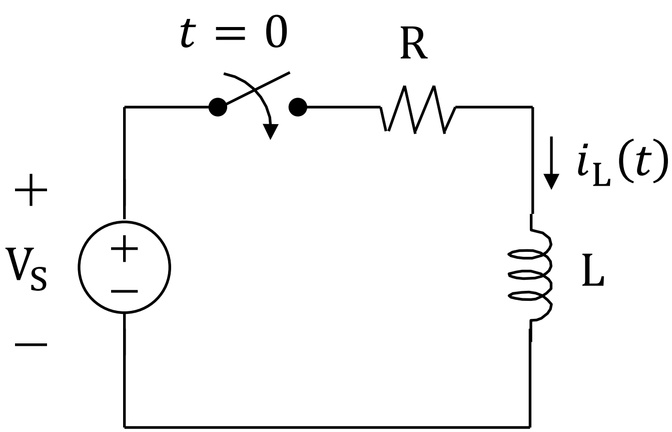
Capacitor $i_C = \mathrm{C}\dfrac{dv_C}{dt}$ $v_\mathrm{C} = $ constant ⇨ $i_\mathrm{C} = 0$



⇨

open circuit

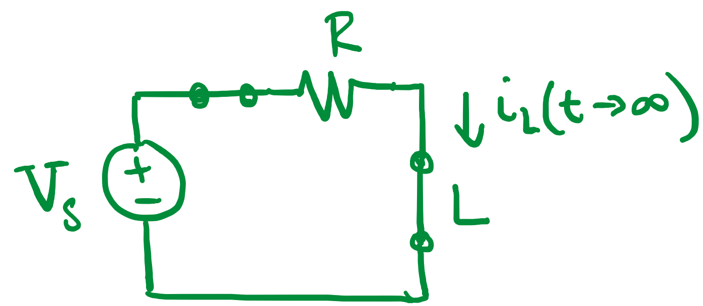
RL Circuit – Solution using Intuitive Approach



$i_L(0^-) = 0$

$i_L(0^+) = i_L(0^-) = 0$

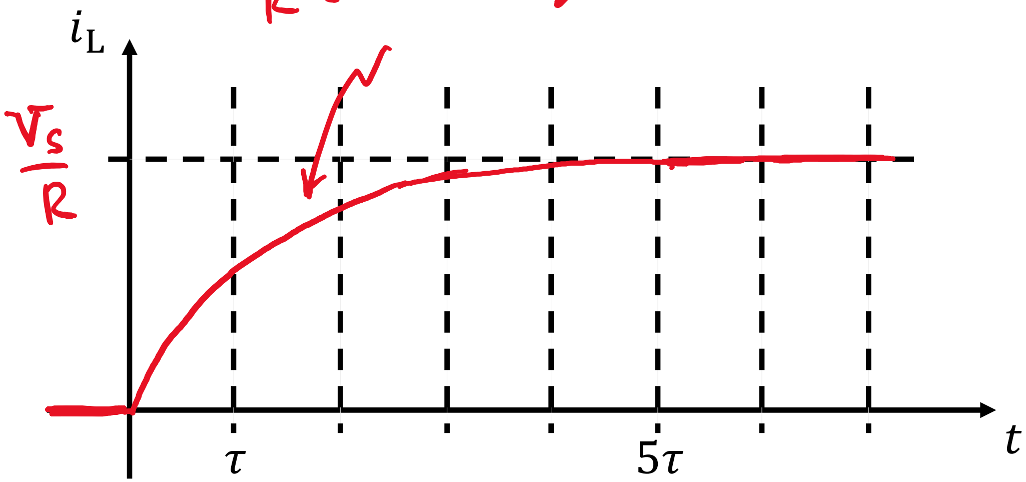
as $t \to \infty$



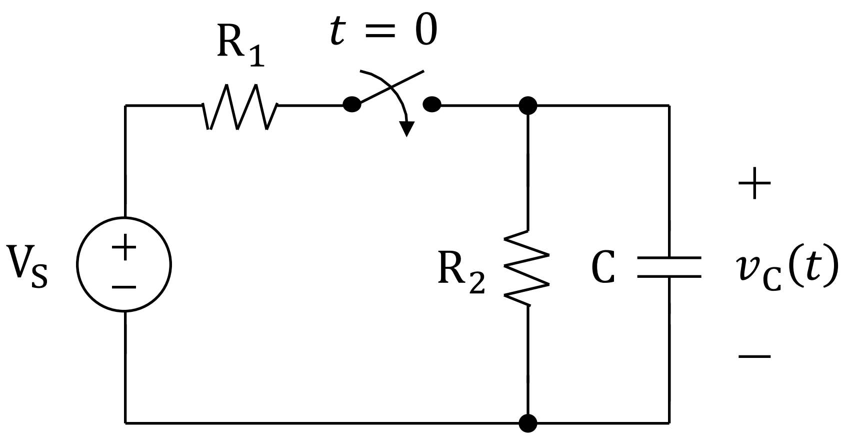
$i_L(t \to \infty) = \frac{V_s}{R}$

$\tau = \frac{L}{R}$

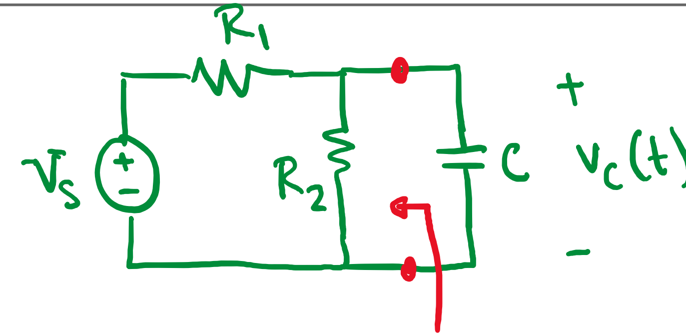
$i_L = \frac{V_s}{R}\left(1 - e^{-t/\tau}\right)$



More Complex Circuit

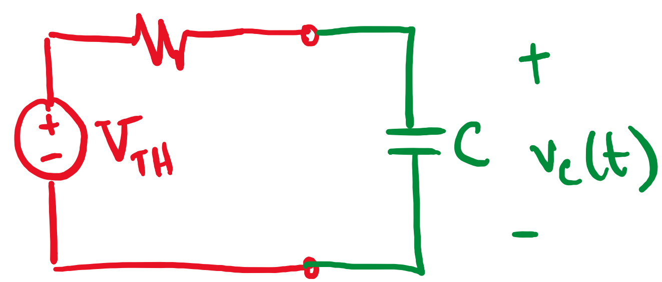


$t \geq 0$

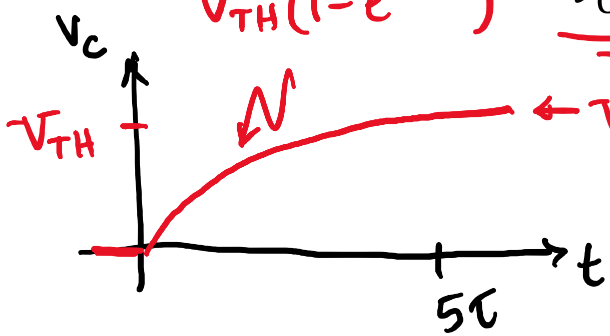


$R_1 \| R_2 = R_{TH}$

$\Downarrow$



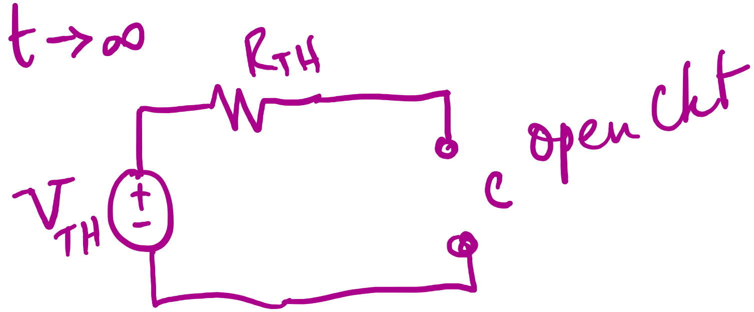
$v_C(0) = 0$



$V_{TH}(1-e^{-t/\tau})$

$\leftarrow V_{TH} = \frac{R_2}{R_1+R_2} V_s$

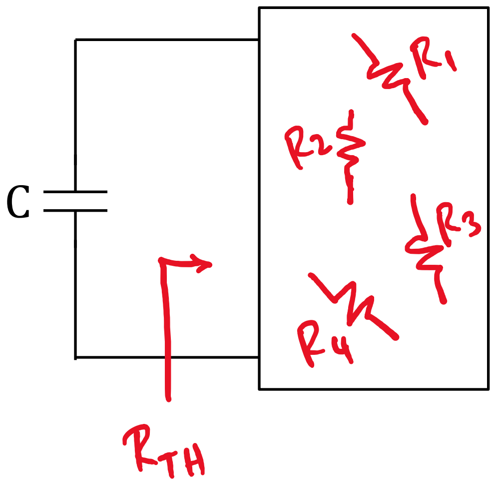
as $t \to \infty$



$$v_c(t) = \frac{R_2}{R_1+R_2} V_s \left(1 - e^{-t/\tau}\right)$$

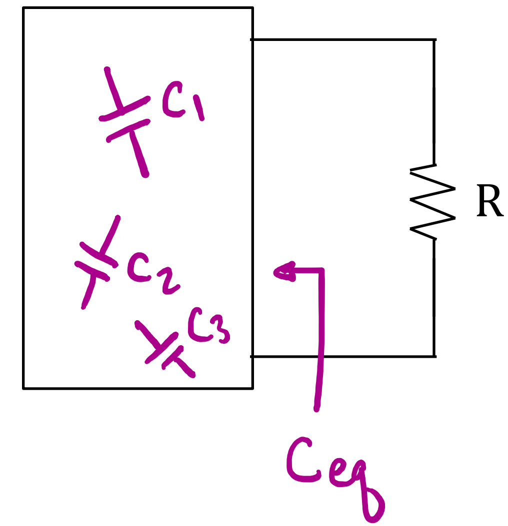
$$\tau = C R_{TH} = \frac{C R_1 R_2}{R_1+R_2}$$

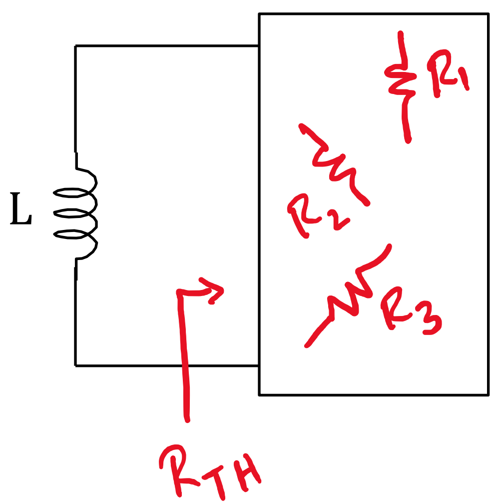
Time Constants for Complex Circuits



$$\tau = R_{TH} C$$

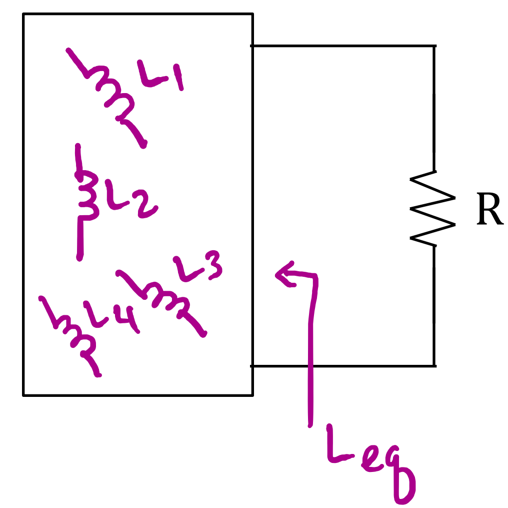
$$\tau = R C_{eq}$$



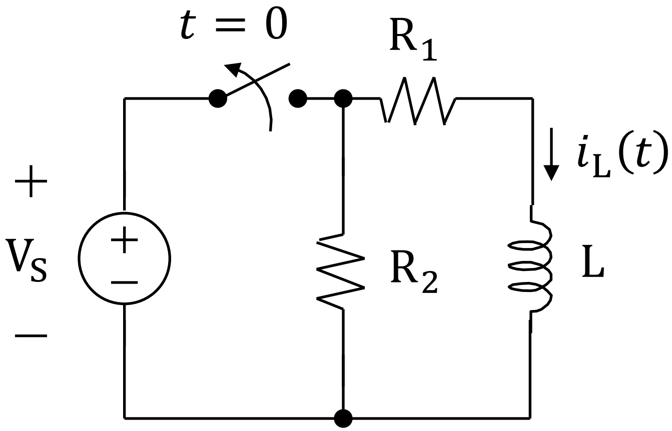


$$\tau = \frac{L}{R_{TH}}$$

$$\tau = \frac{L_{eq}}{R}$$

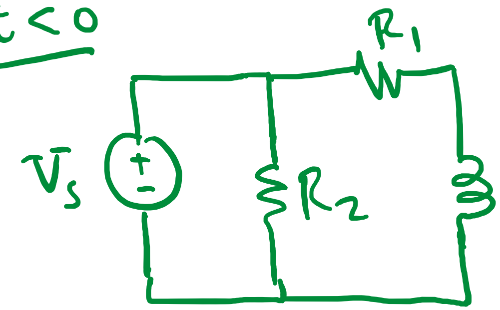


Another RL Circuit Example – Intuitive Approach

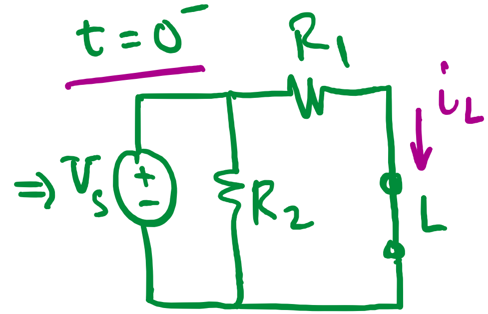


Find $i_L(t)$ for $t > 0$

$t < 0$



$t = 0^-$

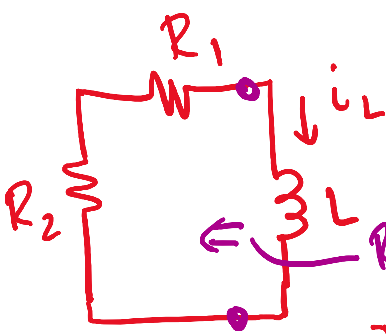


$i_L(0^-) = \frac{V_s}{R_1}$

Initial Condition: $i_L(0^+) = ?$ $\frac{V_s}{R_1}$

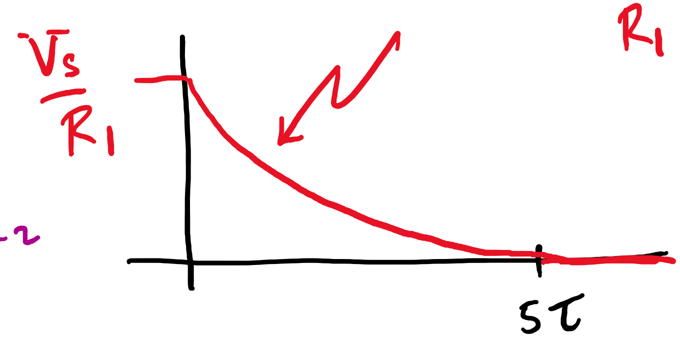
$= i_L(0^-)$

$t > 0$



$R_{TH} = R_1 + R_2$

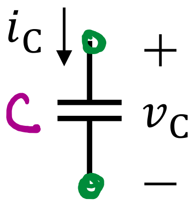
$i_L(0^+) = V_s/R_1$



$i_L(t) = \frac{V_s}{R_1} e^{-t/\tau}$

$\tau = \frac{L}{R_{TH}} = \frac{L}{R_1 + R_2}$

Treating Initial Condition Like Input - Capacitor



$v_C(0) = V_0$

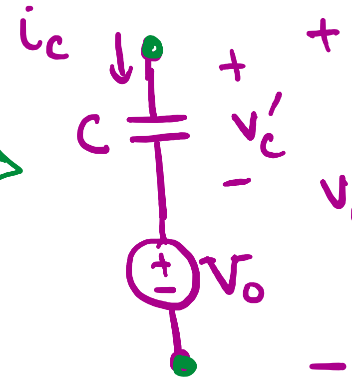
Same

$$i_C = C\frac{dv_C}{dt} \Rightarrow v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t')\,dt'$$

$$\boxed{v_C(t) = \frac{1}{C}\int_0^t i_C(t')\,dt' + v_C(0)}$$

$$v_c(t) = \frac{1}{C}\int_0^t i_c(t')\,dt' + V_0$$

Same



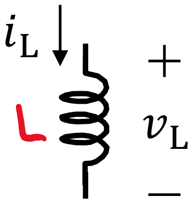
$v_c'(0) = 0$

$$v_c' = \frac{1}{C}\int_0^t i_c(t')\,dt' + \underbrace{v_c'(0)}_{=0}$$

$$v_c = v_c' + V_0$$

$$v_c = \frac{1}{C}\int_0^t i_c(t')\,dt' + V_0$$

Treating Initial Condition Like Input - Inductor



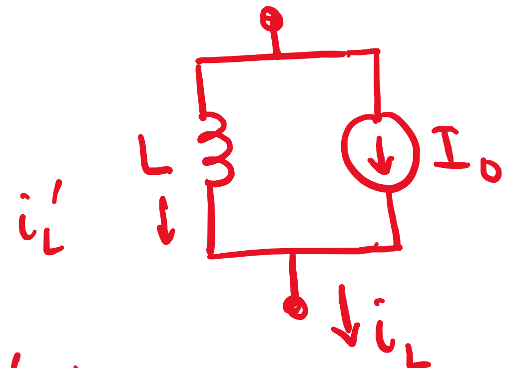
$i_L(0) = I_0$

Same

$$v_L = L\frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t')\,dt' + i_L(0)$$

Same

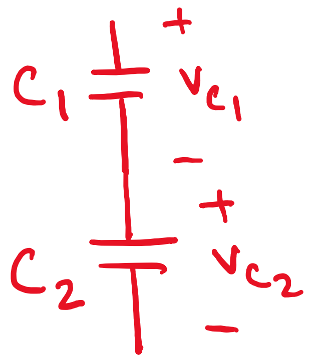


$i_L'(0) = 0$

$i_L = i_L' + I_0$

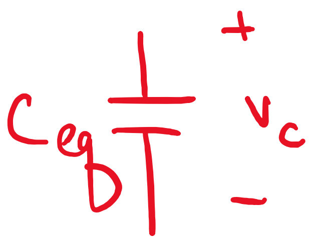
$$i_L = \frac{1}{L}\int_0^t v_L(t')\,dt' + \overset{0}{\underline{\underline{i_L'(0)}}} + I_0$$

More on Combining Capacitors and Inductors



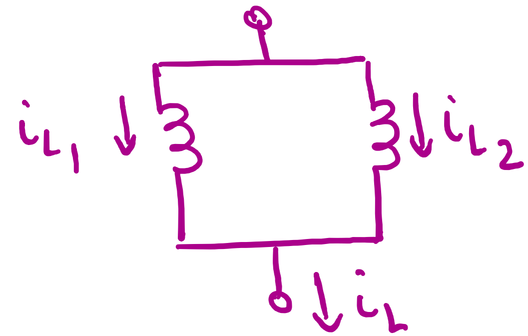
$v_{c_1}(0) = V_1$

$v_{c_2}(0) = V_2$



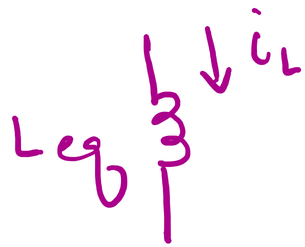
$v_c(0) = V_1 + V_2$

$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$



$i_{L_1}(0) = I_1$

$i_{L_2}(0) = I_2$



$i_L(0) = I_1 + I_2$

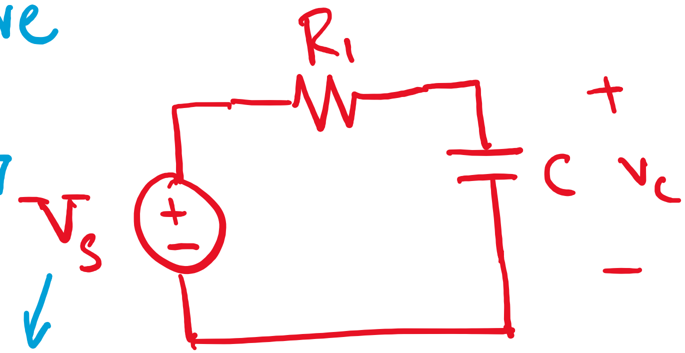
$L_{eq}\left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1}$

ZIR and ZSR Approach (Superposition)

No drive

ZIR – Zero Input Response

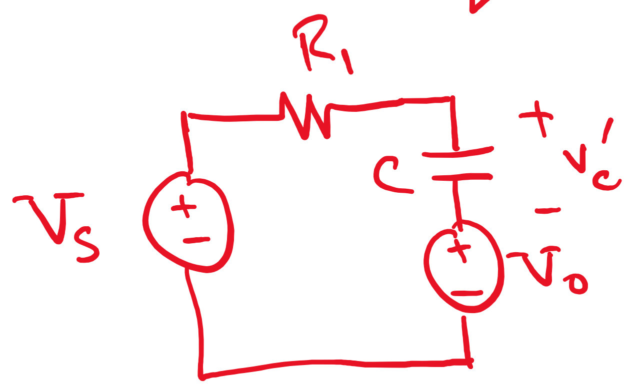
$V_s \to 0$



ZSR – Zero State Response

No initial condition

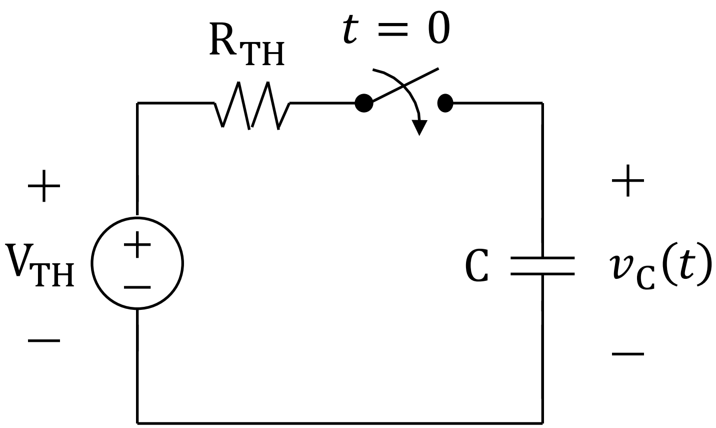
$v_c(0) = V_0 \to 0$



$v_c'(0) = 0$

Total Response = ZIR + ZSR

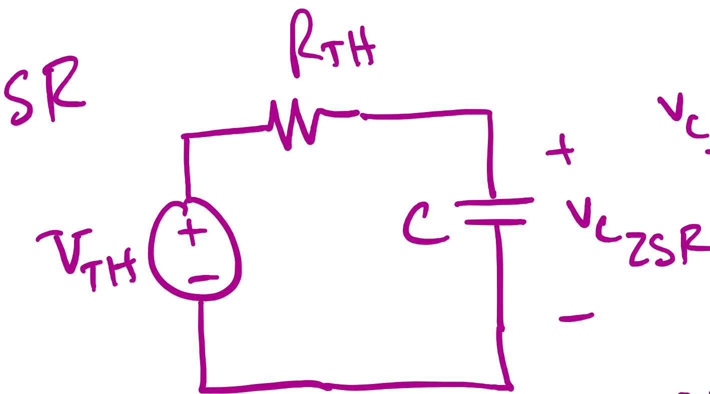
RC Circuit with Non-Zero Initial Condition



$v_C(0^-) = V_0$

$t > 0$

ZSR

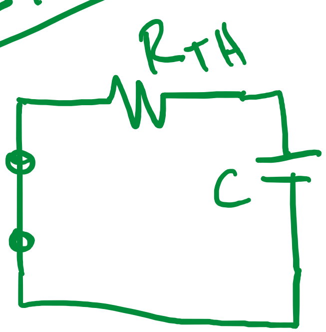


$v_{C_{ZSR}}(0) = 0$

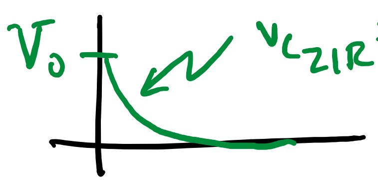
$v_{C_{ZSR}}(t) = V_{TH}(1 - e^{-t/\tau})$

$\tau = R_{TH} C$

ZIR



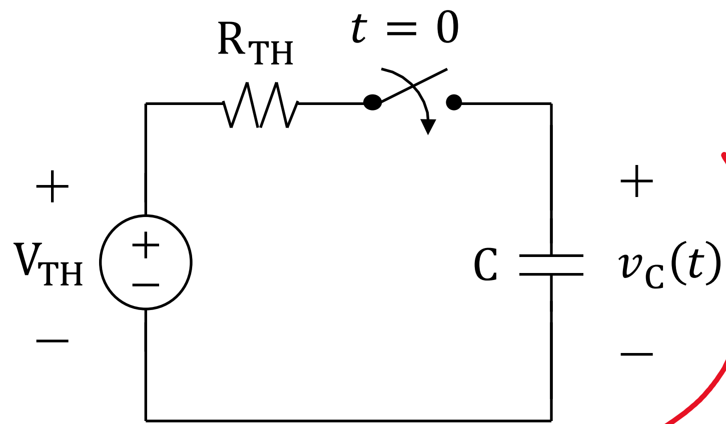
$v_{C_{ZIR}}(0) = V_0$



$v_{C_{ZIR}} = V_0 e^{-t/\tau}$

$\tau = R_{TH} C$

RC Circuit with Non-Zero Initial Condition (Cont.)



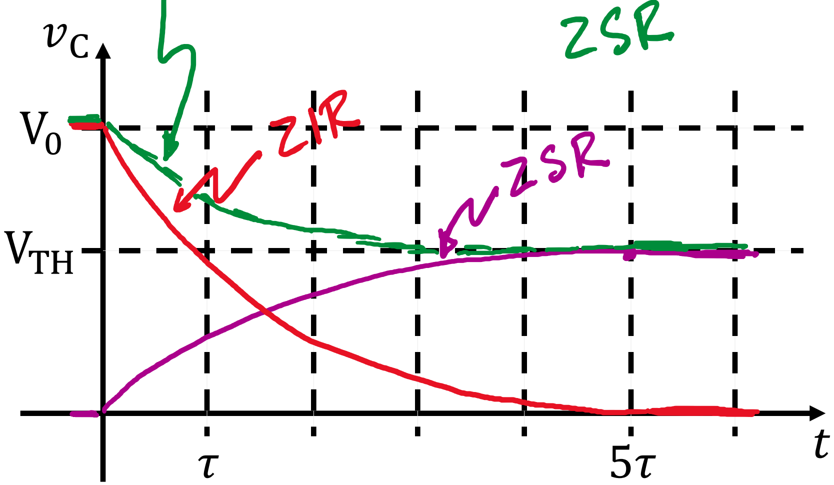
$v_C(0^-) = V_0$

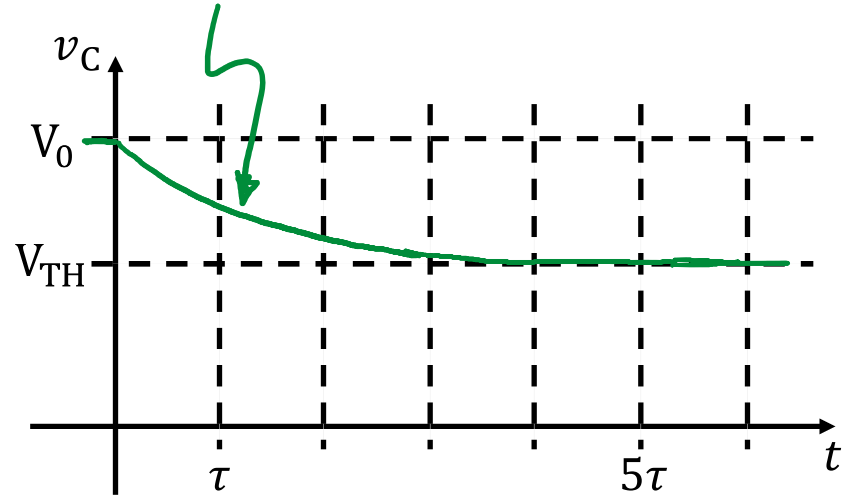
$v_c = v_{c_{ZIR}} + v_{c_{ZSR}}$

$v_c = V_0 e^{-t/\tau} + V_{TH}(1 - e^{-t/\tau})$

Total Response $\quad$ ZIR $\quad$ ZSR

$$\underbrace{v_C(t)}_{\text{Total Response}} = \overbrace{V_0 e^{-t/\tau}}^{\text{ZIR}} + \underbrace{V_{TH}(1 - e^{-t/\tau})}_{\text{ZSR}} \Longrightarrow v_C(t) = \overbrace{(V_0 - V_{TH})e^{-t/\tau}}^{\text{Natural Response}} + \overbrace{V_{TH}}^{\text{Driven Response}}$$





Total Response = ZIR + ZSR

Intuitive Approach with DC Drives - Summary

- Determine time constant $\tau = \begin{cases} \mathrm{R_{TH}C_{eq}} \\ \mathrm{L_{eq}/R_{TH}} \end{cases}$
- Determine initial value, using state variable continuity condition
- Determine final value, using steady state circuit
- Connect initial and final values with exponential $(\mathrm{Y_I} - \mathrm{Y_F})e^{-t/\tau}$

$$\boxed{v_C(t) = \mathrm{V_F} + (\mathrm{V_I} - \mathrm{V_F})e^{-t/\tau}}$$