ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 16

Capacitors and Inductors

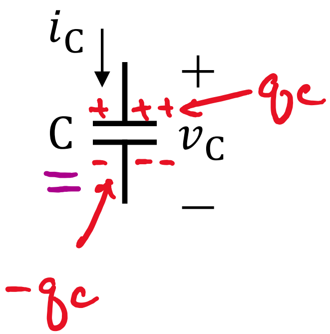
Announcements

- Recommended Reading:
  - Textbook Chapter 6

- Upcoming due dates:
  - Prelab 3 due by 12:20 pm on Tuesday March 5, 2019
  - Homework 3 due by 11:59 pm on Friday March 8, 2019
  - Lab report 3 due by 11:59 pm on Friday March 15, 2019

- Lab 3 is next week (starting Tuesday March 5, 2019)

Capacitors



Linear Capacitor

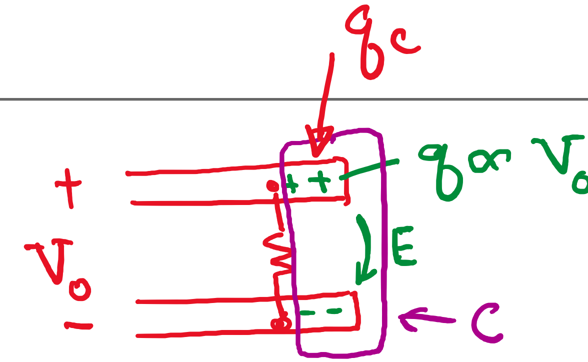
$$\boxed{i_C = C\,\frac{dv_C}{dt}}$$

constant

i_c

$\parallel$

$$q_c = C v_c \Rightarrow \frac{dq_c}{dt} = C\,\frac{dv_c}{dt}$$

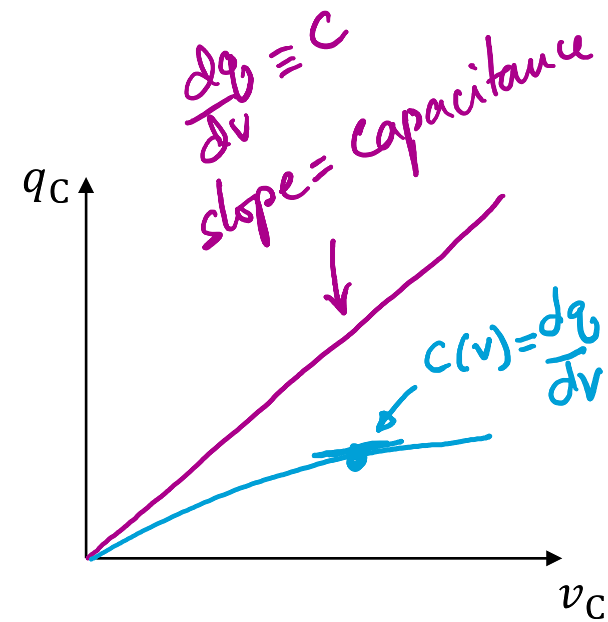


$$\boxed{C \equiv \frac{dq_C}{dv_C}}$$

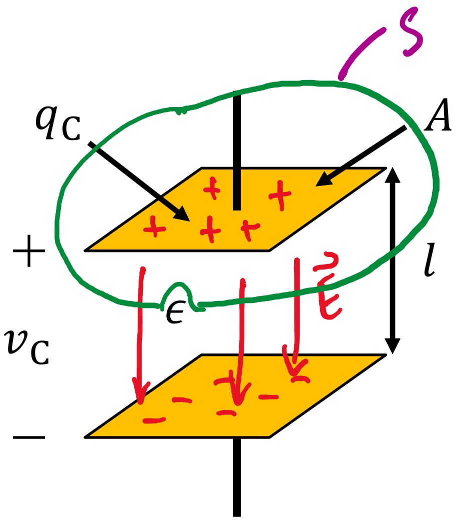
Capacitance measured in Farads [F] $\quad 1\ \text{F} \equiv 1\ \text{C}/\text{V}$

Columb

Volt



Capacitance of Parallel-Plate Capacitor



Gauss' Law: $\oiint \vec{D} \cdot \vec{ds} = \iiint \rho \, dv = q_c$

where: $\vec{D} = \epsilon \vec{E}$

$\vec{E} = \frac{V_c}{l}$

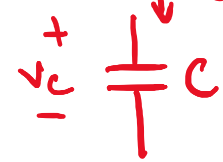
$$\oiint_S \epsilon \frac{V_c}{l} \vec{ds} = q_c$$

$$\frac{\epsilon V_c A}{l} = q_c$$

$$\boxed{C \equiv \frac{dq_c}{dV_c} = \frac{\epsilon A}{l}}$$

Energy Stored in a Capacitor

Power ≡ Time Rate of Change of Energy $\qquad p = \dfrac{dw}{dt}$



$i_c = C\dfrac{dv_c}{dt}$

$t_1 \rightarrow v_c = V_1$

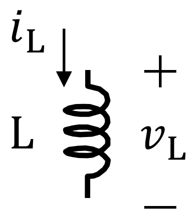
Energy: $\quad w_C(t_1) = \displaystyle\int_{-\infty}^{t_1} p_C\, dt = \int_{-\infty}^{t_1} v_c i_c\, dt = \int_{-\infty}^{t_1} v_c C \frac{dv_c}{\cancel{dt}} \cancel{dt}$

$-\infty \rightarrow v_c = 0$

$$w_c(t_1) = C\int_0^{V_1} v_c\, dv_c = \frac{1}{2} C V_c^2$$

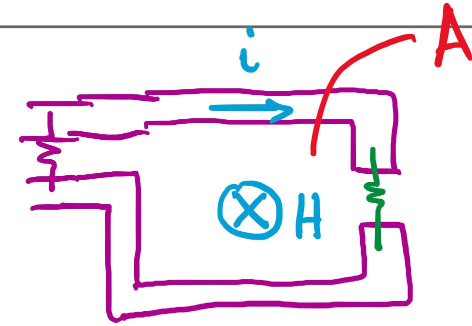
$$\boxed{w_c = \frac{1}{2} C V_c^2}$$

Inductors



Linear Inductor

$$\boxed{v_L = L\frac{di_L}{dt}}$$



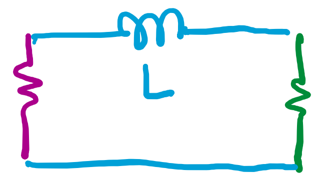
Faraday's Law $v = \frac{d\lambda}{dt}$

$\lambda \propto i \Rightarrow \underline{\lambda = Li}$

Flux Linkage

$\lambda = NA\mu H$

# of turns

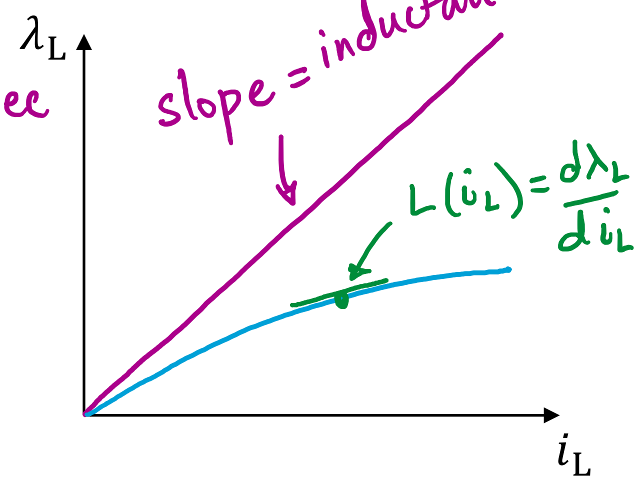


$$\boxed{L \equiv \frac{d\lambda_L}{di_L}}$$

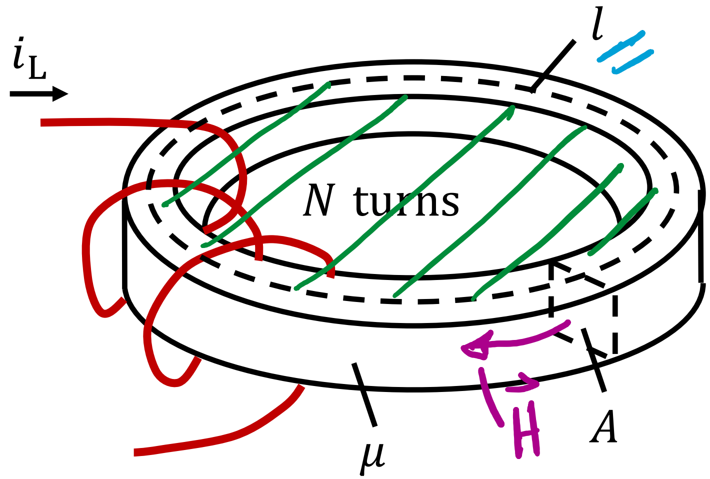
Inductance measured in Henrys [H] $\quad 1\ \text{H} \equiv 1\ ^{\text{Wb}}/_{\text{A}}$

Weber = Volt·sec

Amp



Inductance of Toroidal Inductor



Ampere's Law: $\underbrace{\oint \vec{H}\cdot\vec{dl}}_{H\cdot l} = \underbrace{\iint \vec{J}\cdot\vec{ds}}_{N i_L} + \cancelto{0}{\frac{\partial}{\partial t}\iint \vec{D}\cdot\vec{ds}}$

$\Rightarrow H = \frac{N i_L}{l}$

$B = \mu H = \frac{\mu N i_L}{l}$

$\lambda_L = NBA = \frac{\mu N^2 A i_L}{l}$

$L \equiv \frac{d\lambda_L}{di_L} = \frac{\mu A N^2}{l}$

Compare to: $C = \frac{\epsilon A}{l}$

Energy Stored in an Inductor

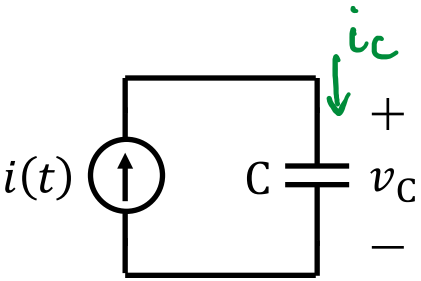
Energy: $w_L(t_1) = \int_{-\infty}^{t_1} p_L \, dt$ ⇨ $w_L(t_1) = \int_{-\infty}^{t_1} i_L v_L \, dt$

$v_L = L \frac{di_L}{dt}$

$I_1 = i_L(t_1)$

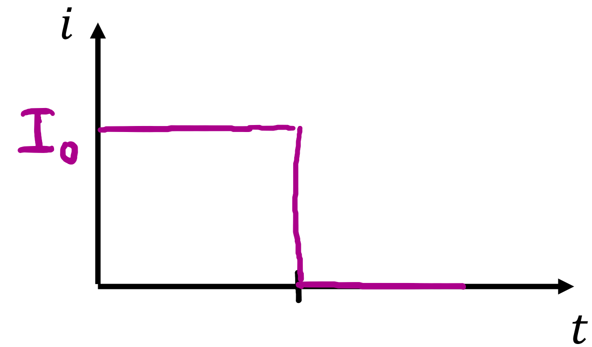
$\Rightarrow \boxed{w_L = \frac{1}{2} L I_1^2}$

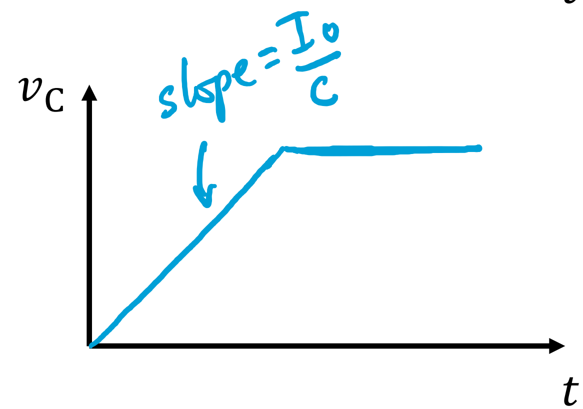
Example Capacitor Circuit



$$i_c = C\frac{dv_c}{dt}$$

$$\frac{dv_c}{dt} = \frac{I_0}{C}$$



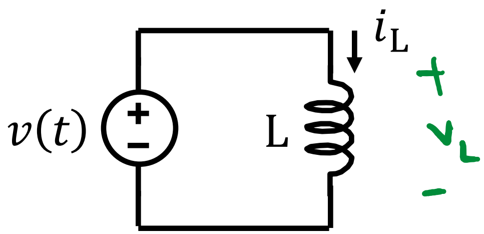


$$\frac{dv_c}{dt} = \frac{i_c}{C}$$

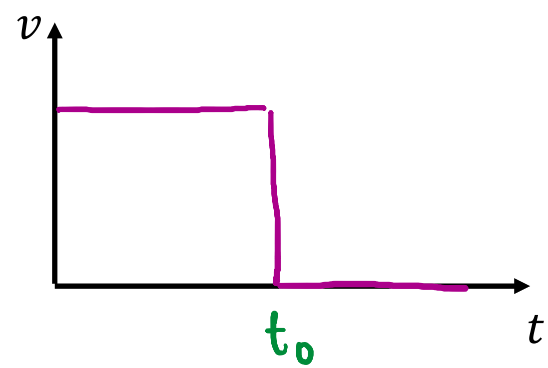
$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c\,dt$$

$$\boxed{v_c(t) = \frac{1}{C}\int_{t_0}^{t} i_c\,dt + v_c(t_0)}$$

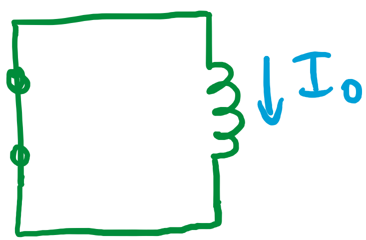
Example Inductor Circuit

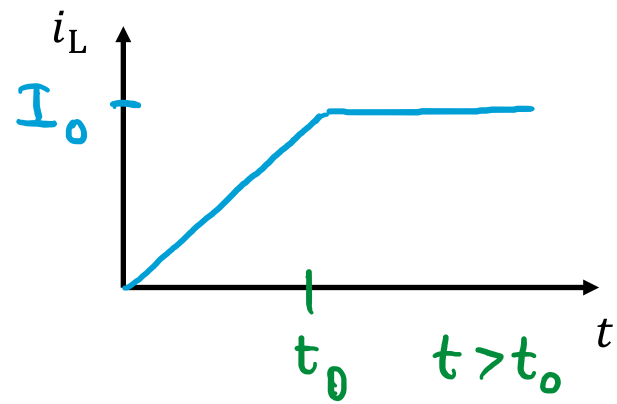


$$v_L = L\frac{di_L}{dt}$$



$t > t_0$





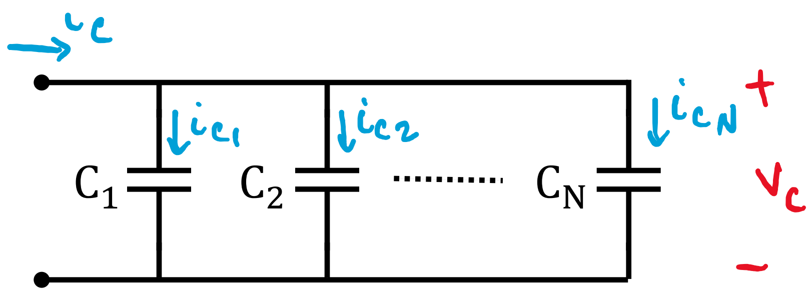
$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L\,dt$$

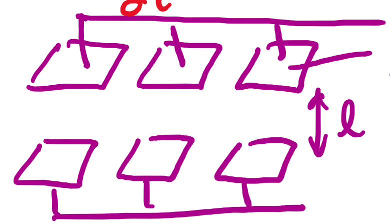
$$\Rightarrow \boxed{i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L\,dt + i_L(t_0)}$$

Series and Parallel Combinations of Capacitors

Parallel

$$i_c = i_{c_1} + i_{c_2} + \cdots + i_{c_N} = C_1\frac{dv_c}{dt} + C_2\frac{dv_c}{dt} + \cdots + C_N\frac{dv_c}{dt}$$



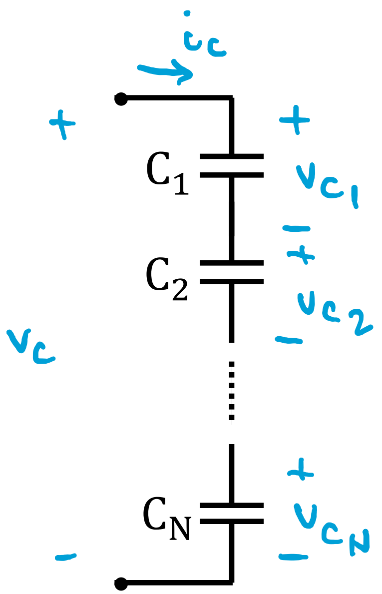


$$C = \frac{\epsilon A}{\ell}$$

$$\boxed{C_{eq} = C_1 + C_2 + \cdots C_N}$$

$$i_c = \underbrace{(C_1 + C_2 + \cdots + C_N)}_{C_{eq}}\frac{dv_c}{dt}$$

Series



$$v_c = v_{c_1} + v_{c_2} + \cdots + v_{c_N}$$

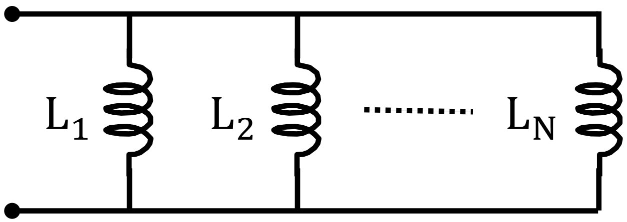
$$v_c = \frac{1}{C_1}\int_{-\infty}^{t} i_c\,dt + \frac{1}{C_2}\int_{-\infty}^{t} i_c\,dt + \cdots + \frac{1}{C_N}\int_{-\infty}^{t} i_c\,dt$$

$$v_c = \underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)}_{\frac{1}{C_{eq}}}\int_{-\infty}^{t} i_c\,dt$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}}$$

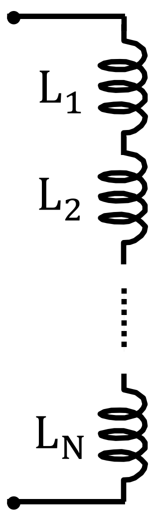
Series and Parallel Combinations of Inductors

Parallel



$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_N}}$$

Series



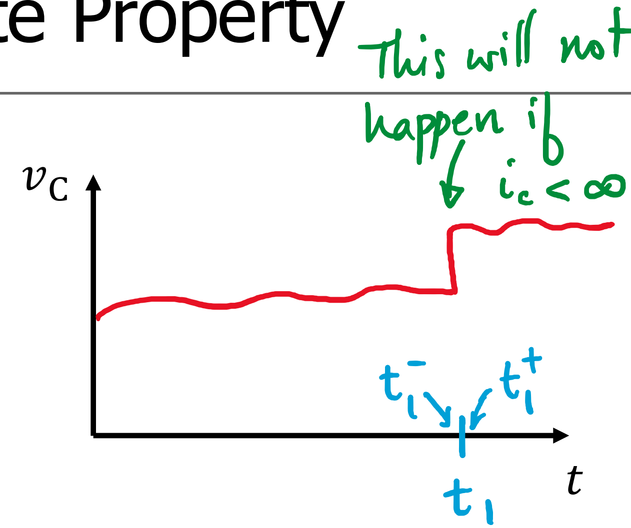
$$\boxed{L_{eq} = L_1 + L_2 + \cdots + L_N}$$

Continuity Condition – State Property

Capacitor

$$i_C = C\frac{dv_C}{dt}$$

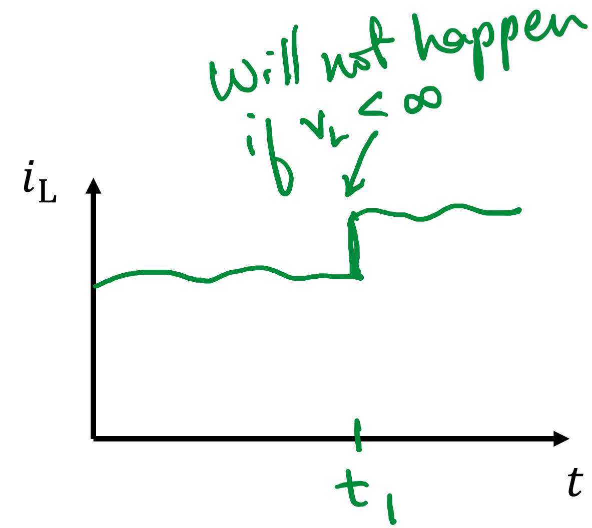
$v_c(t_1^+) = v_c(t_1^-)$ if $i_c < \infty$



Inductor

$$v_L = L\frac{di_L}{dt}$$

$i_L(t_1^+) = i_L(t_1^-)$ if $v_L < \infty$



State Property of Capacitors and Inductors

Capacitor

$$i_C = C\frac{dv_C}{dt}$$

$$v_C(t_1) = v_C(t_0) + \frac{1}{C}\int_{t_0}^{t_1} i_C(t)\,dt$$

Inductor

$$v_L = L\frac{di_L}{dt}$$

$$i_L(t_1) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t_1} v_L(t)\,dt$$

- Capacitors and inductors exhibit memory
  - Future capacitor voltage depends on past capacitor voltage
  - Future inductor current depends on past inductor current
- Circuits that contain capacitors and/or inductors exhibit dynamic behavior
  - Solving such circuits requires solving differential equations

State Variables

State Variables

Capacitor

Capacitor voltage is a state variable
– it has memory (remembers its previous state)

$v_C(t)$

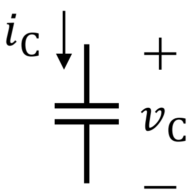
Inductor

Inductor current is a state variable
– it has memory (remembers its previous state)

$i_L(t)$

Capacitors and Inductors Summary

Capacitor $\mathrm{C} \equiv \dfrac{dq_\mathrm{C}}{dv_\mathrm{C}}$



$$i_\mathrm{C} = \mathrm{C}\frac{dv_\mathrm{C}}{dt}$$

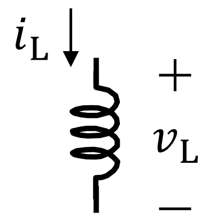
$$v_\mathrm{C}(t_1) = v_\mathrm{C}(t_0) + \frac{1}{\mathrm{C}}\int_{t_0}^{t_1} i_\mathrm{C}(t)\,dt$$

$$v_\mathrm{C}(t_1^+) = v_\mathrm{C}(t_1^-) \quad \text{if} \quad i_\mathrm{C}(t_1) < \infty$$

Energy Stored

$$w_\mathrm{C} = \frac{1}{2}\mathrm{C}v_\mathrm{C}^2$$

Inductor $\mathrm{L} \equiv \dfrac{d\lambda_\mathrm{L}}{di_\mathrm{L}}$



$$v_\mathrm{L} = \mathrm{L}\frac{di_\mathrm{L}}{dt}$$

$$i_\mathrm{L}(t_1) = i_\mathrm{L}(t_0) + \frac{1}{\mathrm{L}}\int_{t_0}^{t_1} v_\mathrm{L}(t)\,dt$$

$$i_\mathrm{L}(t_1^+) = i_\mathrm{L}(t_1^-) \quad \text{if} \quad v_\mathrm{L}(t_1) < \infty$$

Energy Stored

$$w_\mathrm{L} = \frac{1}{2}\mathrm{L}i_\mathrm{L}^2$$