

ECE/ENGRD 2100

Introduction to Circuits for ECE

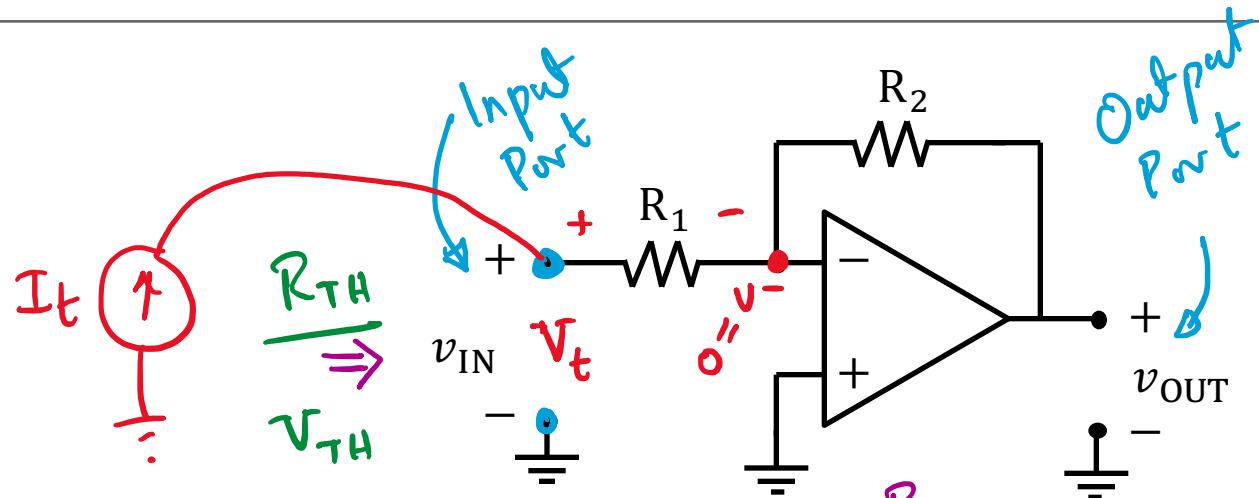
Lecture 15

Input and Output Resistance,
Differential and Common Mode Signals
and Op-Amp Non-Idealities

Announcements

- Recommended Reading:
 - Textbook Chapter 5
- Upcoming due dates:
 - Lab report 2 due by 11:59 pm on Wednesday 27, 2019
 - Prelab 3 due by 12:20 pm on Tuesday March 5, 2019
 - Homework 3 due by 11:59 pm on Friday March 8, 2019
 - Lab report 3 due by 11:59 pm on Friday March 15, 2019
- Lab 3 is next week (starting Tuesday March 5, 2019)

Input Resistance of Inverting Amplifier

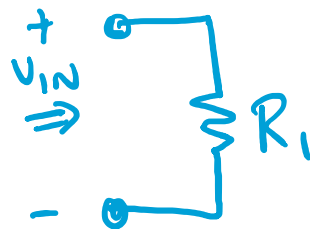


$$V_t = R_1 I_t$$

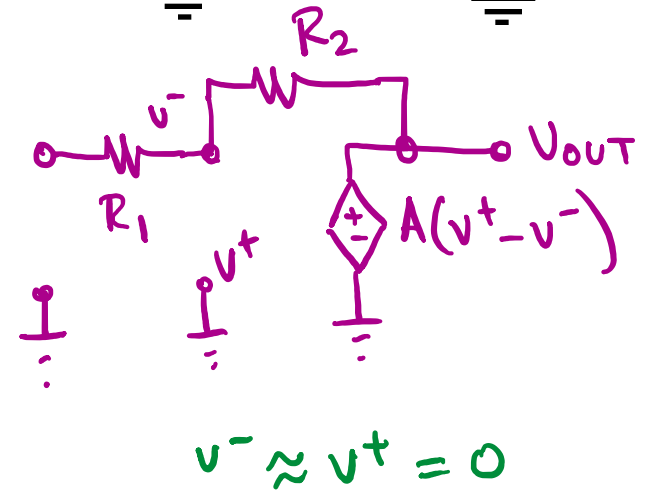
$$\Rightarrow R_{TH} \equiv \frac{V_t}{I_t} = R_1$$

$$R_{TH} = R_1$$

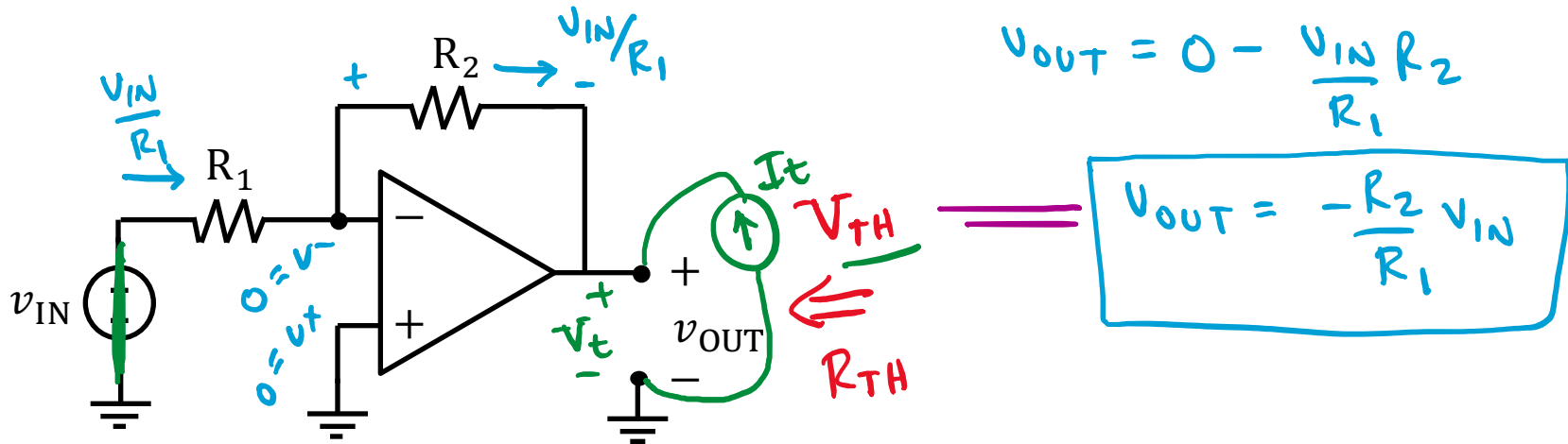
Input Port



$$V_{TH} = V_{OC} = 0$$

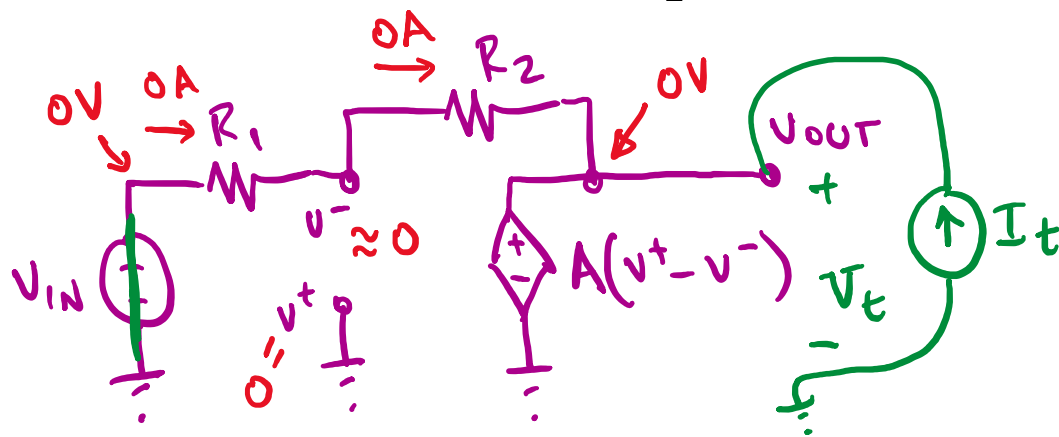


Output Resistance of Inverting Amplifier



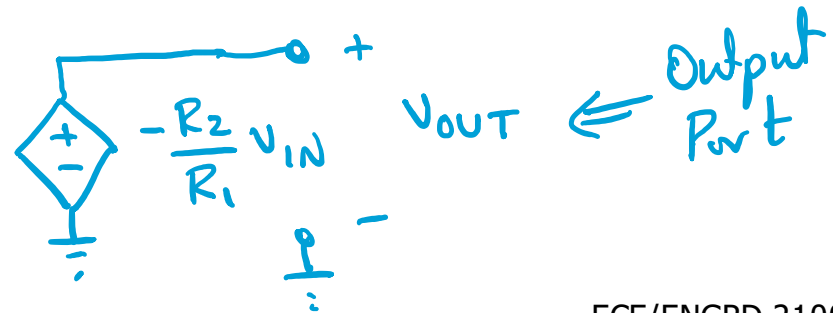
$$v_{OUT} = 0 - \frac{v_{IN} R_2}{R_1}$$

$$v_{OUT} = -\frac{R_2}{R_1} v_{IN}$$

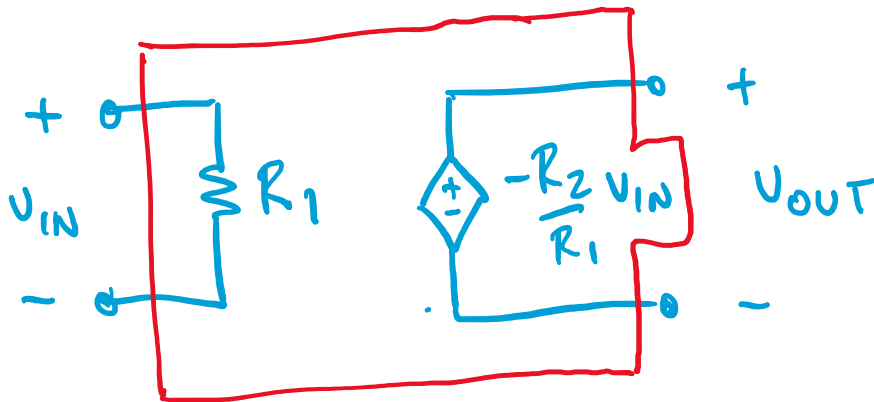
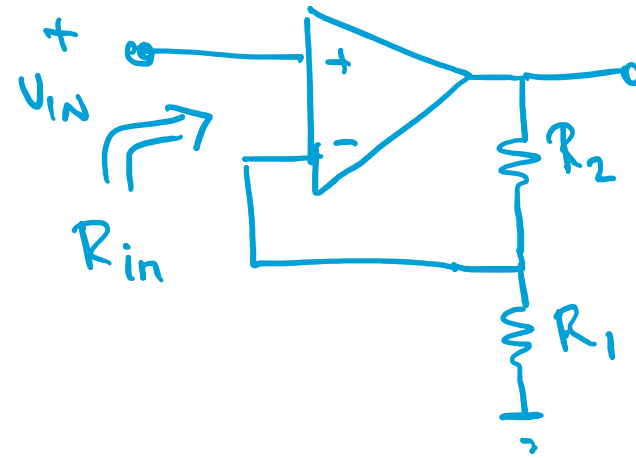
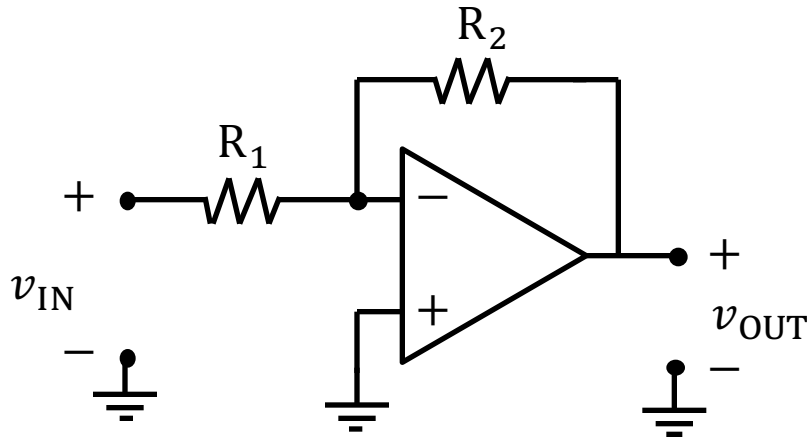


$$V_t = 0$$

$$\Rightarrow R_{TH} \equiv \frac{V_t}{I_t} = 0$$

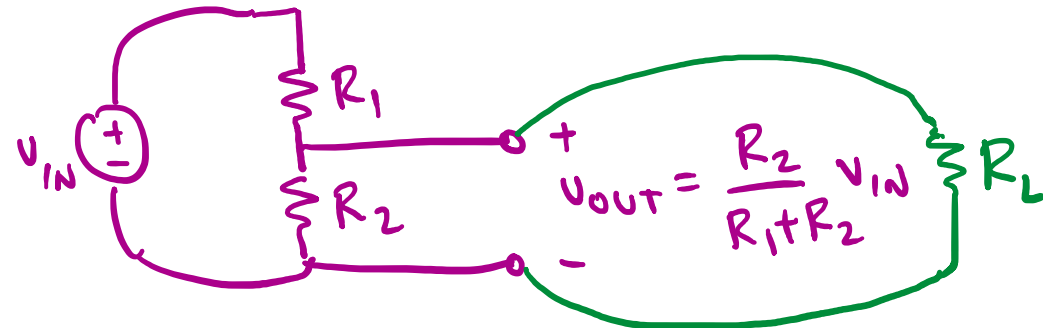
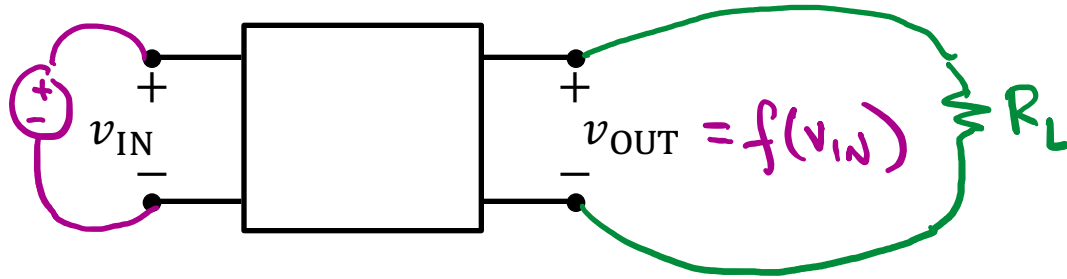


Two-Port Model of Inverting Amplifier

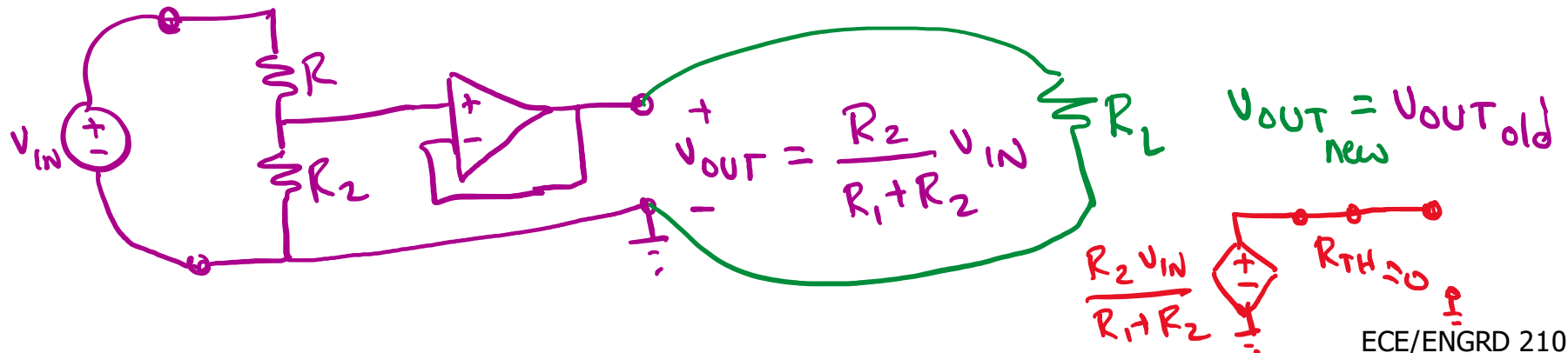


Inverting Amp

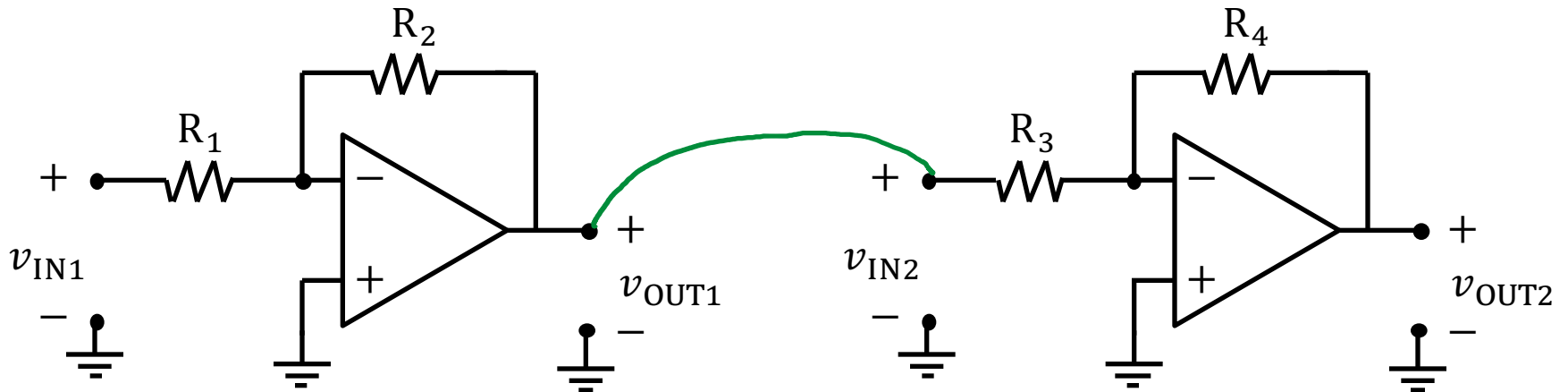
Loading Effect



$$v_{OUT} = \frac{R_2 \parallel R_L}{R_1 + R_2 \parallel R_L} \cdot v_{IN}$$



Cascaded Op-Amp Circuits



$$v_{OUT1} = g(v_{IN1})$$

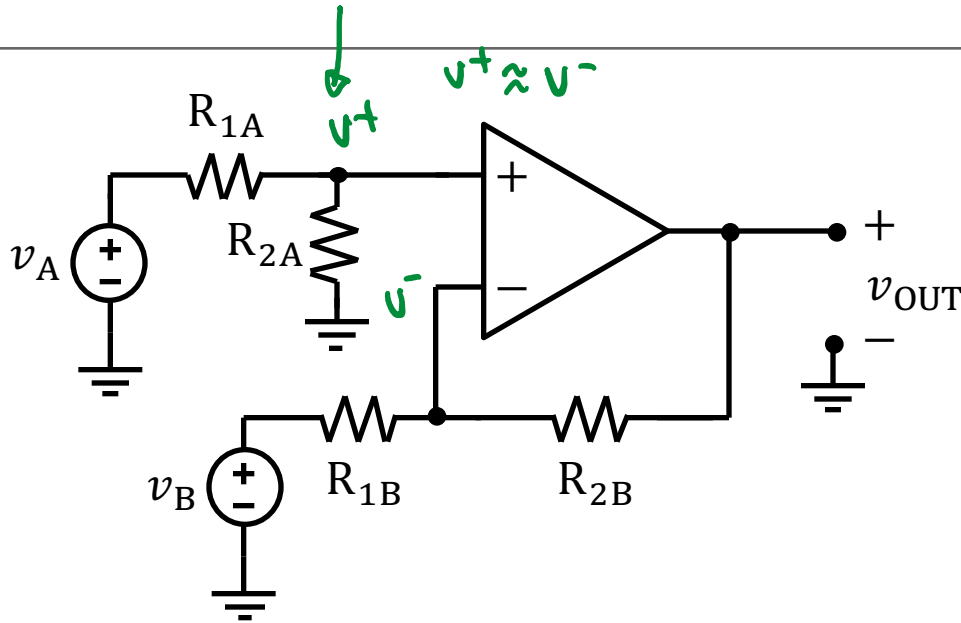
$$v_{OUT1} = -\frac{R_2}{R_1} v_{IN1}$$

$$v_{OUT2} = f(v_{IN2})$$

$$v_{OUT2} = -\frac{R_4}{R_3} v_{IN2} = v_{OUT1}$$

$$\Rightarrow v_{OUT2} = -\frac{R_4}{R_3} \left(-\frac{R_2}{R_1}\right) v_{IN1} = \frac{R_4 R_2}{R_3 R_1} v_{IN1}$$

Differential Amplifier



$$R_{1A} = R_{1B} \quad \& \quad R_{2A} = R_{2B}$$

$$\parallel \quad \parallel$$

$$R_1 \quad R_2$$

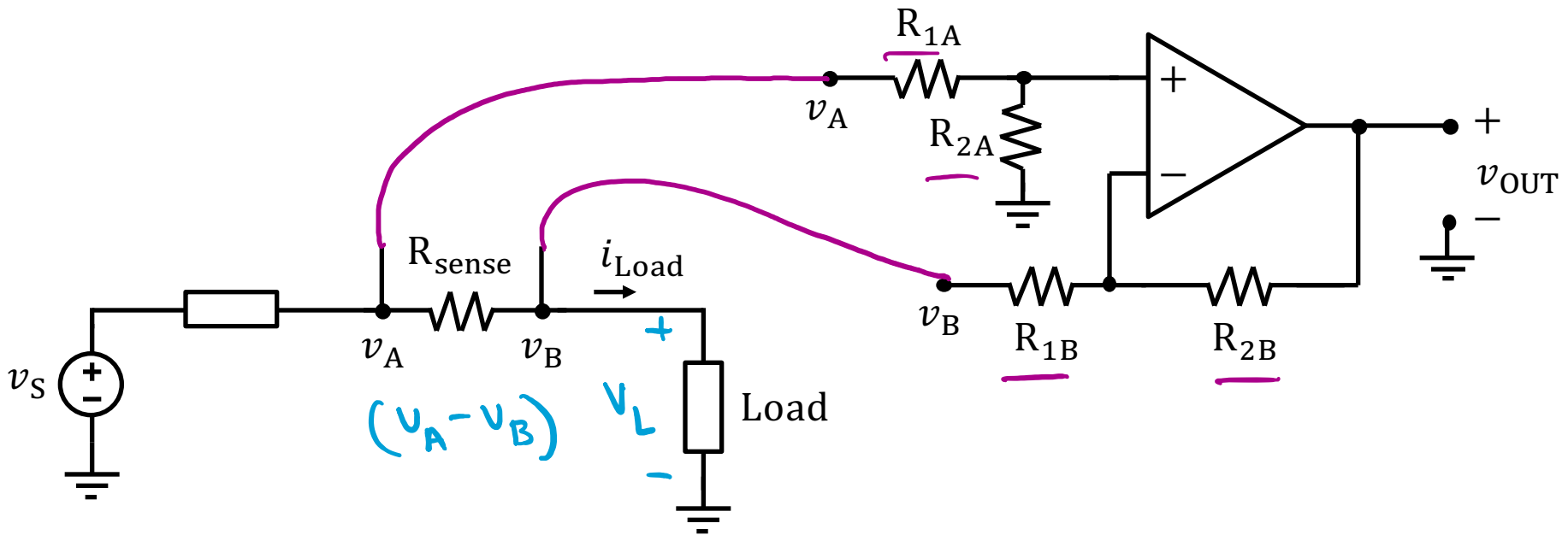
$$v^+ = \frac{R_{2A}}{R_{1A} + R_{2A}} v_A$$

$$v^- = \frac{R_{2B}}{R_{1B} + R_{2B}} v_B + \frac{R_{1B}}{R_{1B} + R_{2B}} v_{OUT}$$

$$v^+ \approx v^-$$

$$\Rightarrow v_{OUT} = \frac{R_{1B} + R_{2B}}{R_{1B}} \left(\frac{R_{2A}}{R_{1A} + R_{2A}} \cdot v_A - \frac{R_{2B}}{R_{1B} + R_{2B}} \cdot v_B \right)$$

Differential Mode and Common Mode Signals



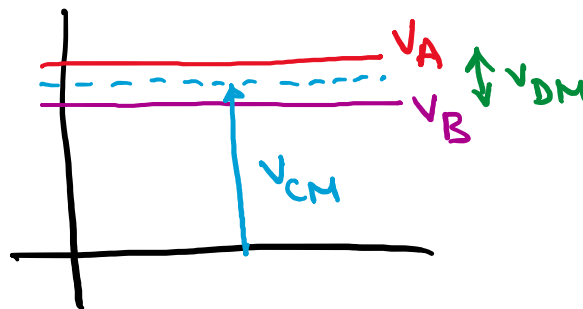
Differential mode signal

$$v_{DM} = v_A - v_B$$

Common mode signal

$$v_{CM} = \frac{v_A + v_B}{2}$$

$$v_L \gg (v_A - v_B)$$



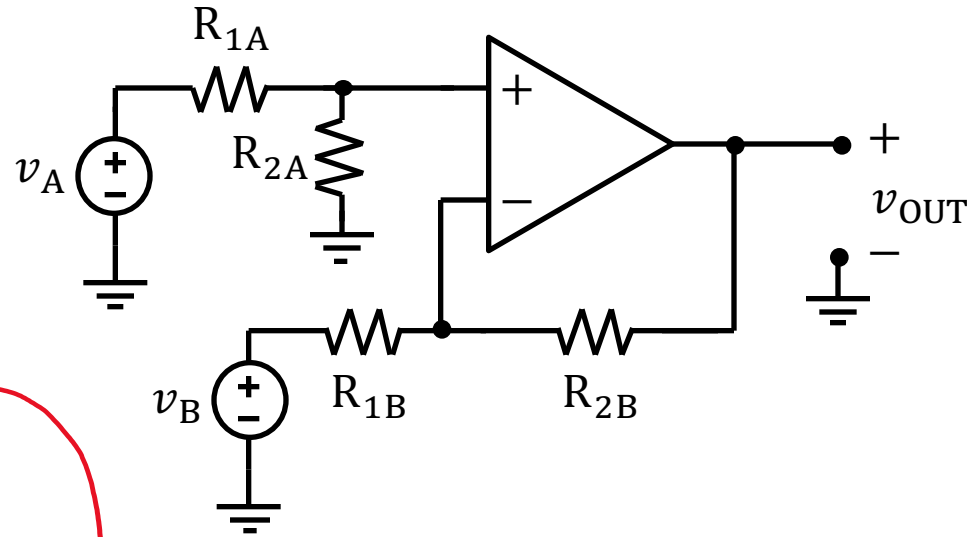
$$v_A = v_{CM} + \frac{v_{DM}}{2}$$

$$v_B = v_{CM} - \frac{v_{DM}}{2}$$

Differential Mode and Common Mode Gain

$$v_A = v_{CM} + \frac{v_{DM}}{2}$$

$$v_B = v_{CM} - \frac{v_{DM}}{2}$$



$$v_{OUT} = \frac{R_{1B} + R_{2B}}{R_{1B}} \left[\frac{R_{2A}}{R_{1A} + R_{2A}} v_A - \frac{R_{2B}}{R_{1B} + R_{2B}} v_B \right]$$

want to keep

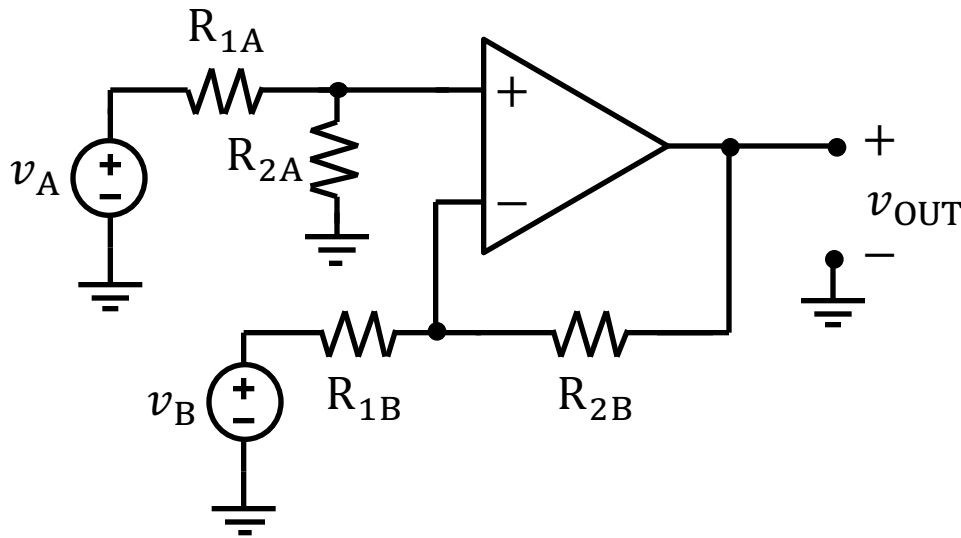
$$v_{OUT} = \frac{R_{1B} + R_{2B}}{R_{1B}} \left(\frac{R_{2A}}{R_{1A} + R_{2A}} - \frac{R_{2B}}{R_{1B} + R_{2B}} \right) v_{CM} + \frac{R_{1B} + R_{2B}}{R_{1B}} \left(\frac{R_{2A}}{R_{1A} + R_{2A}} + \frac{R_{2B}}{R_{1B} + R_{2B}} \right) \frac{v_{DM}}{2}$$

Want to reject

\equiv
 G_{CM}
 common-mode gain

\equiv
 G_{DM}
 differential-mode gain

Common Mode Rejection Ratio



Common mode rejection ratio

$$\text{CMRR} \equiv \frac{G_{\text{DM}}}{G_{\text{CM}}}$$

$$v_{\text{OUT}} = \underbrace{\frac{R_{1B} + R_{2B}}{R_{1B}} \left(\frac{R_{2A}}{R_{1A} + R_{2A}} - \frac{R_{2B}}{R_{1B} + R_{2B}} \right)}_{G_{\text{CM}}} v_{\text{CM}} + \underbrace{\frac{R_{1B} + R_{2B}}{R_{1B}} \left(\frac{R_{2A}}{R_{1A} + R_{2A}} + \frac{R_{2B}}{R_{1B} + R_{2B}} \right)}_{G_{\text{DM}}} \frac{v_{\text{DM}}}{2}$$

$$G_{\text{DM}} \equiv \left. \frac{v_{\text{OUT}}}{v_{\text{DM}}} \right|_{v_{\text{CM}}=0}$$

$$= \frac{R_2}{R_1}$$

$$= 0$$

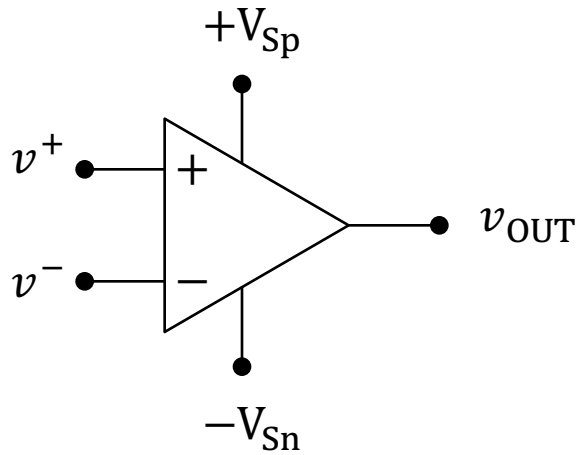
$$G_{\text{CM}} \equiv \left. \frac{v_{\text{OUT}}}{v_{\text{CM}}} \right|_{v_{\text{DM}}=0}$$

$$= 0$$

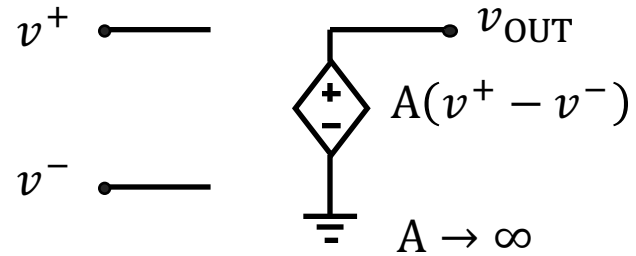
if $R_{1A} = R_{1B} = R_1$
 $R_{2A} = R_{2B} = R_2$
 CMRR = ∞

$$\frac{G_{\text{DM}}}{G_{\text{CM}}} = \frac{\frac{R_2}{R_1}}{0} = \infty$$

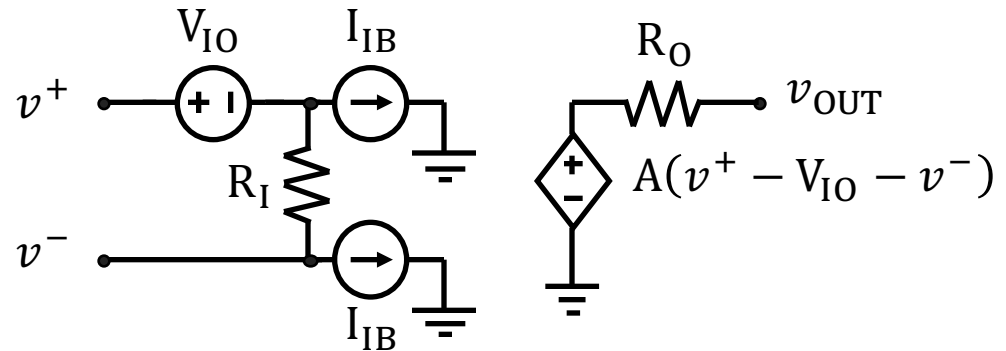
Op-Amp Non-Idealities



Ideal Op-Amp

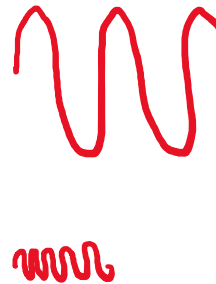
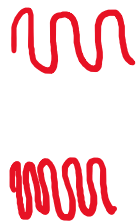
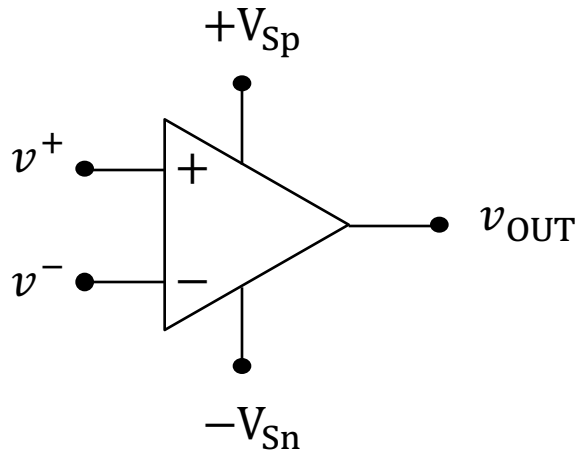


Real Op-Amp



$$-V_{Sn} + V_{Mn} \leq v_{OUT} \leq V_{Sp} - V_{Mp}$$

Op-Amp Bandwidth



Op-Amp gain starts decreasing beyond a certain frequency and crosses 1 at a frequency referred to as its bandwidth



This is caused by capacitive effects