ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 14

Implementing Linear and Non-Linear Functions
Using Op-Amps

Announcements

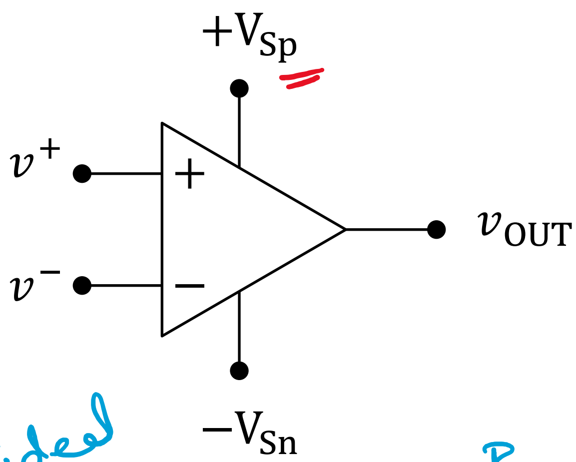
- Recommended Reading:
  - Textbook Chapter 5

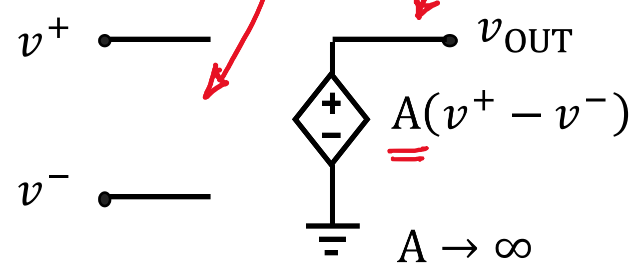
- Upcoming due dates:
  - Lab report 2 due by 11:59 pm on Wednesday 27, 2019

Ideal Op-Amp

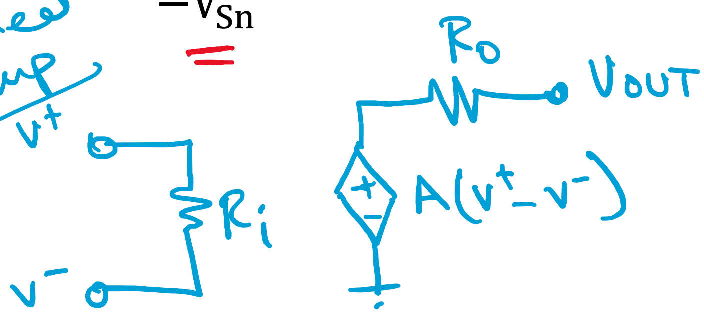
Open ckt

Output resistance zero





Non-ideal Op Amp

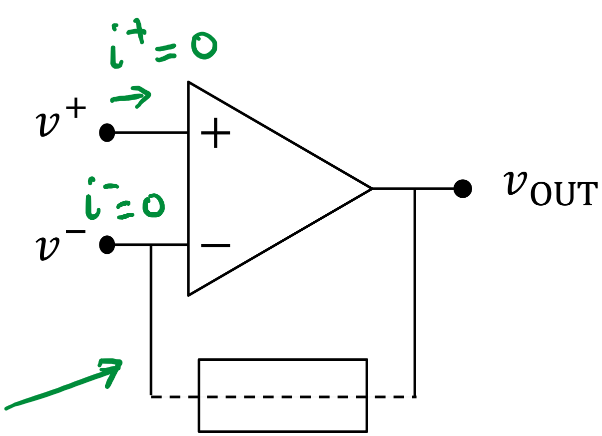


Ideal Op-Amp

- Infinite Gain, i.e., $A \to \infty$
- Infinite Input Resistance
- Zero Output Resistance
- Output v_{OUT} can take any value

Ideal Op-Amp Under Negative Feedback

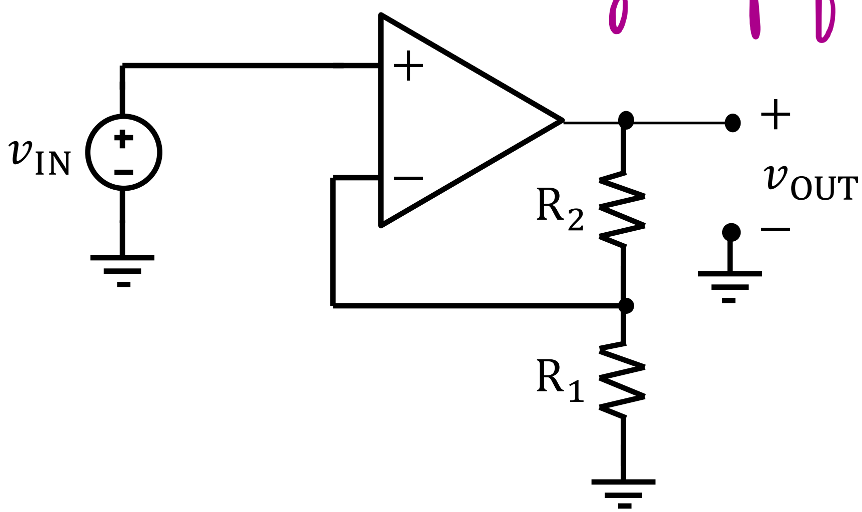
- Virtual Short: $v^- \approx v^+$
- Infinite Input Resistance: $i^- = 0$ and $i^+ = 0$



Negative Feedback

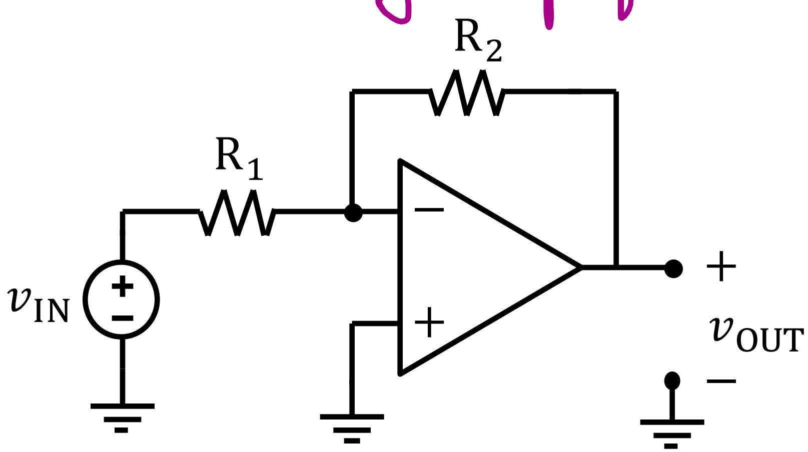
Op-Amp Amplifiers

Non-Inverting Amplifier



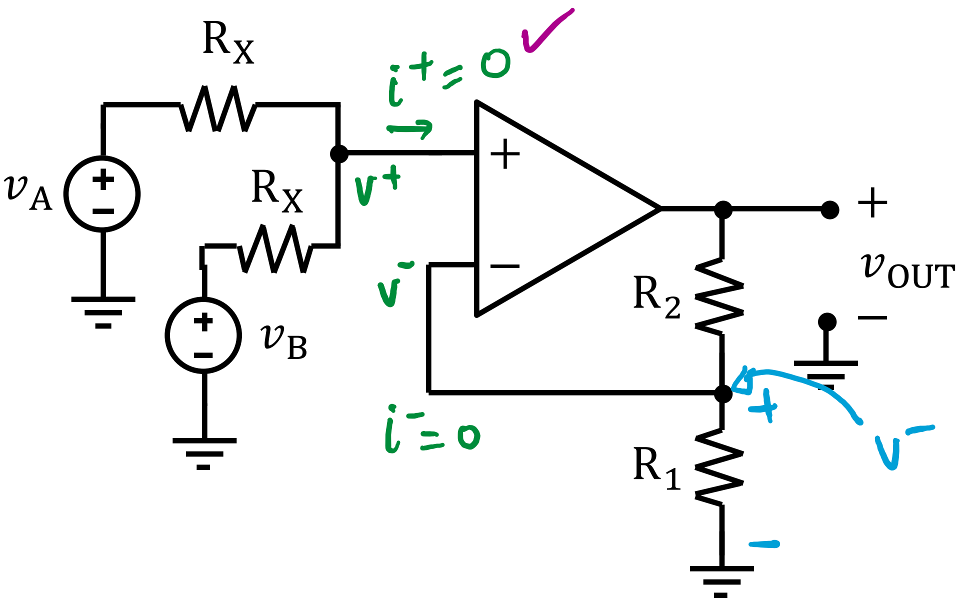
$$v_{OUT} = \left(1 + \frac{R_2}{R_1}\right) v_{IN}$$

Inverting Amplifier



$$v_{OUT} = -\frac{R_2}{R_1} v_{IN}$$

Another Linear Operation with Op-Amp



$v^+ \approx v^-$

$$v^+ = \frac{R_x}{R_x + R_x} v_A + \frac{R_x}{R_x + R_x} v_B = \frac{v_A}{2} + \frac{v_B}{2}$$

$$v^- = \frac{R_1}{R_1 + R_2} v_{OUT}$$

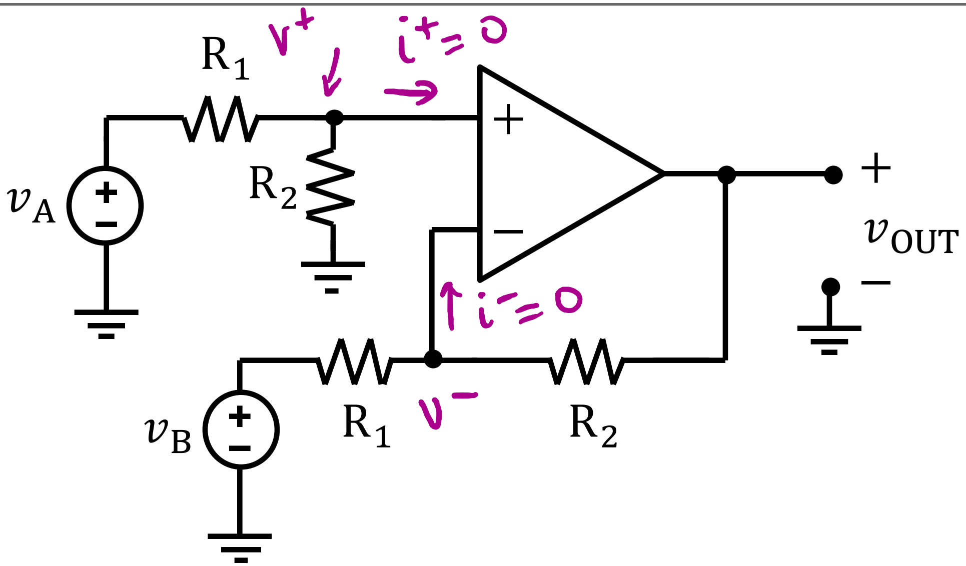
$v^+ \approx v^-$

$$\Rightarrow \frac{v_A + v_B}{2} = \frac{R_1}{R_1 + R_2} v_{OUT}$$

$$\Rightarrow v_{OUT} = \underbrace{\left(1 + \frac{R_2}{R_1}\right)\left(\frac{v_A + v_B}{2}\right)}_{\text{Averaging Ckt}}$$

if Pick $R_1 = R_2 \Rightarrow$ $\boxed{v_{OUT} = v_A + v_B}$ ← Adder

Yet Another Linear Operation with Op-Amp



$$v^+ = \frac{R_2}{R_1+R_2} v_A$$

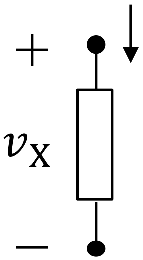
$$v^- = \frac{R_1}{R_1+R_2} v_{OUT} + \frac{R_2}{R_1+R_2} v_B$$

$$v^+ \approx v^- \Rightarrow \frac{R_1}{\cancel{R_1+R_2}} v_{OUT} + \frac{R_2}{\cancel{R_1+R_2}} v_B = \frac{R_2}{\cancel{R_1+R_2}} v_A$$

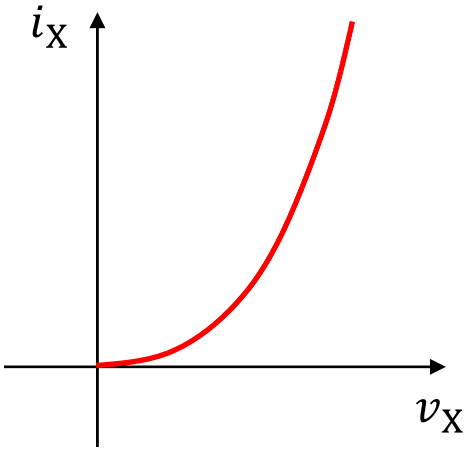
$$\Rightarrow \boxed{v_{OUT} = \frac{R_2(v_A - v_B)}{R_1}} \leftarrow \text{Difference Amplifier}$$

If Pick $R_1 = R_2 \Rightarrow \boxed{v_{OUT} = v_A - v_B}$ ← Subtraction Ckt

Implementing Non-Linear Functions

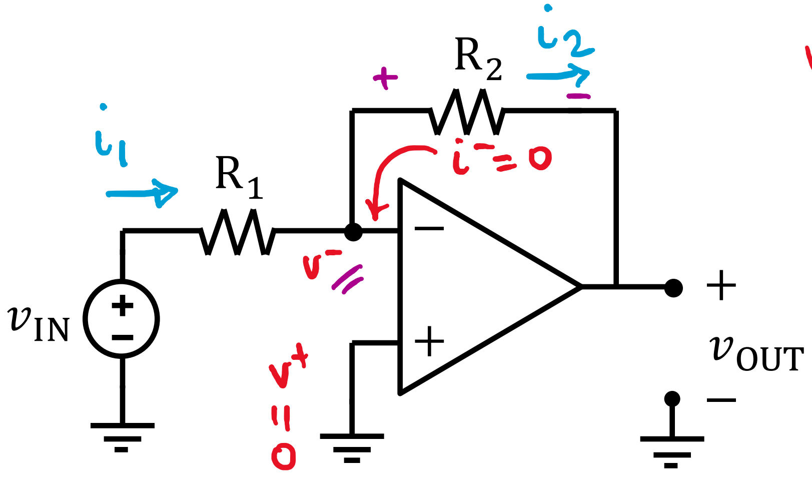


$i_X = f(v_X)$



Want to implement: $v_{OUT} = f(v_{IN})$

Recall: Inverting Amplifier



$v^- \approx v^+ = 0$

$v_{OUT} = 0 - R_2 i_2 = -R_2 i_1$

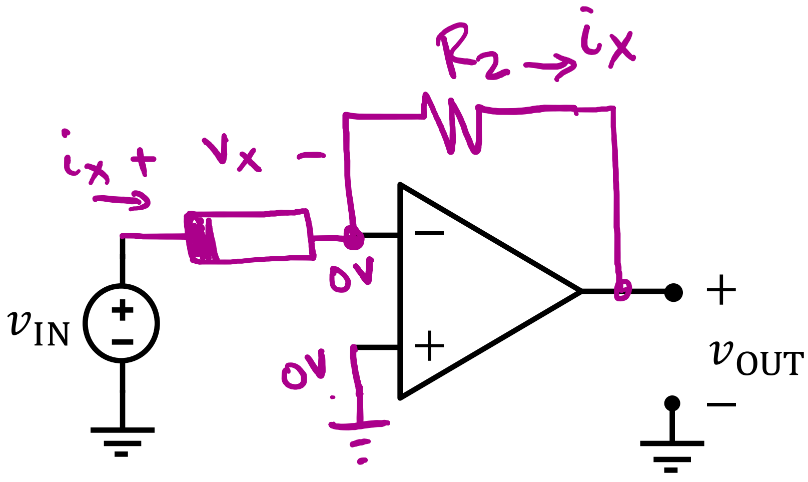
$v_{OUT} = -R_2 i_X$

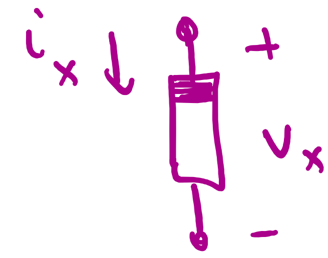
$\Rightarrow \boxed{v_{OUT} = -R_2 \frac{v_{IN}}{R_1}}$

$i_1 = \frac{v_{IN}}{R_1}$

$i_2 \approx i_1$

Op-Amp with a Non-Linear Device





$i_x = f(v_x)$

$i_x = f(v_x)$

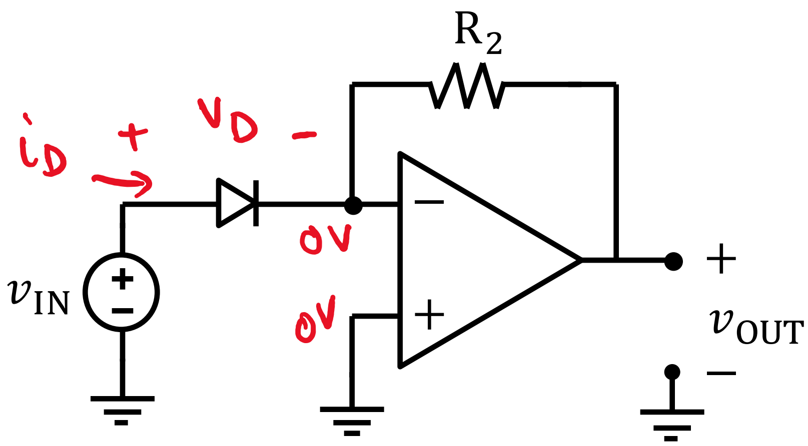
$\Rightarrow \quad i_x = f(v_{IN})$

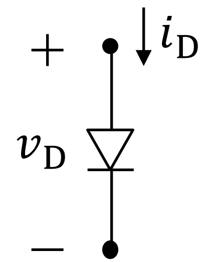
$v_{OUT} = -R_2 i_x = -R_2 f(v_{IN})$

Pick $R_2 = 1 \Rightarrow v_{OUT} = -f(v_{IN})$

Cascade an inverting Amp with gain of "−1" to get $v_{OUT} = f(v_{IN})$

Op-Amp with a Non-Linear Device Example





$$i_D = I_S\left(e^{v_D/V_{TH}} - 1\right)$$

$$V_{TH} = \frac{kT}{q}$$

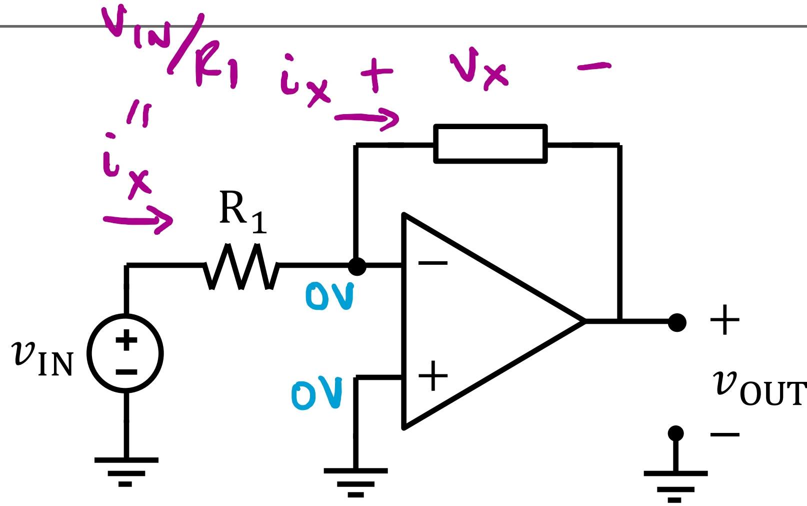
$$V_{OUT} = -R_2 i_D = -R_2 I_S\left(e^{v_D/V_{TH}} - 1\right) = -\underbrace{R_2 I_S}_{=1}\left(e^{v_{IN}/V_{TH}} - 1\right)$$

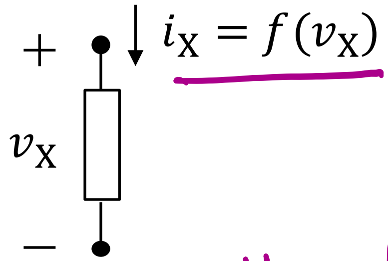
Pick $R_2 I_S = 1$

$$V_{OUT} = -\left(e^{v_{IN}/V_{TH}} - 1\right)$$

$$v_{IN}' \longrightarrow \boxed{V_{TH}} \longrightarrow v_{IN}' V_{TH}$$

Non-Linear Device in Feedback Loop



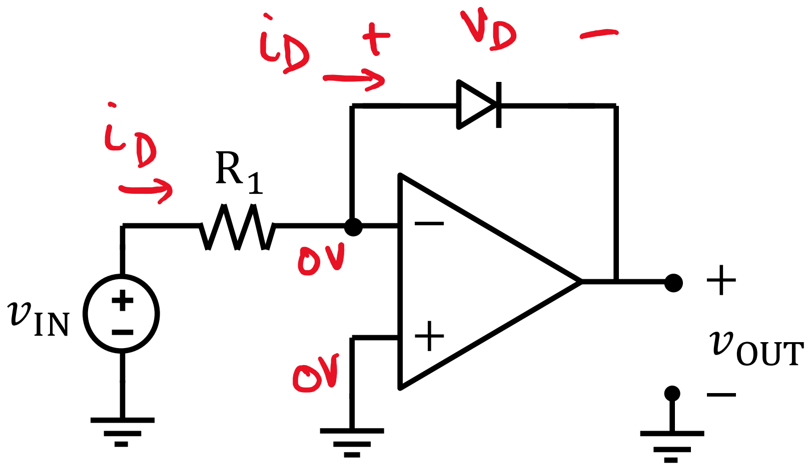


$v_x = f^{-1}(i_x)$

$v_{OUT} = 0 - v_x = -v_x$

$v_{OUT} = -f^{-1}(i_x) = -f^{-1}\left(\frac{v_{IN}}{R_1}\right)$

Logarithmic Amplifier



$$i_D = I_S\left(e^{v_D/V_{TH}} - 1\right)$$

$$\Downarrow$$

$$\frac{i_D}{I_S} + 1 = e^{v_D/V_{TH}}$$

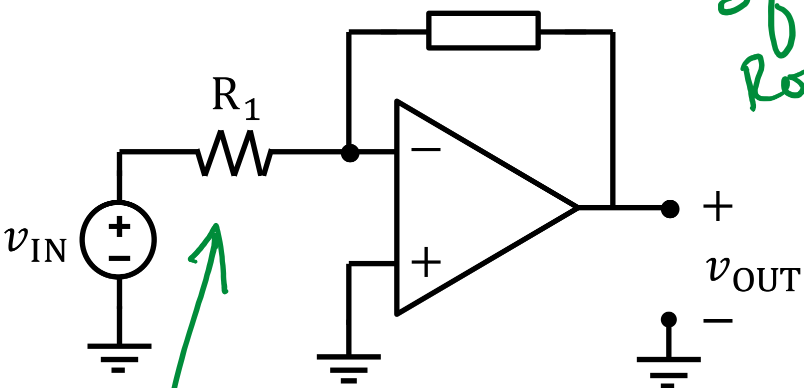
$$v_D = V_{TH} \ln\left(\frac{i_D}{I_S} + 1\right)$$

$$v_{OUT} = -v_D = -V_{TH} \ln\left(\frac{i_D}{I_S} + 1\right)$$

$$i_D = \frac{v_{IN}}{R_1} \Rightarrow \boxed{v_{OUT} = -V_{TH} \ln\left(\frac{v_{IN}}{R_1 I_S} + 1\right)}$$

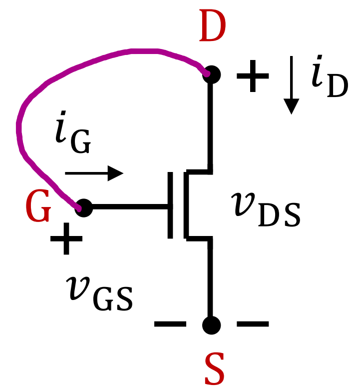
Pick $R_1 I_S = 1$

Square Root Using an Op-Amp



Square Root

For Square Put this device here



$$\boxed{i_D = \frac{K}{2}(v_{GS} - V_T)^2}$$

$$i_D = \frac{K}{2}(v_{DS} - V_T)^2$$

Saturation

$v_{GS} \geq V_T$

$v_{DS} \geq v_{GS} - V_T$

$v_{GS} = v_{DS}$

$v_{DS} \geq V_T$

$v_{GS} \geq v_{GS} - V_T$

True for $V_T > 0$