ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 12

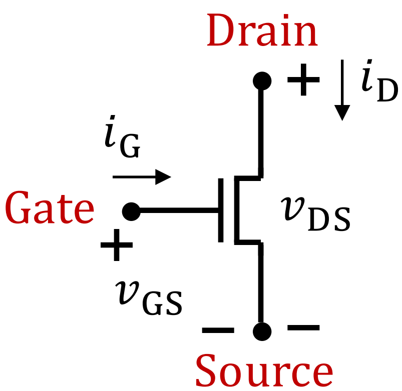
Large and Small Signal Analysis of Transistor Circuits

Announcements

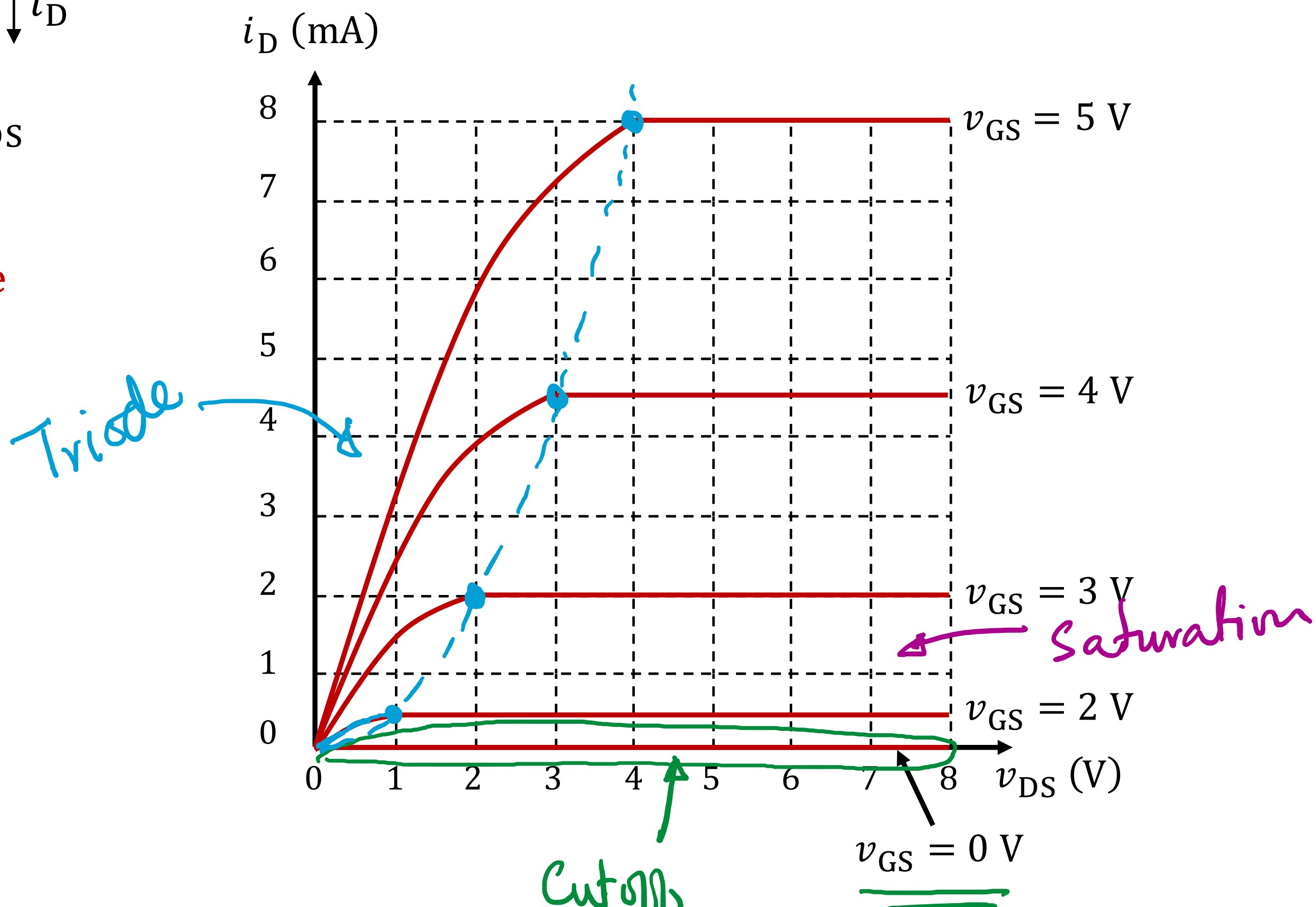
- Upcoming due dates:
  - Lab report 2 due by 11:59 pm on Wednesday 27, 2019
- Prelim 1 on Thursday February 21, 2019 from 7:30 – 9 pm in 203 Phillips
  - Email afridi@cornell.edu if have conflict
  - Make up exams on same day (Thursday February 21):
    - 10 – 11:30 am in 307 Phillips
    - 2:30 – 4 pm in 228 Phillips
  - Will cover material through Lecture 11
  - Prelim is closed-book and closed-notes
  - One double-sided page formula sheet is allowed
  - Bring a calculator

MOSFET i-v Characteristics

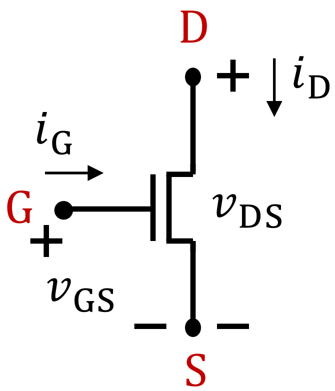
Metal-Oxide-Semiconductor Field-Effect Transistor (MOSFET) – n-channel

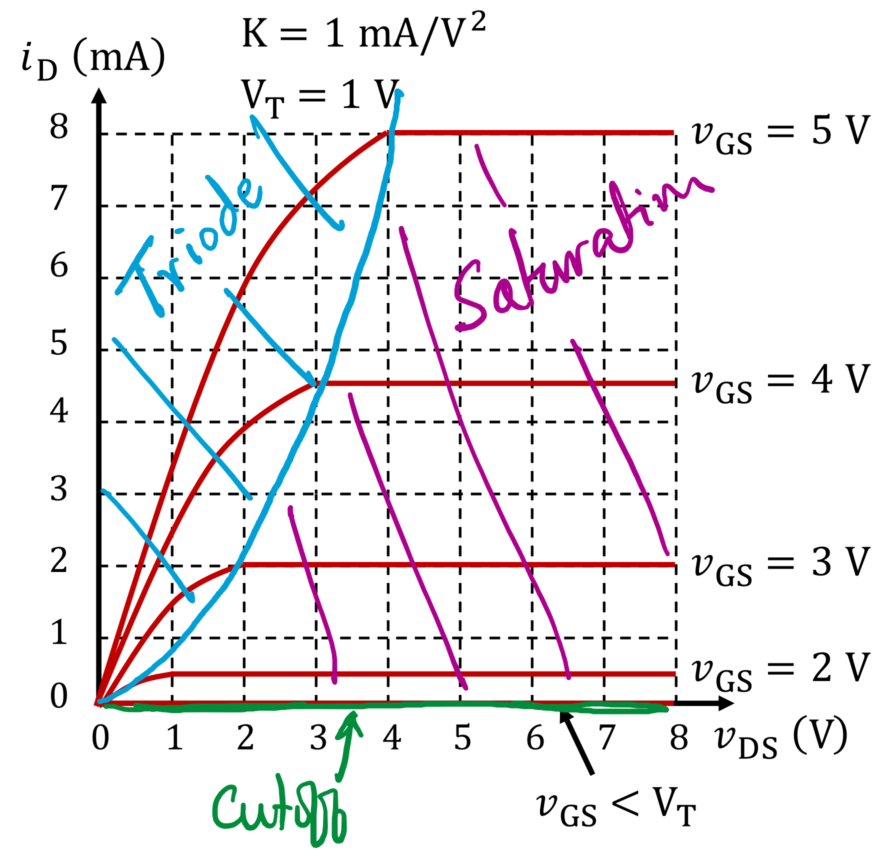


$i_G = 0$



MOSFET i-v Characteristics (Cont.)



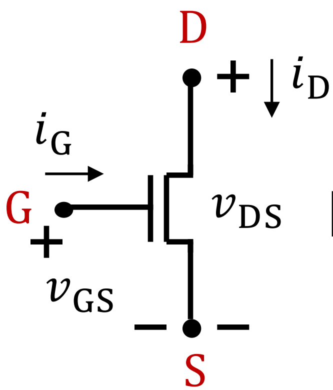


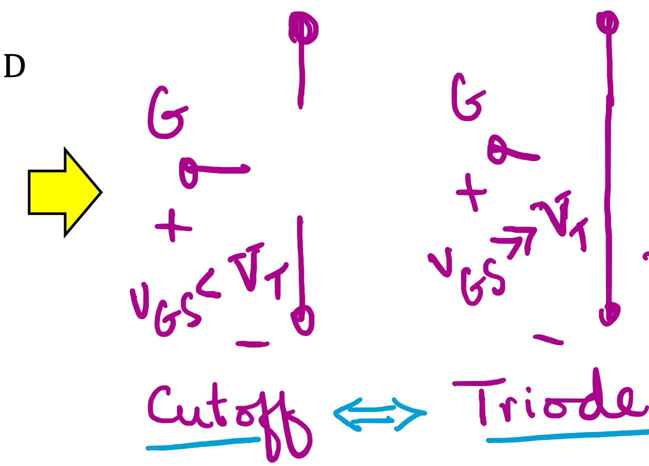
$i_G = 0$

$$i_D = \begin{cases} 0 & \text{if } v_{GS} < V_T \quad \textbf{Cutoff} \\ K\left[(v_{GS} - V_T)v_{DS} - \dfrac{v_{DS}^2}{2}\right] & \text{if } v_{GS} \geq V_T \text{ and } v_{DS} < v_{GS} - V_T \quad \textbf{Triode} \\ \dfrac{K}{2}(v_{GS} - V_T)^2 & \text{if } v_{GS} \geq V_T \text{ and } v_{DS} \geq v_{GS} - V_T \quad \textbf{Saturation} \end{cases}$$

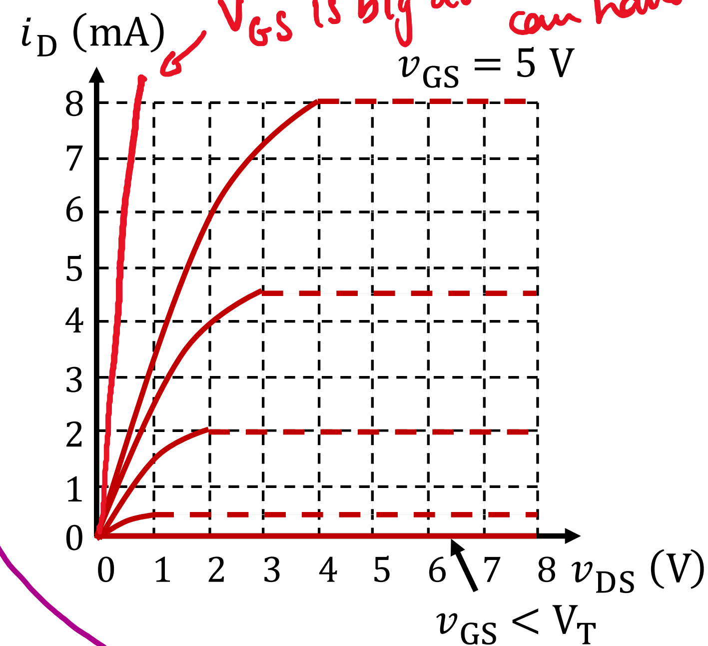
MOSFET as a Switch

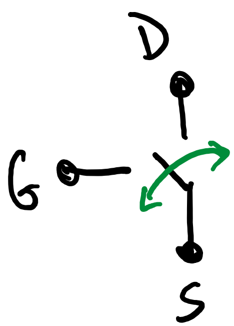
- MOSFET can be used as a switch (e.g., for digital logic) by going between cutoff and triode regions

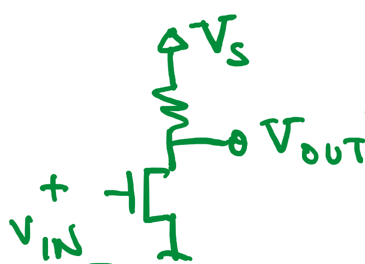




V_{GS} is big as the device can handle

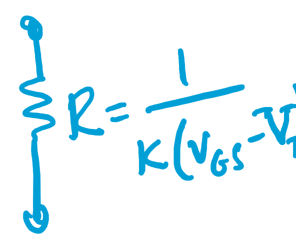






⟸ Inverter

v_{DS} very small

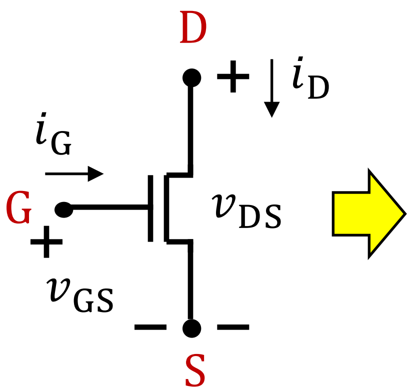


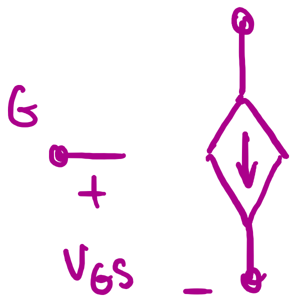
$$
i_D = \begin{cases} 0 & \textbf{Cutoff} \\ K\left[(v_{GS} - V_T)v_{DS} - \dfrac{v_{DS}^2}{2}\right] & \textbf{Triode} \end{cases}
$$

$\approx K(v_{GS} - V_T)v_{DS}$

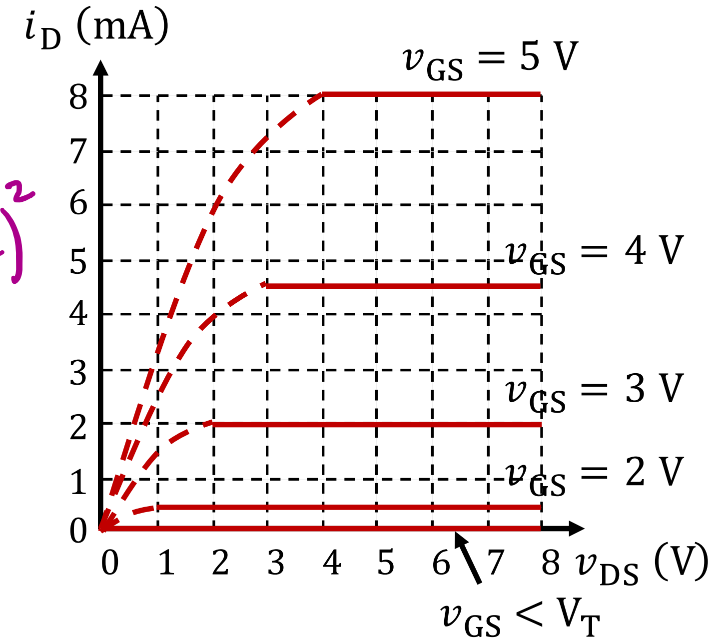
MOSFET as a Controlled Current Source

- MOSFET can be used as a Voltage Controlled Current Source by operating it in its saturation region





$i_D = \frac{K}{2}(V_{GS} - V_T)^2$

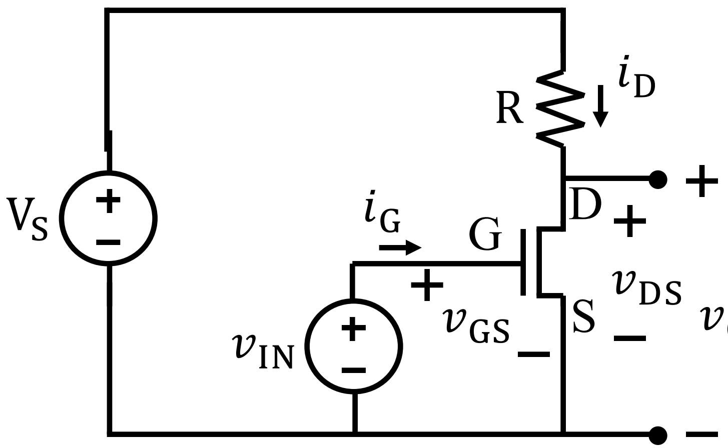


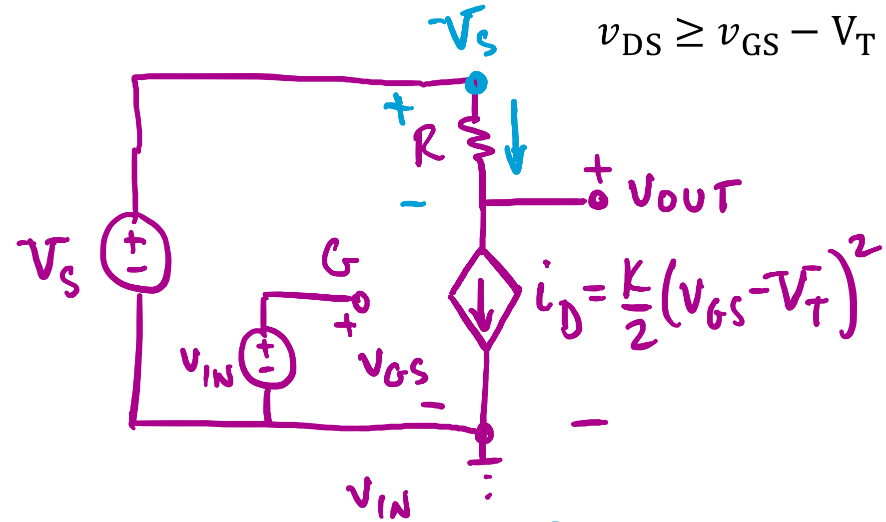
$$i_D = \frac{K}{2}(v_{GS} - V_T)^2$$ **Saturation**

MOSFET Circuit – Large-Signal Analysis

Given: MOSFET operating in Saturation $\quad v_{GS} \geq V_T$ and

$v_{DS} \geq v_{GS} - V_T$





$V_S = 8$ V

$R = 1$ kΩ

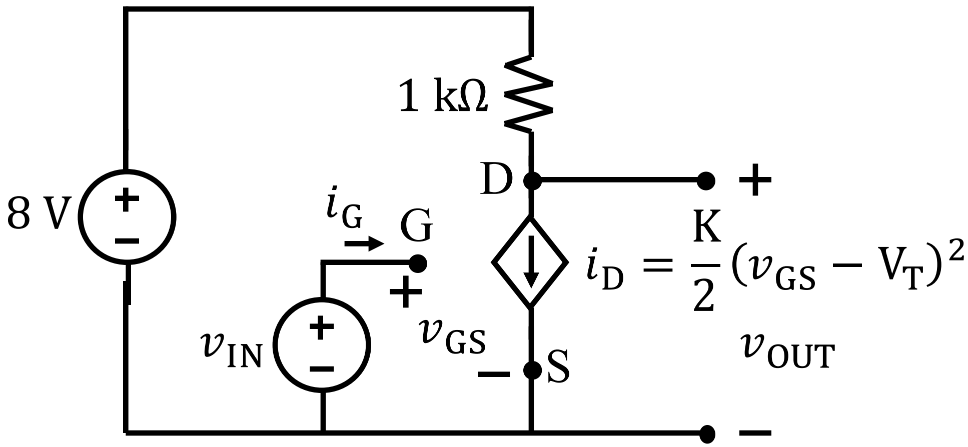
$K = 1$ mA/V^2

$V_T = 1$ V

$$v_{OUT} = V_S - R i_D = V_S - \frac{RK}{2}\left(\overset{v_{IN}}{v_{GS}} - V_T\right)^2$$

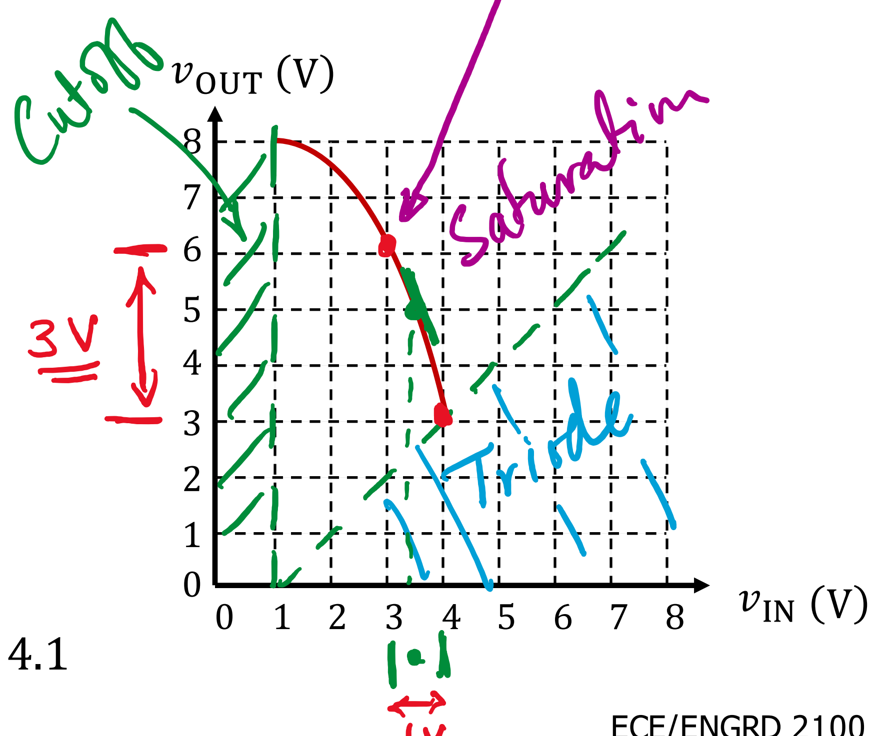
$$\boxed{v_{OUT} = 8 - \frac{1}{2}\left(v_{IN} - 1\right)^2}$$

MOSFET Circuit – Large-Signal Analysis (Cont.)



MOSFET in Saturation

$$\boxed{v_{OUT} = 8 - \frac{1}{2}(v_{IN} - 1)^2}$$



$K = 1\ \mathrm{mA/V^2}$

$V_T = 1\ V$

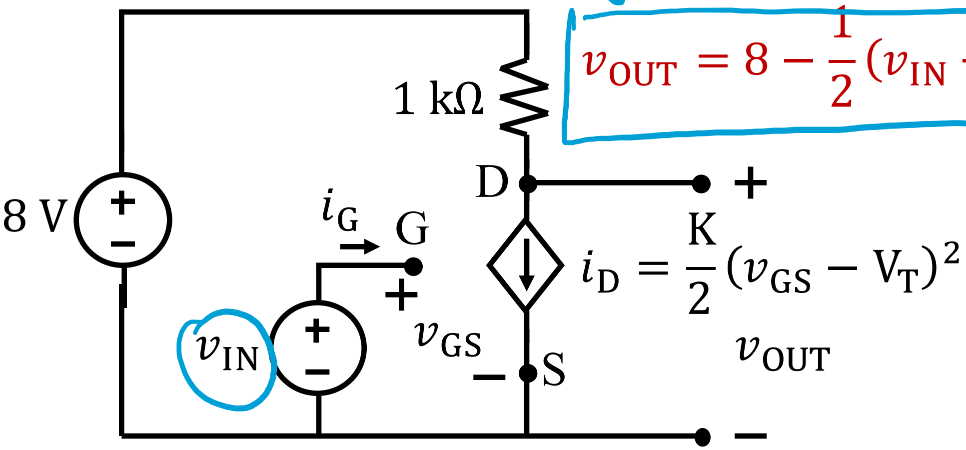
MOSFET is in Saturation when:

$v_{GS} \geq V_T \Rightarrow v_{IN} \geq 1$

$v_{DS} \geq v_{GS} - V_T \Rightarrow v_{OUT} \geq v_{IN} - 1$

$\Rightarrow 8 - \frac{1}{2}(v_{IN} - 1)^2 \geq v_{IN} - 1 \Rightarrow v_{IN} \leq 4.1$

MOSFET Circuit – Small-Signal Analysis



$$\boxed{v_{OUT} = 8 - \frac{1}{2}(v_{IN} - 1)^2}$$

small signal

$$v_{IN} = V_{IN} + v_{in}(t)$$

$$v_{OUT} = V_{OUT} + v_{out}(t)$$

$$v_{OUT} = f(v_{IN})$$

$$\Rightarrow\ V_{OUT} + v_{out} = f(V_{IN} + v_{in})$$

ignore

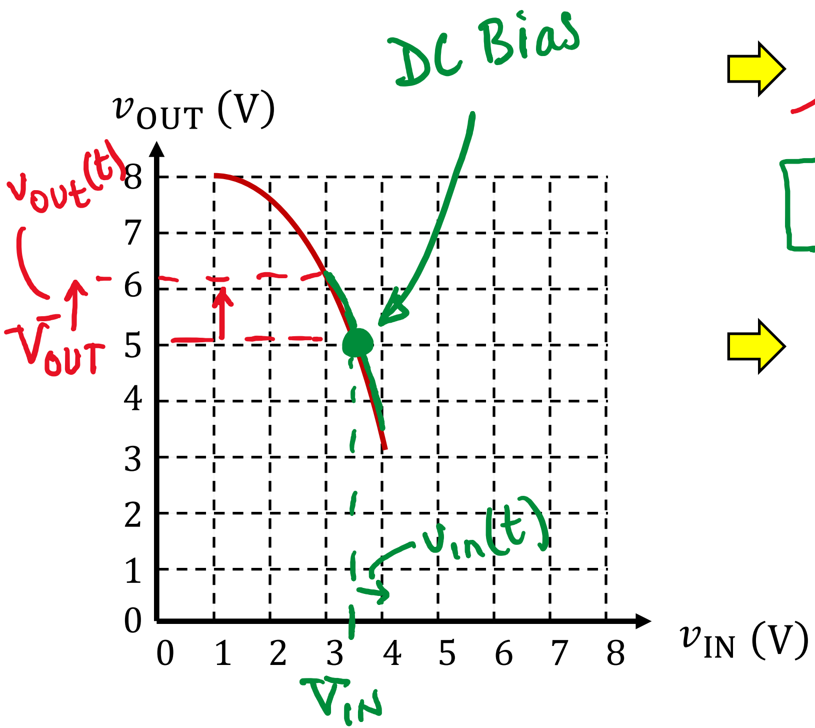
$$\Rightarrow\ \cancel{V_{OUT}} + v_{out} = \cancel{f(V_{IN})} + \left.\frac{\partial f}{\partial v_{IN}}\right|_{v_{IN}=V_{IN}} \cdot v_{in} + \cdots$$

higher order terms

$$\boxed{V_{OUT} = f(V_{IN})}$$

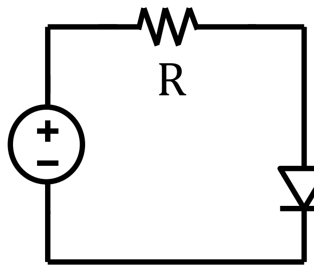
$$\Rightarrow\ v_{out} = \left.\frac{\partial f}{\partial v_{IN}}\right|_{v_{IN}=V_{IN}} \cdot v_{in}$$

$$\boxed{v_{out}(t) = -(V_{IN} - 1) \cdot v_{in}(t)}$$



Small-Signal Circuit Method

$v_{\mathrm{IN}}(t) = \mathrm{V_{IN}} + v_{\mathrm{in}}(t)$

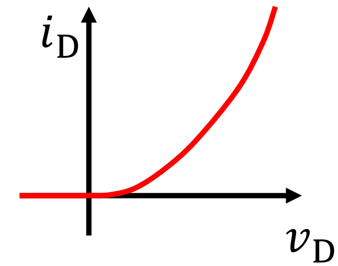


$i_{\mathrm{D}}(t) = \mathrm{I_D} + i_{\mathrm{d}}(t)$

$+$

$v_{\mathrm{D}}(t) = \mathrm{V_D} + v_{\mathrm{d}}(t)$

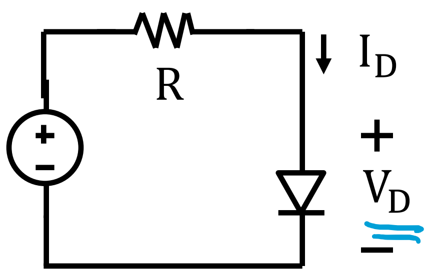
$-$



same circuit

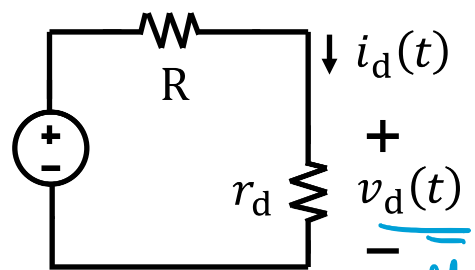
Breakup solution of nonlinear circuit into two parts

Large-Signal Circuit



DC values

Small-Signal Circuit



small signal

$$r_{\mathrm{d}} = \frac{1}{\left.\dfrac{\partial i_{\mathrm{D}}}{\partial v_{\mathrm{D}}}\right|_{v_{\mathrm{D}}=\mathrm{V_D}}}$$

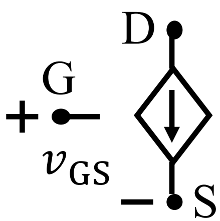
1) Find DC Bias Point using Large-Signal Circuit – i.e., using non-linear large-signal constitutive relations for the non-linear elements

2) Create Small-Signal Circuit by replacing non-linear elements by their linearized (small-signal) models, with values appropriate for DC Bias Point

3) Solve the resulting Small-Signal Circuit – which is linear

Small-Signal Model of MOSFET in Saturation

Large-Signal Model



$$i_D = \frac{\overset{1\,\mathrm{mA/V^2}}{K}}{2}(v_{GS} - \overset{1\,\mathrm{V}}{V_T})^2$$

Small-Signal Model

small signal response

small signal input

$$i_d = \underbrace{\left.\frac{\partial i_D}{\partial v_{GS}}\right|_{v_{GS}=V_{GS}}}_{\text{Slope at } v_{GS}=V_{GS}} \cdot v_{gs}$$

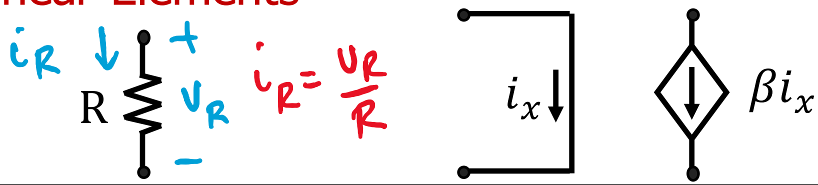
$$\left.\frac{\partial i_D}{\partial v_{GS}}\right|_{v_{GS}=V_{GS}} = \left.\cancel{2}\,\frac{K}{\cancel{2}}(v_{GS}-V_T)\right|_{v_{GS}=V_{GS}}$$

$$\left.\frac{\partial i_D}{\partial v_{GS}}\right|_{v_{GS}=V_{GS}} = K(V_{GS}-V_T)$$

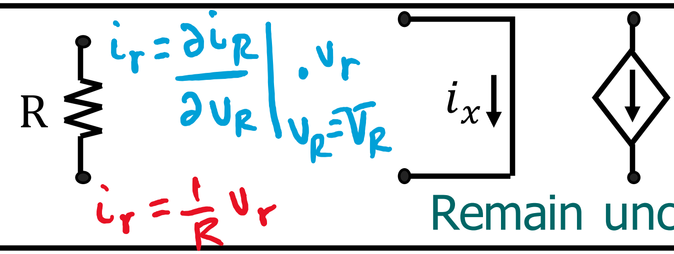
Small-Signal Models for Various Circuit Elements

Large-Signal Model	Small-Signal Model

Linear Elements



$i_R = \frac{v_R}{R}$



$i_r = \left.\frac{\partial i_R}{\partial v_R}\right|_{v_R = V_R} \cdot v_r$

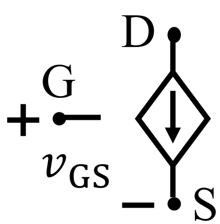
$i_r = \frac{1}{R} v_r$

Remain unchanged

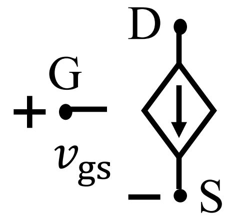
Constant Voltage/Current Sources

Set to zero

MOSFET in Saturation



$i_D = \frac{K}{2}(v_{GS} - V_T)^2$



$i_d = \underbrace{K(V_{GS} - V_T)}_{g_m} \cdot v_{gs}$

Linear Dependent Source

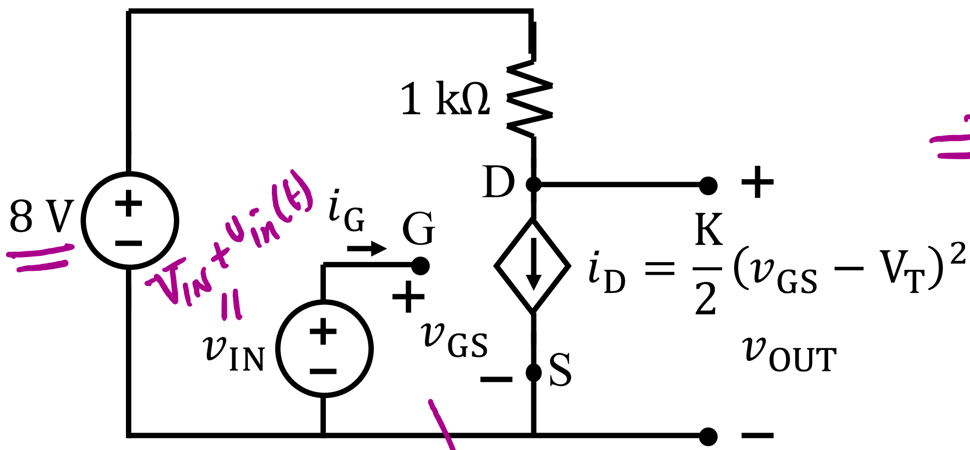
Small Signal Sources

inputs for small signal

Remain unchanged

MOSFET Amplifier – Small-Signal Circuit

Large-Signal Circuit



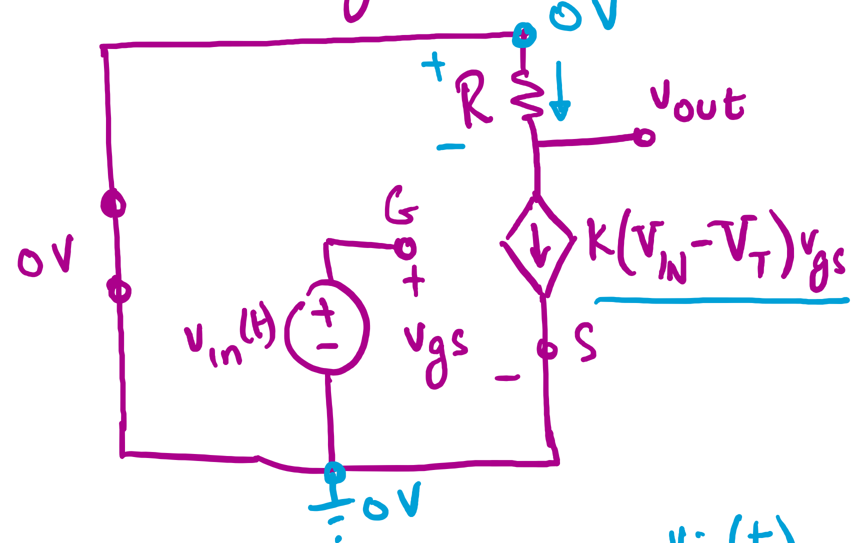
$K = 1\ \text{mA/V}^2$

$V_T = 1\ \text{V}$

$v_{IN} = V_{IN} + v_{in}(t)$

$V_{GS} = V_{IN}$

Small Signal Circuit

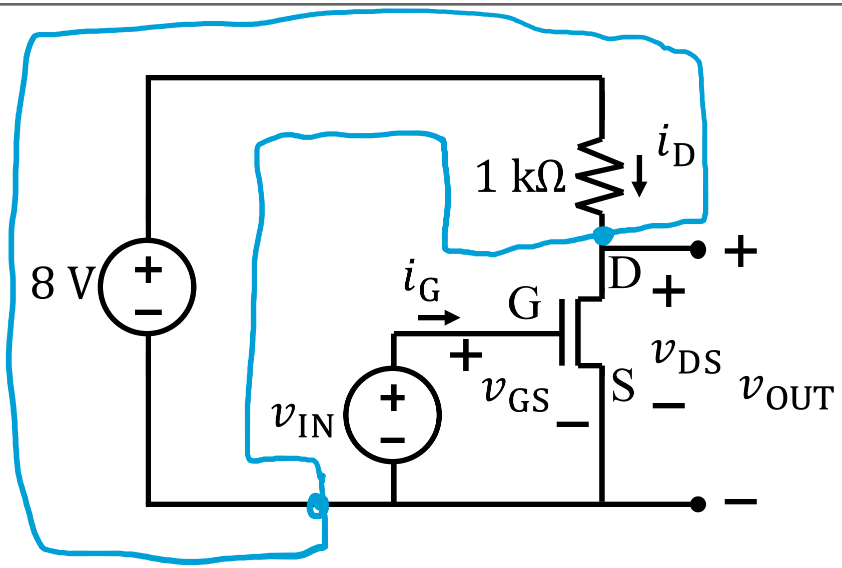


$$v_{out}(t) = 0 - RK(V_{IN} - V_T)\underbrace{v_{gs}}_{=\,v_{in}(t)}$$

$$v_{out}(t) = -RK(V_{IN} - V_T)\,v_{in}(t)$$

$$\boxed{v_{out}(t) = -(V_{IN} - 1)\,v_{in}(t)}$$

MOSFET Amplifier – Graphical Analysis



$v_{OUT} = 8V - (1k)\, i_D$