ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 11

Circuits with Dependent Sources

Announcements

- Recommended Reading:
  - Textbook Sections 2.1, 2.5, 4.3, 4.4, 4.6, 4.7, 4.8, 4.11, 4.13
- Upcoming due dates:
  - Homework 2 due by 11:59 pm on Friday February 15, 2019
  - Lab report 2 due by 11:59 pm on Wednesday 27, 2019
- Prelim 1 on Thursday February 21, 2019 from 7:30–9 pm in 203 Phillips
  - Email afridi@cornell.edu if have conflict
  - Make up exams on same day: 10–11:30 am and 2:30–4 pm, venue TBD
  - Will cover material through Lecture 11
  - Prelim is closed-book and closed-notes
  - One double-sided page formula sheet is allowed
  - Bring a calculator

Circuit Elements Covered So Far

So far we have seen two types of elements:

- Independent Sources (Voltage and Current)
  - Impose a voltage or current that does not depend on other constraints
  - Treated as system inputs

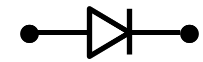
- Resistive Elements (Linear and Non-linear)
  - Impose a relationship between their terminal voltage and current

$v = \mathrm{R}i$ $\qquad$ $v = f(i)$

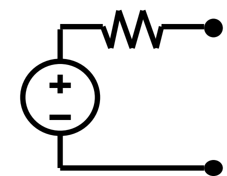
With these we can model many real components

Resistor

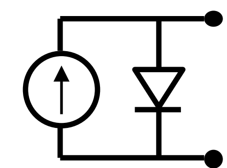
Diode



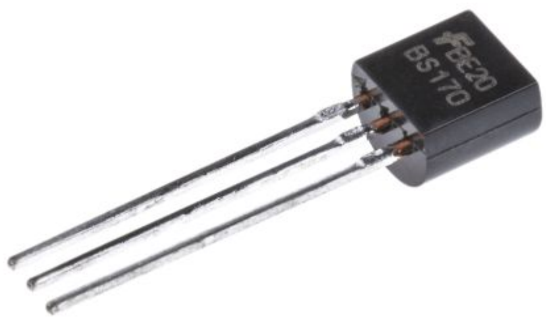
Battery

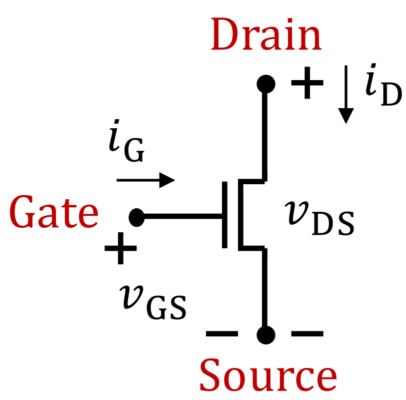


Solar Cell

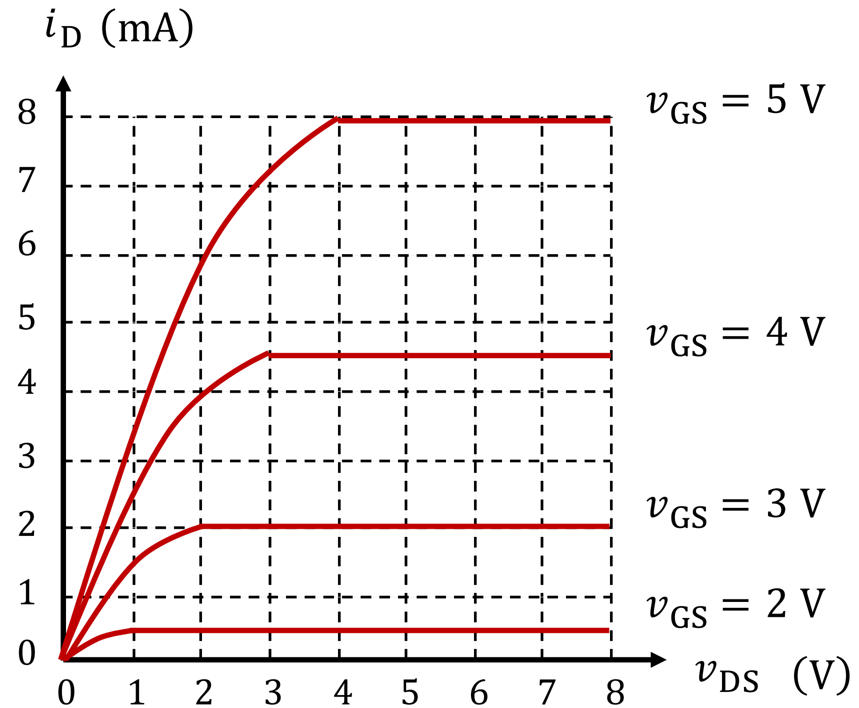


Transistor





MOSFET

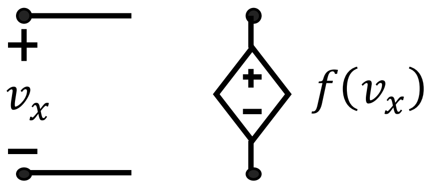


Dependent Sources

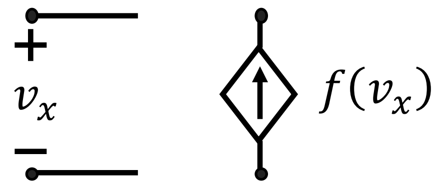
Dependent Sources are another important category of circuit elements where the voltage or current at one place determines the voltage or current at another place in the circuit

Four Types

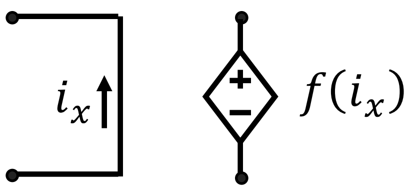
Voltage-Controlled Voltage Source



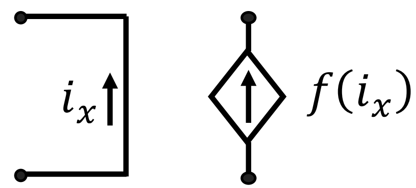
Voltage-Controlled Current Source



Current-Controlled Voltage Source



Current-Controlled Current Source

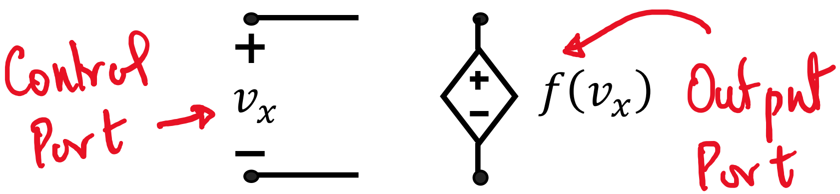


Dependent Sources have a huge range of uses, and are especially useful for modeling transistors and other devices with more than two terminals (e.g., Op-Amps)

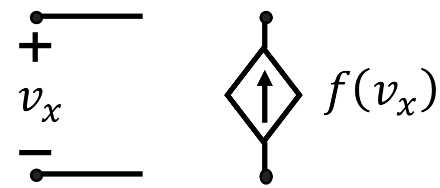
Dependent Sources (Cont.)

- Dependent sources are "two port" devices, where a "port" is a pair of terminals
  - The "control port" measures a voltage or a current without disturbing it
  - The "output port" imposes a voltage or current at its terminals that depends on the measured variable at the control port

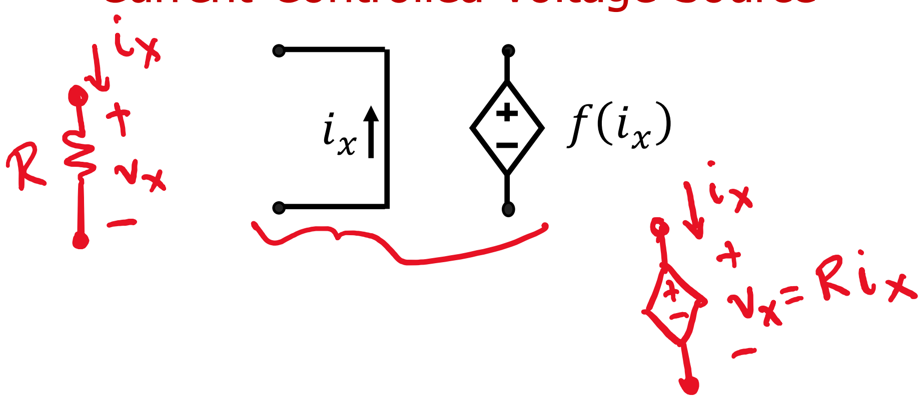
Voltage-Controlled Voltage Source



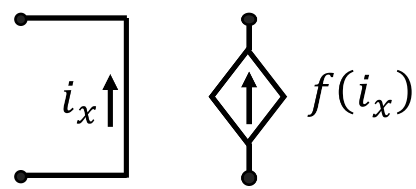
Voltage-Controlled Current Source



Current-Controlled Voltage Source



Current-Controlled Current Source



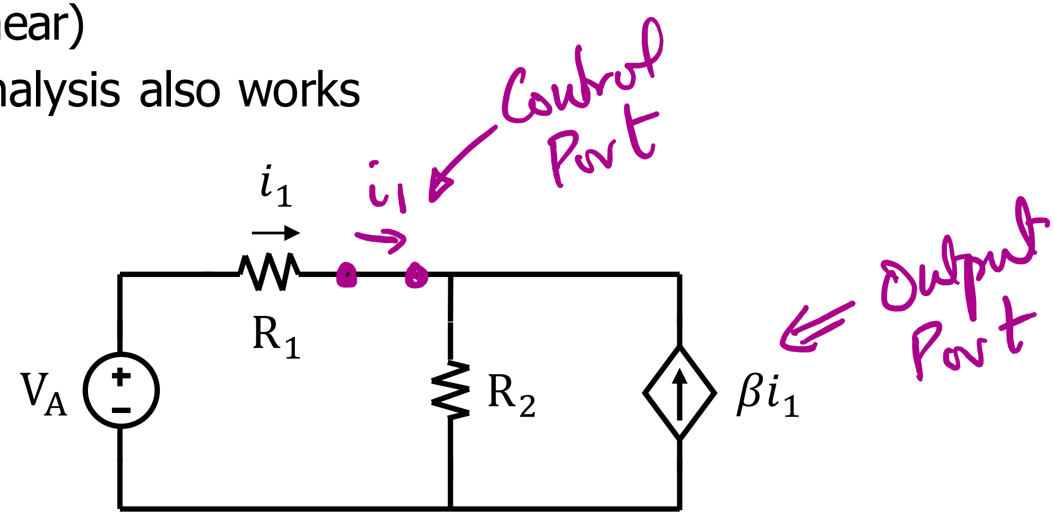
- In a linear dependent source: $f(x) = Kx$, where K is a constant

Independent Sources vs. Dependent Sources

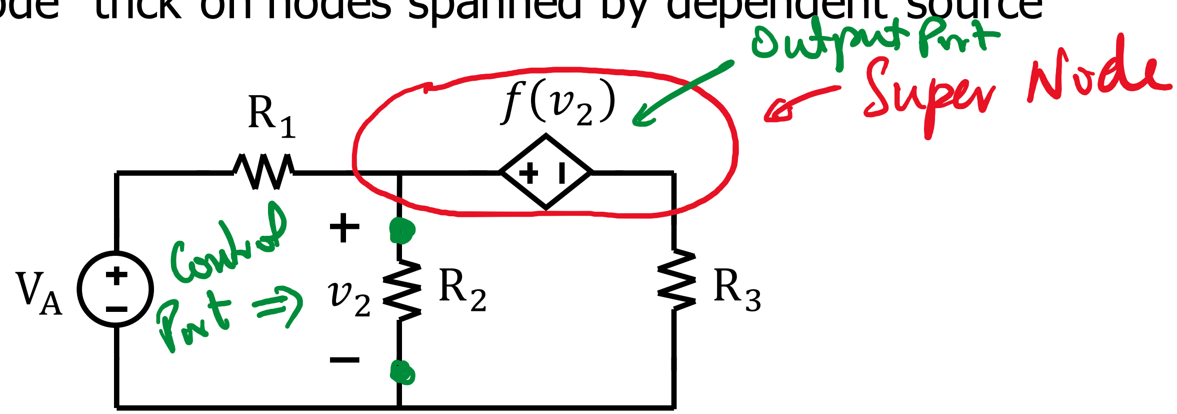
- The voltage or current value of an Independent Source is independent of the circuit in which it is connected } Treat as inputs
- The voltage or current value of a Dependent Source depends on voltage or current in some other part of the circuit] Do not treat as input
- In some sense the Dependent Source is similar to a Resistive Element
  - In Resistive Element (linear or non-linear), voltage across its port depends on current through its port
  - In Dependent Source (linear or non-linear), voltage (current) across (through) its output port depends on voltage (current) across (through) its control port

Analysis of Circuits with Dependent Sources

- Node Analysis is a general method, so it also works with Dependent Sources (linear or non-linear)
  - Mesh/Loop Analysis also works



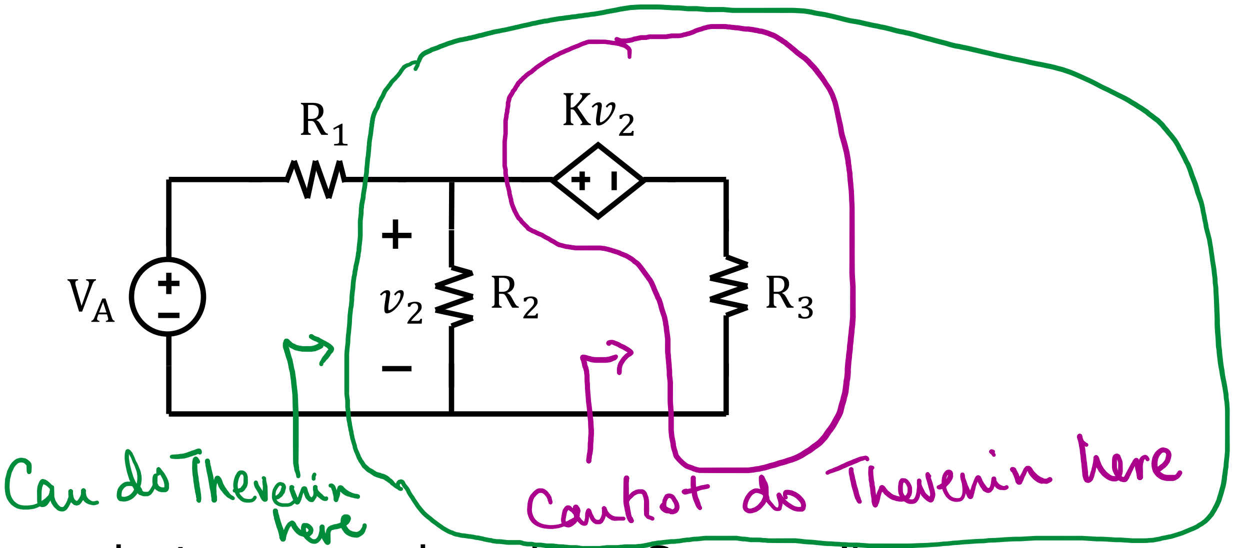
- Can use "super node" trick on nodes spanned by dependent source



Analysis of Circuits with Dependent Sources (Cont.)

- Superposition and Thevenin/Norton will only work if the Dependent Sources and the rest of the circuit is linear

- If dependent sources are linear, can do Thevenin and Norton equivalents of circuits with dependent sources, provided the control and output ports of the dependent sources are together inside the circuit being modeled

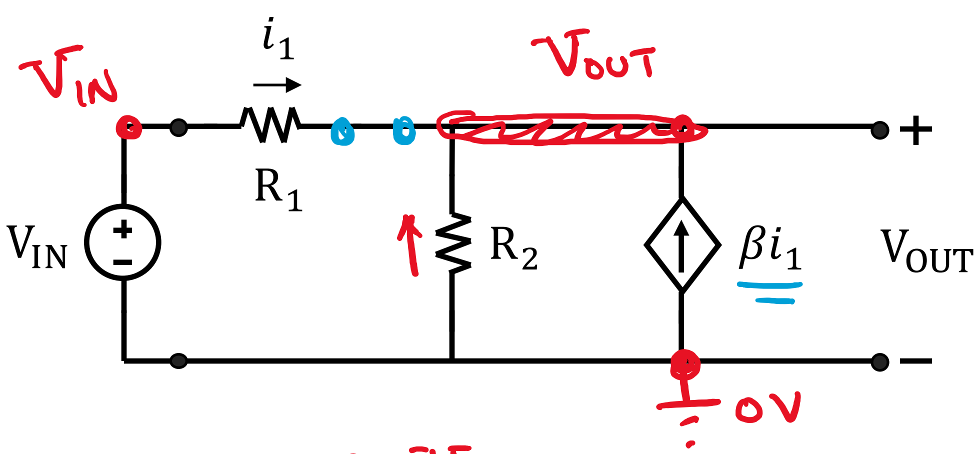


- Do not "kill" dependent sources when doing Superposition

- Do not "kill" dependent sources when finding Thevenin Resistance

Dependent Source Circuit Analysis Example

Find V_{OUT}



$$i_1 + \frac{0 - V_{OUT}}{R_2} + \beta i_1 = 0 \Rightarrow V_{OUT} = (1+\beta) i_1 R_2$$

$$i_1 = \frac{V_{IN} - V_{OUT}}{R_1} \Rightarrow V_{OUT} = (1+\beta) R_2 \left[\frac{V_{IN} - V_{OUT}}{R_1}\right]$$

$$R_1 V_{OUT} + \frac{(1+\beta) R_2 V_{OUT}}{\cancel{R_1}} = \frac{(1+\beta) R_2 V_{IN}}{\cancel{R_1}}$$

$$\boxed{V_{OUT} = \frac{(1+\beta) R_2}{R_1 + (1+\beta) R_2} \cdot V_{IN}}$$

$\beta \gg 1$

$\Rightarrow V_{OUT} \approx V_{IN}$

Thevenin Equivalent Example – Input Port

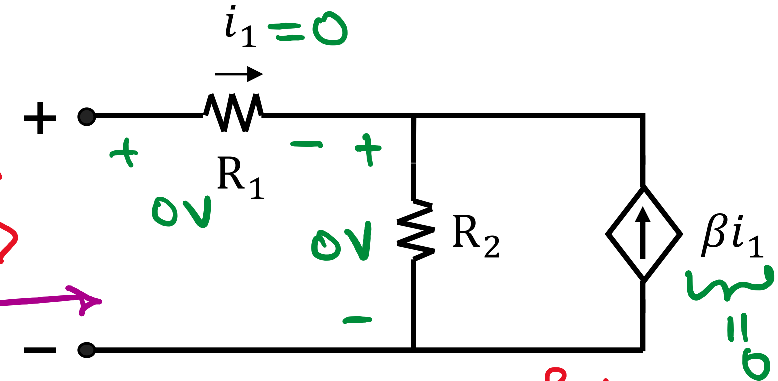
$V_{TH} = V_{oc}$

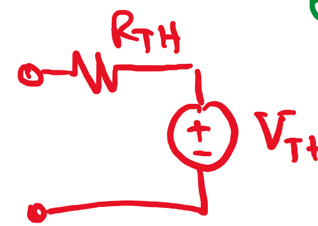
$V_{TH} = V_{oc} = \underline{\underline{0V}}$

Thevenin ⇒

$V_{oc} = 0V$

R_{TH}





Find R_{TH}

① $R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{0V}{0A}$ ✗ cannot use this

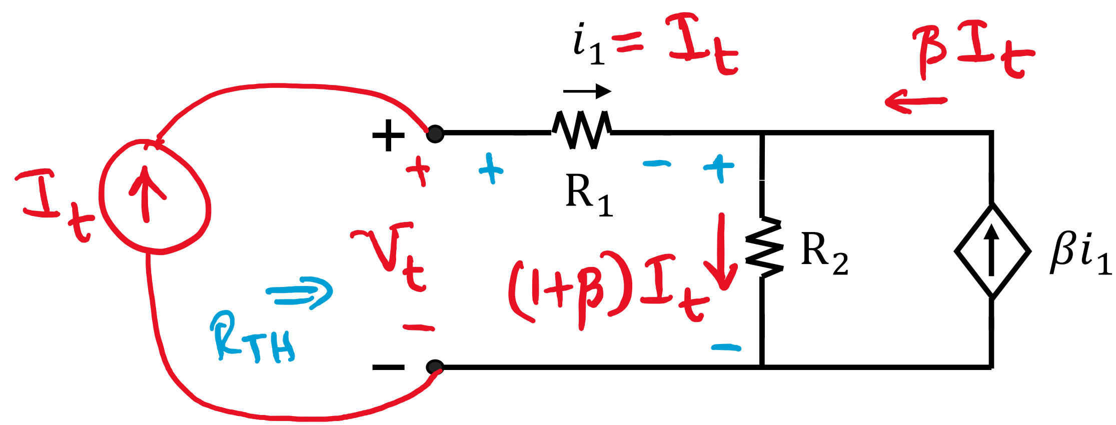
② Shut off Independent Sources & do series & parallel resistors ✗

Cannot do this when have dependent sources

③ Shut off Independent Sources & use test source method

This method always works

Thevenin Equivalent Example – Input Port (Cont.)



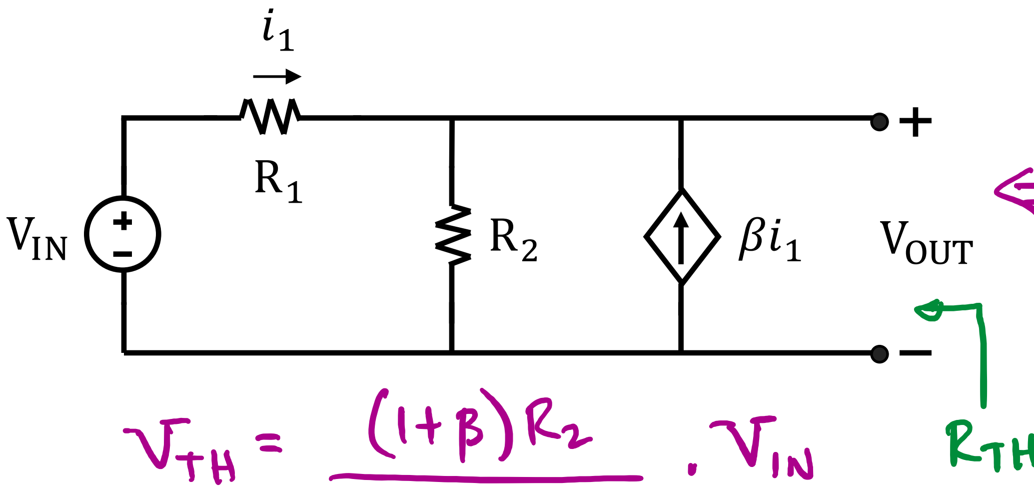
$$V_t = (1+\beta) I_t R_2 + I_t R_1$$

$$R_{TH} \equiv \frac{V_t}{I_t} = \boxed{(1+\beta)R_2 + R_1 = R_{TH}}$$

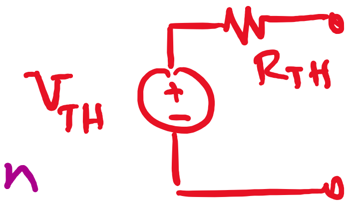
If $\beta \gg 1$ → make it large

⟹ R_{TH} → becomes large

Thevenin Equivalent Example – Output Port

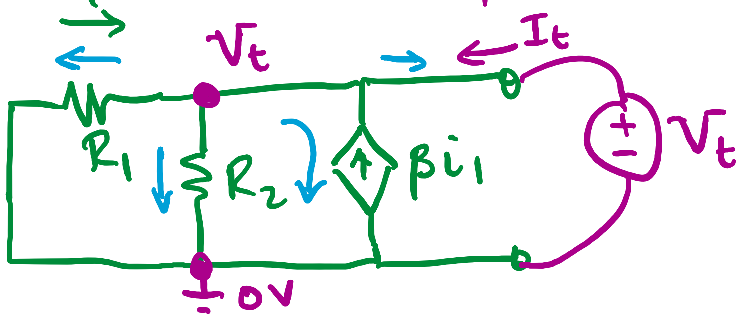


⇐ Thevenin



$$V_{TH} = V_{OC}$$

$$V_{TH} = \frac{(1+\beta)R_2}{R_1 + (1+\beta)R_2} \cdot V_{IN}$$



$$\frac{V_t}{R_1} + \frac{V_t}{R_2} - \beta i_1 - I_t = 0$$

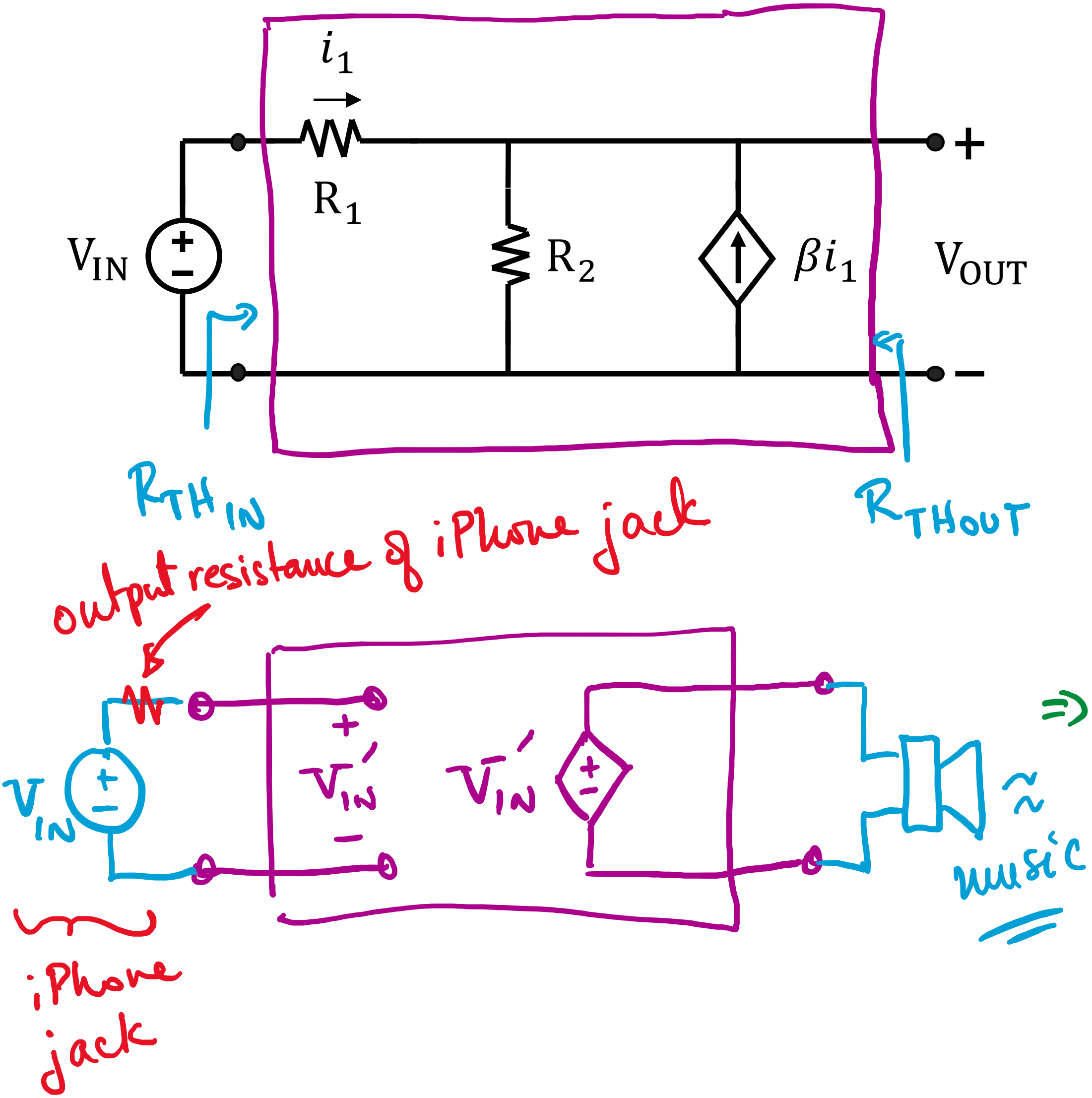
$$i_1 = -\frac{V_t}{R_1}$$

$$\Rightarrow I_t = \frac{V_t}{R_1} + \frac{V_t}{R_2} + \frac{\beta V_t}{R_1} \Rightarrow \frac{I_t}{V_t} = \frac{1+\beta}{R_1} + \frac{1}{R_2} \Rightarrow \boxed{R_{TH} = R_2 \| \frac{R_1}{1+\beta}}$$

$$\frac{I_t}{V_t} = \frac{1}{R_{TH}}$$

If $\beta >> 1$ → large $\Rightarrow R_{TH} \approx \frac{R_1}{1+\beta}$

Dependent Source Circuit Example



$$V_{OUT} = \frac{R_2(1+\beta)}{R_1 + R_2(1+\beta)} \cdot V_{IN}$$

$$R_{TH_{IN}} = R_1 + R_2(1+\beta)$$

$$R_{TH_{OUT}} = R_2 \,\|\, \frac{R_1}{(1+\beta)}$$

$\beta \rightarrow$ Large

$\Rightarrow V_{OUT} \approx V_{IN}$

$R_{TH_{IN}} \rightarrow$ large $\rightarrow \infty$

$R_{TH_{OUT}} \rightarrow$ small $\rightarrow 0$