ECE/ENGRD 2100

Introduction to Circuits for ECE

Lecture 10

Small-Signal Analysis of Nonlinear Circuits

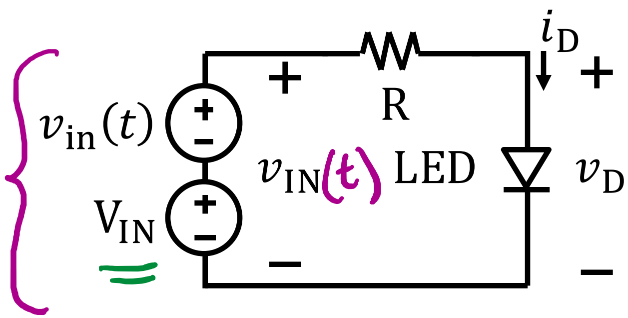
and

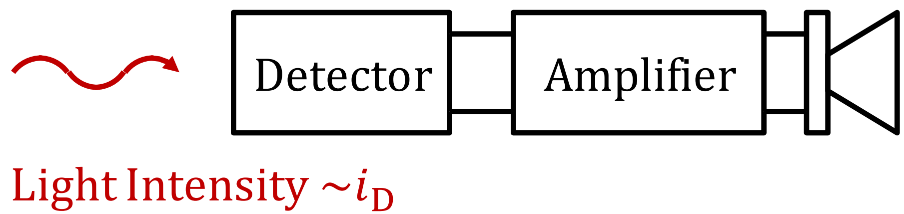
Transistors and Dependent Sources

Announcements

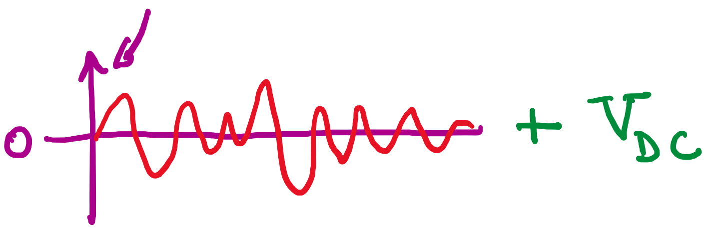
- Recommended Reading:
  - Textbook Sections 2.1, 2.5, 4.3, 4.4, 4.6, 4.7, 4.8, 4.11, 4.13
- Upcoming due dates:
  - Homework 2 due by 11:59 pm on Friday February 15, 2019
  - Lab report 2 due by 11:59 pm on Wednesday 27, 2019
- Prelim 1 on Thursday February 21, 2019 from 7:30 – 9 pm in 203 Phillips
  - Will cover material through Lecture 11
  - Prelim is closed-book and closed-notes
  - One double-sided page formula sheet is allowed
  - Bring a calculator
  - Email afridi@cornell.edu if have conflict

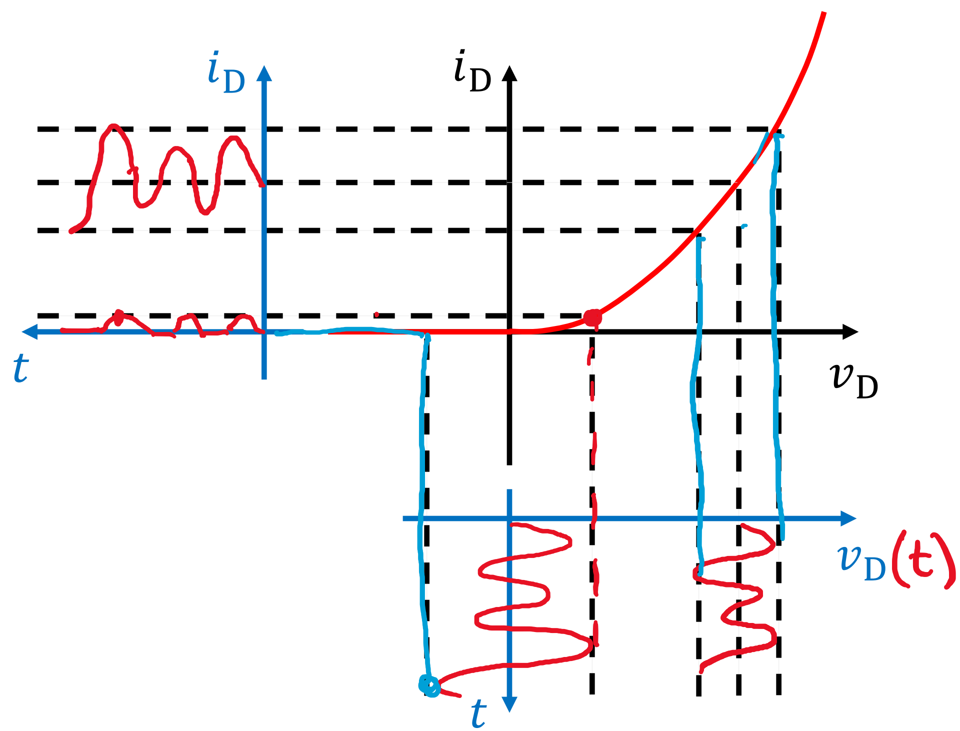
Small-Signal Application



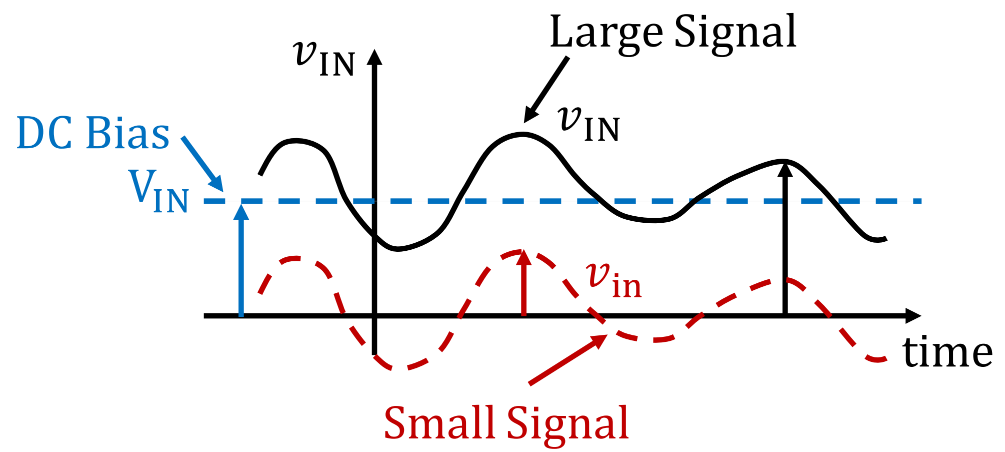


$$v_{IN}(t) = V_{IN} + v_{in}(t)$$





Large-Signal and Small-Signal Notation



lower case

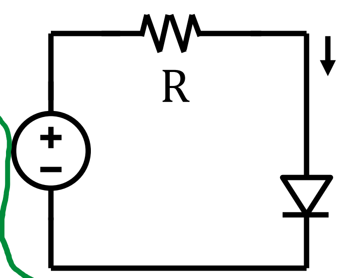
DC Bias

$v_{IN}(t) = V_{IN} + v_{in}(t)$

Large signal

Capital

small-signal

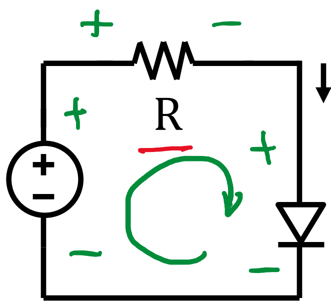


$i_D(t) = I_D + i_d(t)$

$v_D(t) = V_D + v_d(t)$

Circuit Laws for Small-Signals

$v_{IN}(t) = V_{IN} + v_{in}(t)$



$i_D(t) = I_D + i_d(t)$

$v_D(t) = V_D + v_d(t)$

Large-Signal quantities must satisfy circuit laws (KVL/KCL):

$$v_{IN}(t) = Ri_D(t) + v_D(t) \Rightarrow V_{IN} + v_{in}(t) = R\left(I_D + i_d(t)\right) + V_D + v_d(t) \rightarrow A$$

DC Bias quantities must also satisfy circuit laws (KVL/KCL):

$$V_{IN} = RI_D + V_D \rightarrow B$$

$$A - B =$$

Small-Signal quantities also satisfy circuit laws (KVL/KCL):

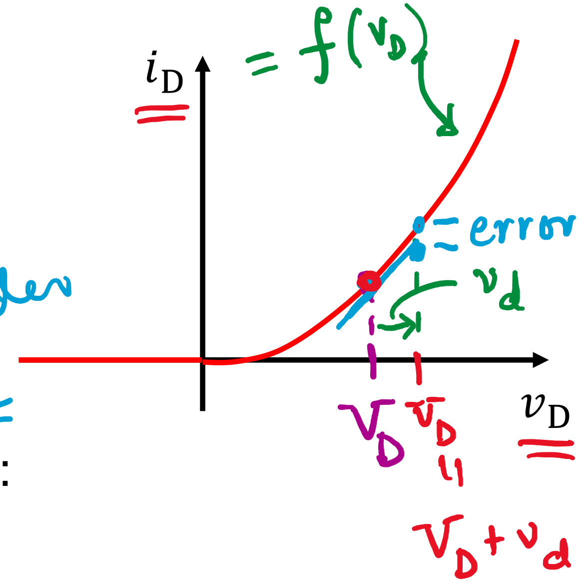
$$v_{in}(t) = Ri_d(t) + v_d(t)$$

Constitutive Relationships for Small-Signals

Large-Signal quantities must satisfy element laws:

$$i_D = f(v_D) \Rightarrow I_D + i_d = f(V_D + v_d)$$

$$\Rightarrow I_D + i_d = f(V_D) + \left.\frac{\partial f}{\partial v_D}\right|_{v_D = V_D} \cdot v_d + \cdots$$



DC Bias quantities must also satisfy same element laws:

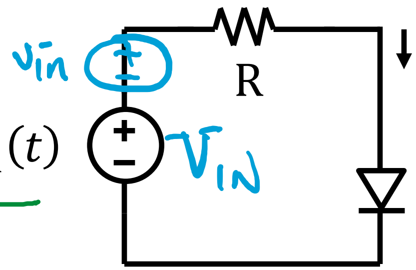
$$I_D = f(V_D)$$

Small-Signal quantities satisfy incremental relationship:

$$i_d \approx \left.\frac{\partial f}{\partial v_D}\right|_{v_D = V_D} \cdot v_d \Rightarrow r_d \equiv \frac{v_d}{i_d} = \frac{1}{\left.\frac{\partial f}{\partial v_D}\right|_{v_D = V_D}}$$

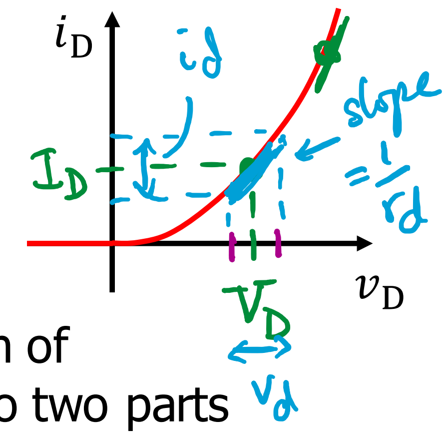
Small-Signal Analysis Summary

$$v_{IN}(t) = V_{IN} + v_{in}(t)$$

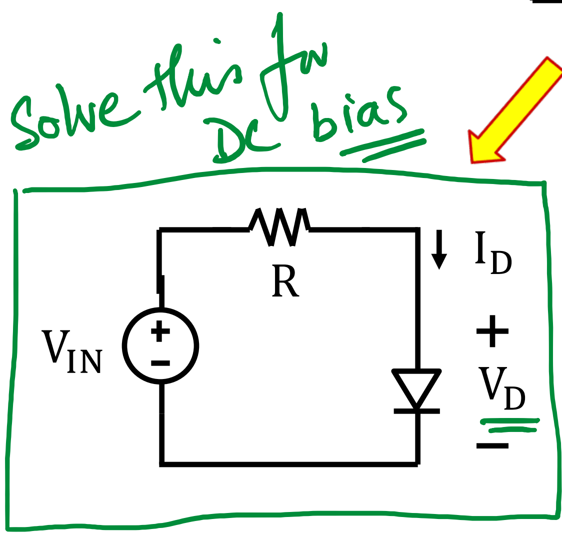


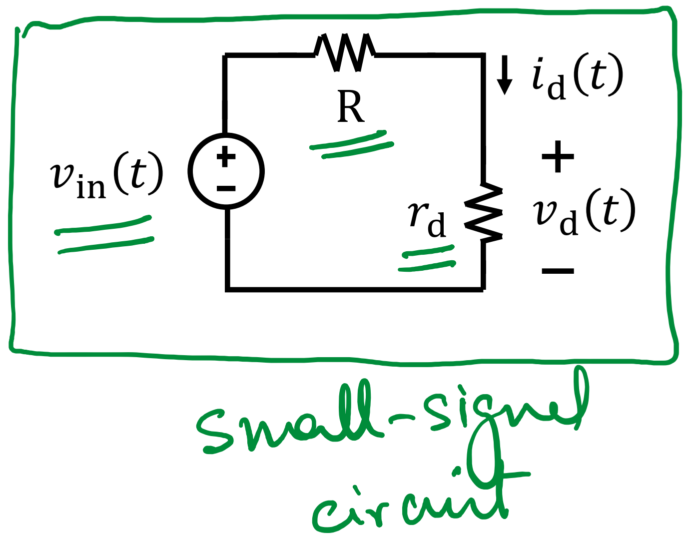
$$\downarrow i_D(t) = I_D + i_d(t)$$

$$+ \; v_D(t) = V_D + v_d(t) \; -$$



Can breakup solution of nonlinear circuits into two parts





$$r_d = \frac{1}{\left.\frac{\partial i_D}{\partial v_D}\right|_{v_D = V_D}}$$

Example:

$$i_D = I_S\left(e^{q v_D / kT} - 1\right)$$

Circuit Elements Covered So Far

So far we have seen two types of elements:

- Independent Sources (Voltage and Current)
  - Impose a voltage or current that does not depend on other constraints
  - Treated as system inputs

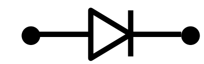
- Resistive Elements (Linear and Non-linear)
  - Impose a relationship between their terminal voltage and current

$v = \mathrm{R}i$ $\quad$ $v = f(i)$

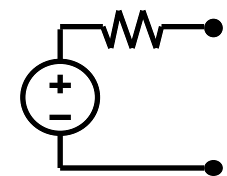
With these we can model many real components

Resistor

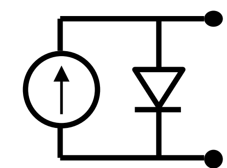
Diode



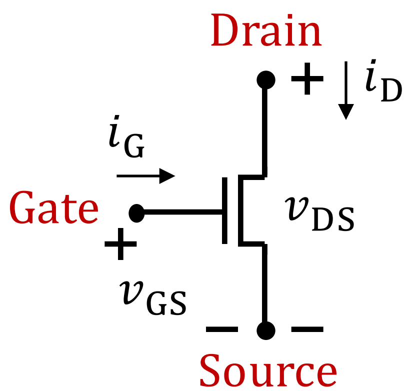
Battery



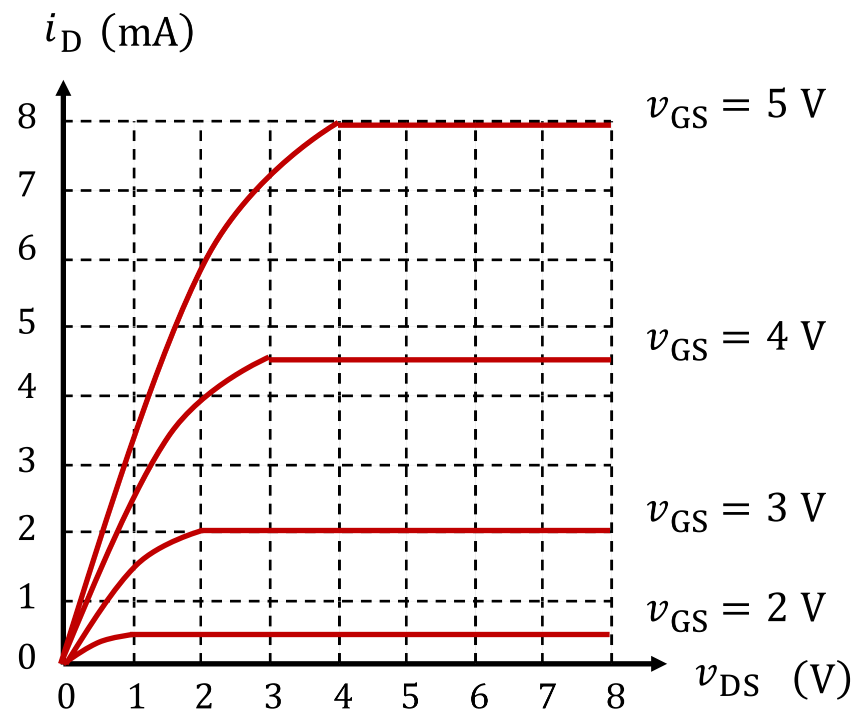
Solar Cell



Transistor



MOSFET



Dependent Sources

Dependent Sources are another important category of circuit elements where the voltage or current at one place determines the voltage or current at another place in the circuit

Four Types

Voltage-Controlled Voltage Source

Voltage-Controlled Current Source

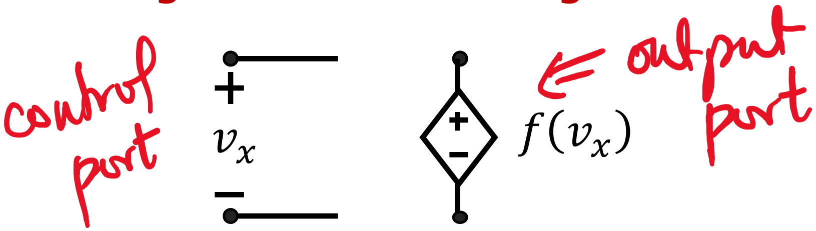
Current-Controlled Voltage Source

Current-Controlled Current Source

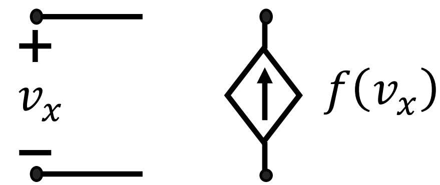
Dependent Sources (Cont.)

- Dependent sources are "two port" devices, where a "port" is a pair of terminals
  - The "control port" measures a voltage or a current without disturbing it
  - The "output port" imposes a voltage or current at its terminals that depends on the measured variable at the control port

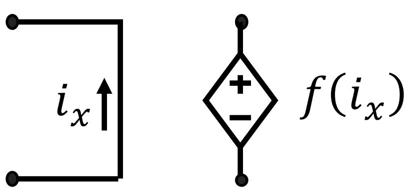
Voltage-Controlled Voltage Source



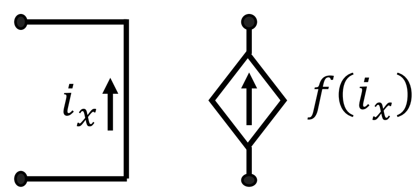
Voltage-Controlled Current Source



Current-Controlled Voltage Source



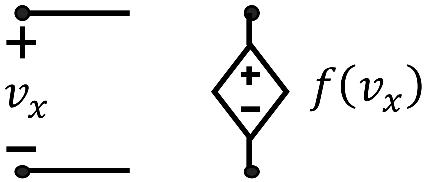
Current-Controlled Current Source



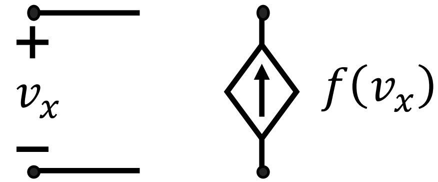
- Dependent Sources have a huge range of uses, and are especially handy for modeling transistors and operational amplifiers

Linear Dependent Sources

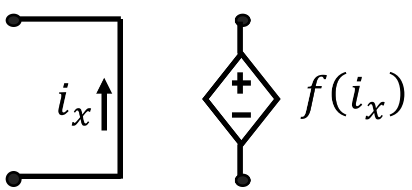
Voltage-Controlled Voltage Source



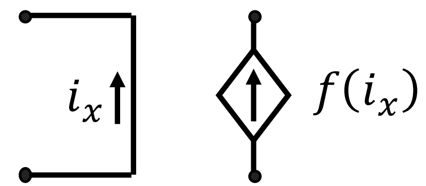
Voltage-Controlled Current Source



Current-Controlled Voltage Source

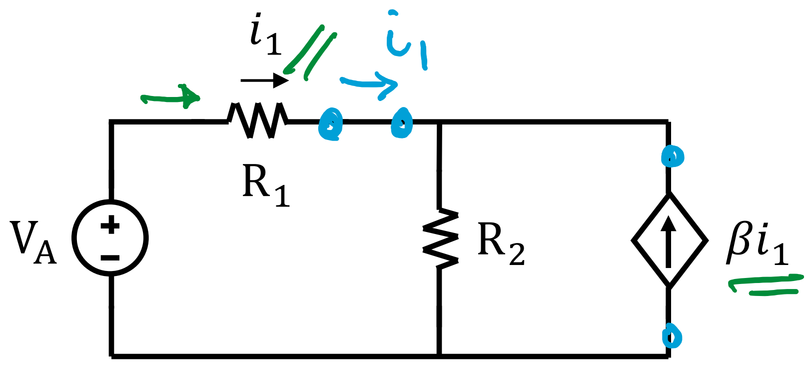


Current-Controlled Current Source



A linear dependent sources has the form: $f(x) = Kx$

Analysis of Circuits with Dependent Sources

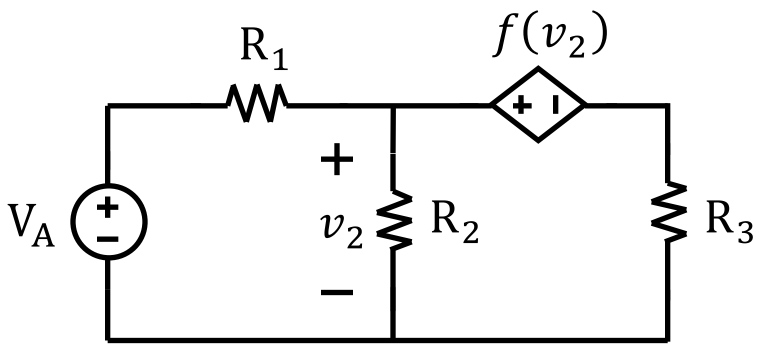


Node Analysis is a general method, so it also works with Dependent Sources (linear or non-linear)

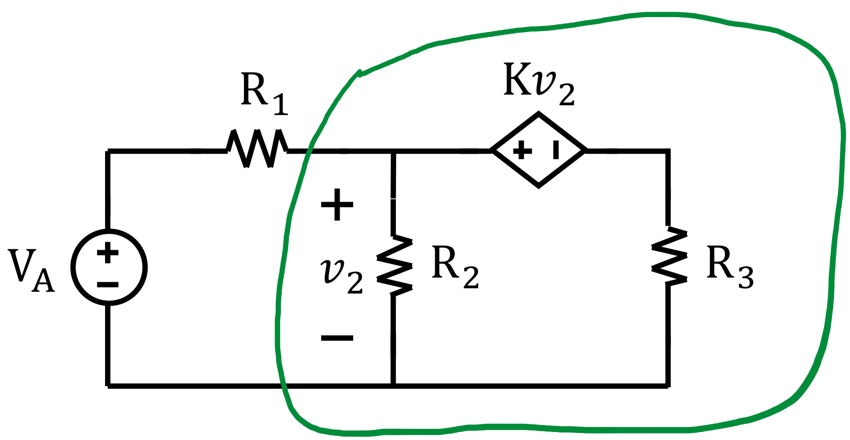
$$e_1 = \frac{(\beta + 1)R_2 V_A}{R_1 + (\beta + 1)R_2}$$

Analysis of Circuits with Dependent Sources (Cont.)

- Can use "super node" trick on nodes spanned by dependent source



- If dependent sources are linear, can do Thevenin and Norton equivalents of circuits with dependent sources, provided the control and output ports of the dependent sources are together inside the circuit being modeled



Independent Sources vs. Dependent Sources

- The voltage or current value of an Independent Source is independent of the circuit in which it is connected

- The voltage or current value of a Dependent Source depends on voltage or current in some other part of the circuit

- In some sense the Dependent Source is similar to a Resistor
  - In Resistor (linear or non-linear), voltage across its port depends on current through its port
  - In Dependent Source (linear or non-linear), voltage (current) across (through) its output port depends on voltage (current) across (through) its control port

- Superposition and Thevenin/Norton will only work if the Dependent Sources and the rest of the circuit is linear
  - Do not ”kill” dependent sources when doing superposition
  - Do not ”kill” dependent sources when finding Thevenin Resistance