

ECE4740: Digital VLSI Design

Lecture 22: Tree adders

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Recap

Carry-skip and carry-select adders

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Carry-skip adder principle

$BP = P_0 P_1 P_2 P_3$ "Block Propagate"

why is this the critical path?

- If $BP = P_0 P_1 P_2 P_3 = 1$ then $C_{O,3} = C_{i,0}$, otherwise block itself generates (or kills) carry internally

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N-bit carry skip adder

no direct path to carry out

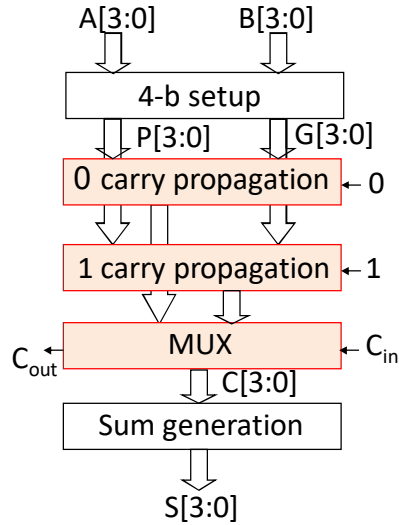
- Set block size to $B = \sqrt{N/2}$
- Delay grows only with \sqrt{N}

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Image taken from: CMOS VLSI Design: A Circuits and Systems Perspective by Weste, Harris

Carry select adder (CSA)

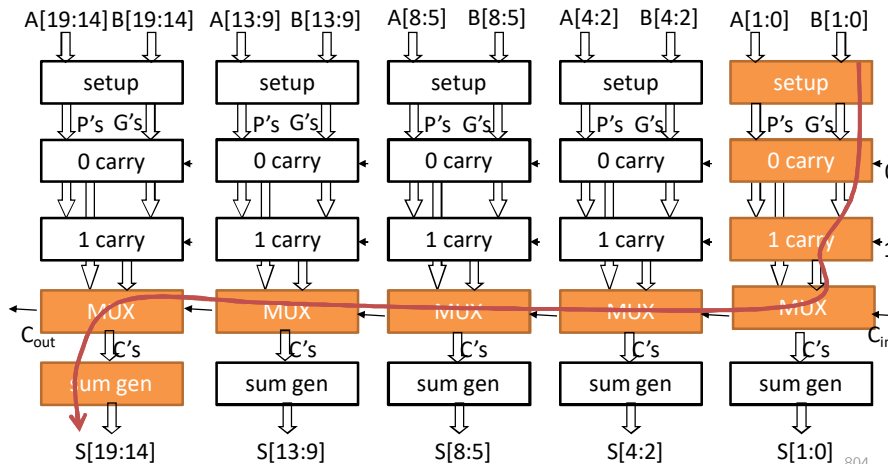
- Pre-compute carry out for each block for $C_{in}=0$ and $C_{in}=1$
- Select correct outputs as soon as C_{in} is ready
- Only a MUX in the critical path!



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Square root carry select adder

- Linearly increasing group sizes: T grows in \sqrt{N}



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Recap and more topologies

Tree adders

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The carry recurrence

- Remember: $C_{i+1} = G_i + P_i C_i$

$$C_1 = G_0 + P_0 C_0$$

- $C_2 = (G_1 + P_1 G_0) + (P_1 P_0) C_0$



new **group generate** and **group propagate** signals

- Can be modeled as an operation on a tuple:

$$(G_i, P_i) \bullet (G_{i-1}, P_{i-1}) = (G_i + P_i * G_{i-1}, P_i * P_{i-1})$$

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PG diagram notation

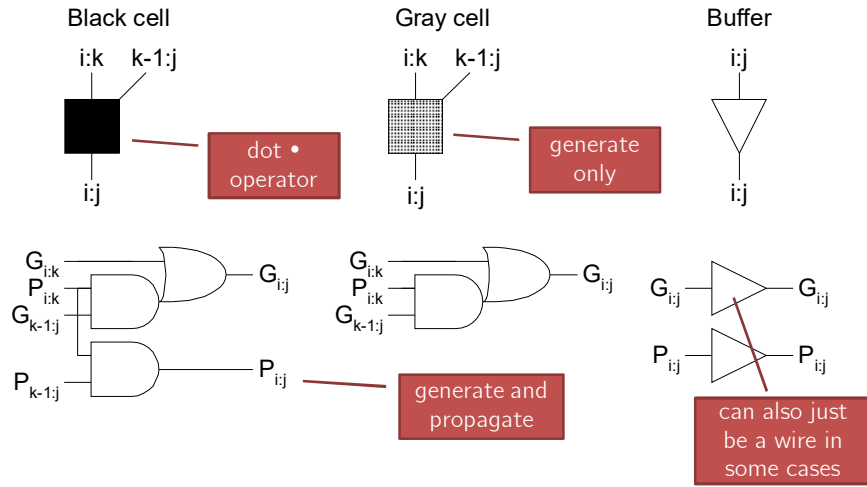


Image Adapted From: CMOS VLSI Design: A Circuits and Systems Perspective by Weste, Harris

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Brent-Kung adder (1982)

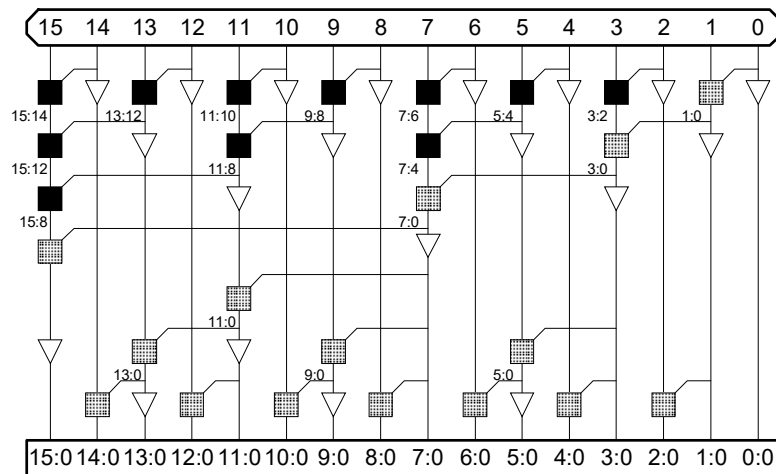


Image taken from: CMOS VLSI Design: A Circuits and Systems Perspective by Weste, Harris

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Brent-Kung adder (cont'd)

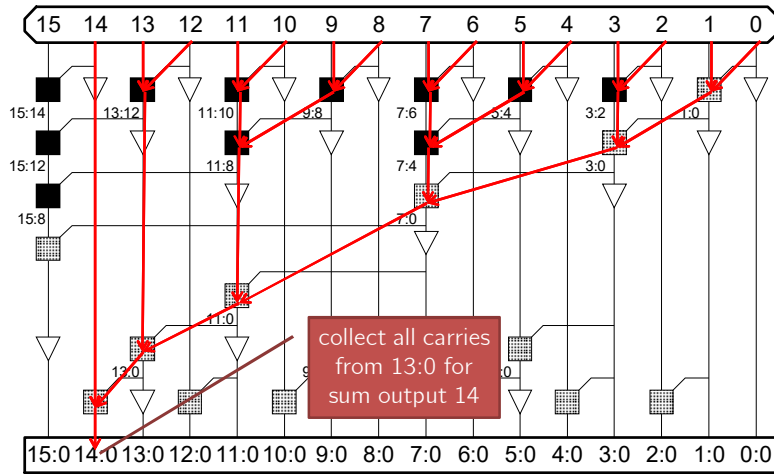
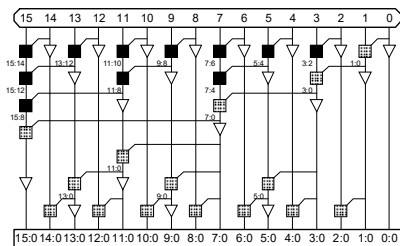


Image taken from: CMOS VLSI Design: A Circuits and Systems Perspective by Weste, Harris 809

Summary: Brent-Kung adder

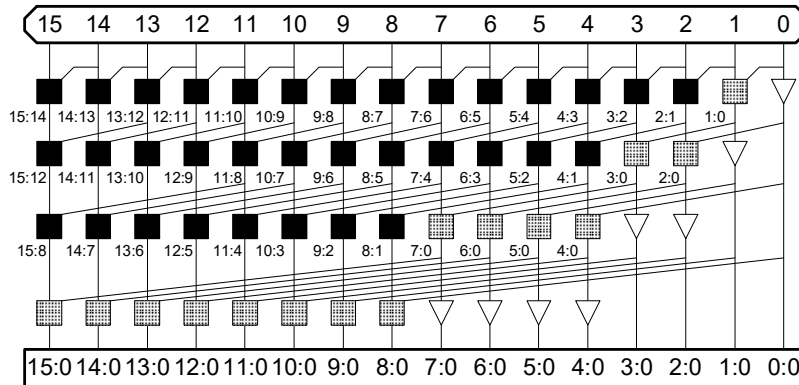
- $A_c = 2N - \log(N) - 2$
- $T_c = 2\log(N) - 2$
- $FO_{max} = \log(N)$
- Pros:
 - regular structure (...really?)
 - limited fan-in for all gates
- Cons:
 - FO is an issue: grows $\log(N)$
 - Power? _____



uneven path lengths
→ glitches

Image taken from: CMOS VLSI Design: A Circuits and Systems Perspective by Weste, Harris 810

Kogge-Stone adder (1973)

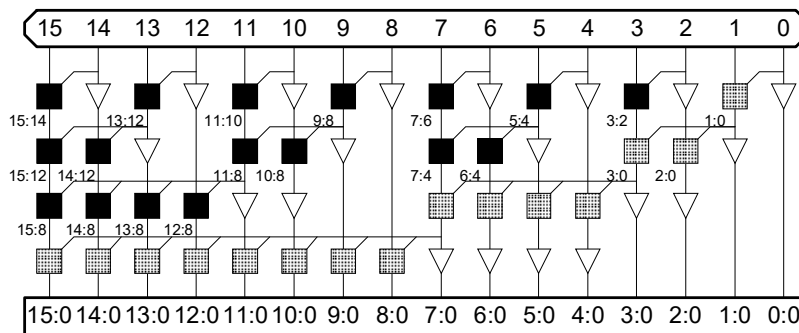


- $A_s = N \log N - N + 1$
- $T_s = \log(N)$
- $FO_{max} = 2$
- High wiring overhead

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Image taken from: CMOS VLSI Design: A Circuits and Systems Perspective by Weste, Harris

Sklansky adder (1960)



- $A_s = 0.5N \log(N)$
- $T_s = \log(N)$
- $FO_{max} = N/2$



Images taken from: CMOS VLSI Design: A Circuits and Systems Perspective by Weste, Harris; <https://www.the-scientist.com/?articles.view/articleNo/18705/title/New-Technology-Weighs-In-On-Mammography-Debate/> Jack Sklansky, UC Irvine

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Which tree adder should I pick?

Name	Area [A.]	Time [T.]	Max. FO	Wiring
Ripple carry	$N-1$	$N-1$	2	☹☹
Sklansky	$N/2 \cdot \log(N)$	$\log(N)$	$N/2$	☹
Brent-Kung	$2N - \log(N) - 2$	$2 \cdot \log(N) - 2$	$\log(N)$	😊
Kogge-Stone	$N \cdot \log(N) - N + 1$	$\log(N)$	2	☹☹
Carry increment	$2N - \sqrt{2N}$	$\sqrt{2N}$	$\sqrt{2N}$	😊

- Trade-off between area, propagation delay, etc.
- For adders with a small number of bits, do not forget carry select and carry skip adders!

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What are CAD tools doing?

Adder design with Synopsys DC

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Synopsys design compiler (DC)

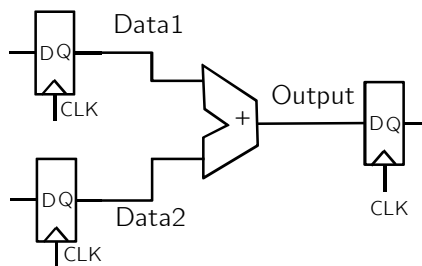
- One of the leading CAD tools for digital integrated circuits and FPGAs
- How do you use it?
 - You write hardware description language
 - You provide constraints and then compile it
 - The tool generates a gate-level netlist
- Automatic logic optimization, sizing, etc.

SYNOPSYS[®]

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Simple example

- Faraday 90nm CMOS technology
- 16-bit adder design
 - 16 bit inputs (Data1 and Data2)
 - 17 bit outputs (Output)



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Synthesis script example

```
[...]
analyze -library WORK -format vhdl {chris_adder.vhd}
elaborate chris_adder -architecture arch1 -library WORK
create_clock -name "ClkxCI" -period 5 -waveform {0 2.5} {ClkxCI}
compile_ultra
```

set the clock
period to 5ns
= constraint

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Area results for T=5ns

WHY?

- 49 sequential cells (D-flip-flops)
- 18 combinational cells
- **Combinational area = 463um²**
- Noncombinational area = 882um²

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```

*****
Report : area
Design : chris_adder
Version: ██████████
Date   : Thu Apr 9 03:59:42 2015
*****
Library(s) Used:
████████████████████████████████████████████████████████████████████████████████
Number of ports:          51
Number of nets:          116
Number of cells:         67
Number of combinational cells: 18
Number of sequential cells: 49
Number of macros/black boxes: 0
Number of buf/inv:       0
Number of references:    4
Combinational area:      463.000000
Buf/Inv area:            0.000000
Noncombinational area:  882.000000
Macro/Black Box area:   0.000000
Net Interconnect area:  undefined (Wire load has zero net area)
Total cell area:         1345.000000
Total area:              undefined

Hierarchical area distribution
-----

```

how Synopsys reports
the circuit area

usually comes without
units → depends on library

Hierarchical cell	Global cell area		Local cell area			Design
	Absolute Total	Percent Total	Combi-national	Noncombi-national	Black-boxes	
chris_adder	1345.0000	100.0	463.0000	882.0000	0.0000	chris_adder
Total			463.0000	882.0000	0.0000	

Timing results for 5ns

- Critical path: $t_{pdmax} = 4.76ns$
 - Startpoint: LSB of Data1
 - Endpoint: carry output (bit 16)
- Most likely a simple ripple carry adder
- Carry out (= bit 16) is critical!

Part 1

```

*****
Report : timing
        -path full
        -delay max
        -max_paths 1
Design : chris adder
Version: ██████████
Date   : Thu Apr 9 03:59:42 2015
*****

Operating Conditions: TCCOM Library: ██████████
Wire Load Model Mode: enclosed

Startpoint: Data1regxDR_reg[0]
            (rising edge-triggered flip-flop clocked by ClkxCI)
Endpoint:  OutputregxDR_reg[16]
            (rising edge-triggered flip-flop clocked by ClkxCI)
Path Group: ClkxCI
Path Type: max

Des/Clust/Port      Wire Load Model      Library
-----
chris_adder        enG5K                            ██████████
    
```

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Part 2

AN2RLX1
= 2-input
AND gate
w/ drive
strength 1

Point	Incr	Path
clock ClkxCI (rise edge)	0.00	0.00
clock network delay (ideal)	0.00	0.00
Data1regxDR_reg[0]/CK (QDFFRBRLX1)	0.00	0.00 r
Data1regxDR_reg[0]/Q (QDFFRBRLX1)	0.33	0.33 f
U5/Q (AN2RLX1)	0.16	0.49 f
intadd_0/U16/CO (FA1RLX1)	0.22	0.71 f
intadd_0/U15/CO (FA1RLX1)	0.23	0.93 f
intadd_0/U14/CO (FA1RLX1)	0.23	1.16 f
intadd_0/U13/CO (FA1RLX1)	0.23	1.38 f
intadd_0/U12/CO (FA1RLX1)	0.23	1.61 f
intadd_0/U11/CO (FA1RLX1)	0.23	1.84 f
intadd_0/U10/CO (FA1RLX1)	0.23	2.06 f
intadd_0/U9/CO (FA1RLX1)	0.23	2.29 f
intadd_0/U8/CO (FA1RLX1)	0.23	2.51 f
intadd_0/U7/CO (FA1RLX1)	0.23	2.74 f
intadd_0/U6/CO (FA1RLX1)	0.23	2.97 f
intadd_0/U5/CO (FA1RLX1)	0.23	3.19 f
intadd_0/U4/CO (FA1RLX1)	0.23	3.42 f
intadd_0/U3/CO (FA1RLX1)	0.23	3.64 f
intadd_0/U2/CO (FA1RLX1)	0.20	3.84 f
OutputregxDR_reg[16]/D (QDFFRBRLX1)	0.00	3.84 f
data arrival time		3.84
clock ClkxCI (rise edge)	5.00	5.00
clock network delay (ideal)	0.00	5.00
OutputregxDR_reg[16]/CK (QDFFRBRLX1)	0.00	5.00 r
library setup time	-0.24	4.76
data required time		4.76
data required time		4.76
data arrival time		-3.84
slack (MET)		0.92

4.76ns -
3.84ns =
0.92 ns

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The reason (VHDL ftw!)

```

architecture arch1 of chris_adder is
    signal Data1regxDR: std_logic_vector(number_of_bits-1 downto 0);
    signal Data2regxDR: std_logic_vector(number_of_bits-1 downto 0);
    signal OutputregxDR: std_logic_vector(number_of_bits downto 0);

begin
    process(ClkxCI, RstxCi)
    begin
        if RstxCi = '0' then
            Data1regxDR <= (others => '0');
            Data2regxDR <= (others => '0');
            OutputregxDR <= (others => '0');
        elsif ClkxCI'event and ClkxCI = '1' then
            Data1regxDR <= Data1xDI;
            Data2regxDR <= Data2xDI;
            OutputregxDR <= std_logic_vector(unsigned('0' & Data1regxDR) + unsigned('0' & Data2regxDR));
        end if;
    end process;

    OutputxD0 <= OutputregxDR;
end arch1;

```

asynchronous reset

do not write data-path
stuff in combinational
blocks → bad practice

Can you see it now?

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What happens if we set $T=1\text{ns}$?

- Very aggressive delay constraint!
- Combinational area = $1323\mu\text{m}^2$
- Noncombinational area = $987\mu\text{m}^2$
- (Remember $463\mu\text{m}^2$, $882\mu\text{m}^2$ of ripple carry adder)

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Part 1

```

*****
Report : timing
        -path full
        -delay max
        -max_paths 1
Design : chris_adder
Version: ██████████
Date   : Thu Apr 9 04:02:00 2015
*****

Operating Conditions: TCCOM  Library: ██████████
Wire Load Model Mode: enclosed

Startpoint: Data2regxDR_reg[5]
             (rising edge-triggered flip-flop clocked by ClkxCI)
Endpoint:   OutputregxDR_reg[13]
             (rising edge-triggered flip-flop clocked by ClkxCI)
Path Group: ClkxCI
Path Type:  max

Des/Clust/Port  Wire Load Model  Library
-----
chris_adder    enG5K                fsd0k_a_generic_core_1d0vtc
    
```

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Part 2

Point	Incr	Path
clock ClkxCI (rise edge)	0.00	0.00
clock network delay (ideal)	0.00	0.00
Data2regxDR_reg[5]/CK (QDFFBRLX3)	0.00	0.00 r
Data2regxDR_reg[5]/Q (QDFFBRLX3)	0.28	0.28 f
U76/0 (NR2RLX3)	0.09	0.37 r
U75/0 (AOI12RLX2)	0.06	0.43 f
U96/0 (AOI12RLX2)	0.18	0.61 r
U84/0 (ND2RLX6)	0.13	0.74 f
U69/0 (ND3RLX2)	0.07	0.82 r
U113/0 (ND3RLX2)	0.07	0.89 f
U127/0 (XOR2RLX1)	0.11	1.00 f
OutputregxDR_reg[13]/D (QDFFBRLX1)	0.00	1.00 f
data arrival time		1.00
clock ClkxCI (rise edge)	1.00	1.00
clock network delay (ideal)	0.00	1.00
OutputregxDR_reg[13]/CK (QDFFBRLX1)	0.00	1.00 r
library setup time	-0.24	0.76
data required time		0.76
data required time		0.76
data arrival time		-1.00
slack (VIOLATED)		-0.24

1.00ns -
0.76ns =
-0.24 ns



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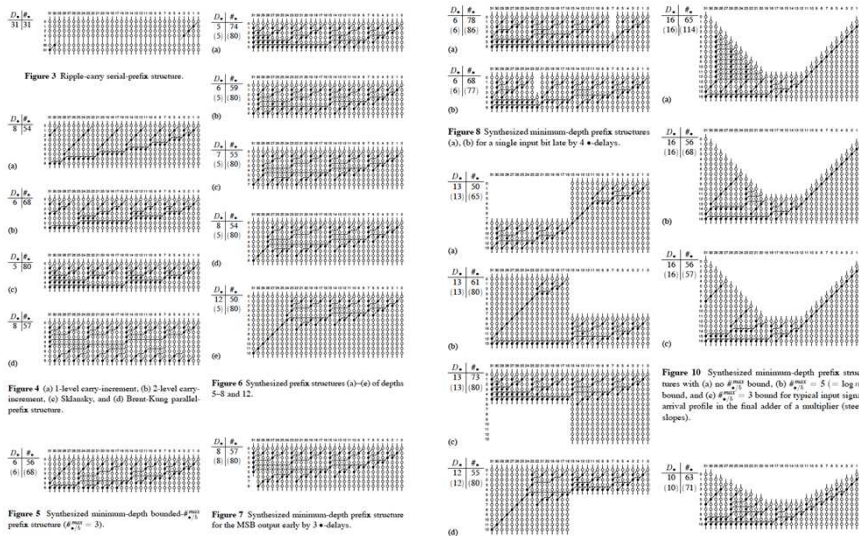
Fastest design

- $t_{pd} = 1.24ns$ (including flip-flop timing)
- $t_{pd} = 720ps$ (without flip-flop timing)
- Old lab 4 constraints:
 - 500ps critical path
 - Area smaller than $800\mu m^2 \rightarrow 5pts$
- Hand-design can be faster & smaller than tool-based design (but it's much more work)!

Note that we are comparing different processes here and in lab 4; flip-flop timing: propagation delay = 280ps=propagation, setup time = 240ps

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CAD tools are smart!



R. Zimmermann, "Non-Heuristic Optimization and Synthesis of Parallel-Prefix Adders," IWLAS 1996

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