

# ECE4740: Digital VLSI Design

## Lecture 20: Pipelining and adders

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### Semiconductor revenue grew by 22.2% in 2017

2017 Rank	2016 Rank	Vendor	2017 Revenue	2017 Market Share (%)	2016 Revenue	2016-2017 Growth (%)
1	2	Samsung Electronics	61,215	14.6	40,104	52.6
2	1	intel	57,712	13.8	54,091	6.7
3	4	SK Hynix	26,309	6.3	14,700	79.0
4	6	Micron Technology	23,062	5.5	12,950	78.1
5	3	Qualcomm	17,063	4.1	15,415	10.7
6	5	Broadcom	15,490	3.7	13,223	17.1
7	7	Texas Instruments	13,806	3.3	11,901	16.0
8	8	Toshiba	12,813	3.1	9,918	29.2
9	17	Western Digital	9,181	2.2	4,170	120.2
10	9	NXP	8,651	2.1	9,306	-7.0
		<b>Others</b>	<b>174,418</b>	<b>41.6</b>	<b>157,736</b>	<b>10.6</b>
		<b>Total Market</b>	<b>419,720</b>	<b>100.0</b>	<b>343,514</b>	<b>22.2</b>

Source: Gartner (January 2018)

<https://www.gartner.com/newsroom/id/3842666>

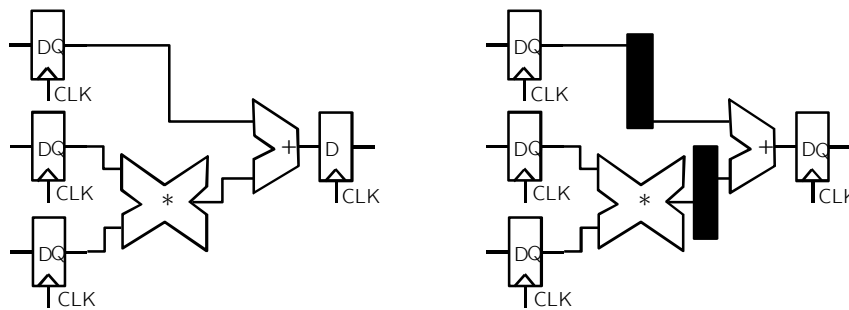
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Recap

## Pipelining and more

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## Pipelining increases throughput

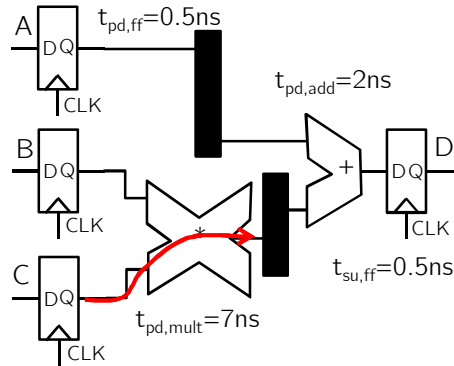


- Process same amount of data per clock cycle
- But clock frequency is higher because critical path has been reduced

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## Can we do better?

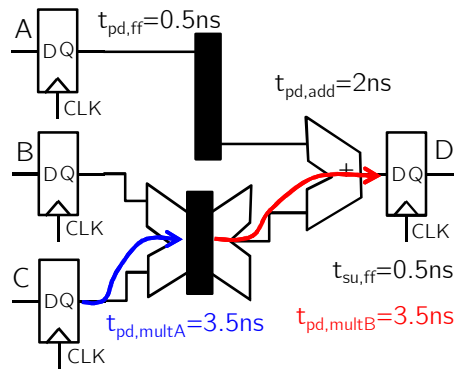
- Assume you want only 1 pipeline stage but faster clock frequency



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## Can we do better? → retiming

- Retiming helps to balance critical paths
- Not always possible ☹

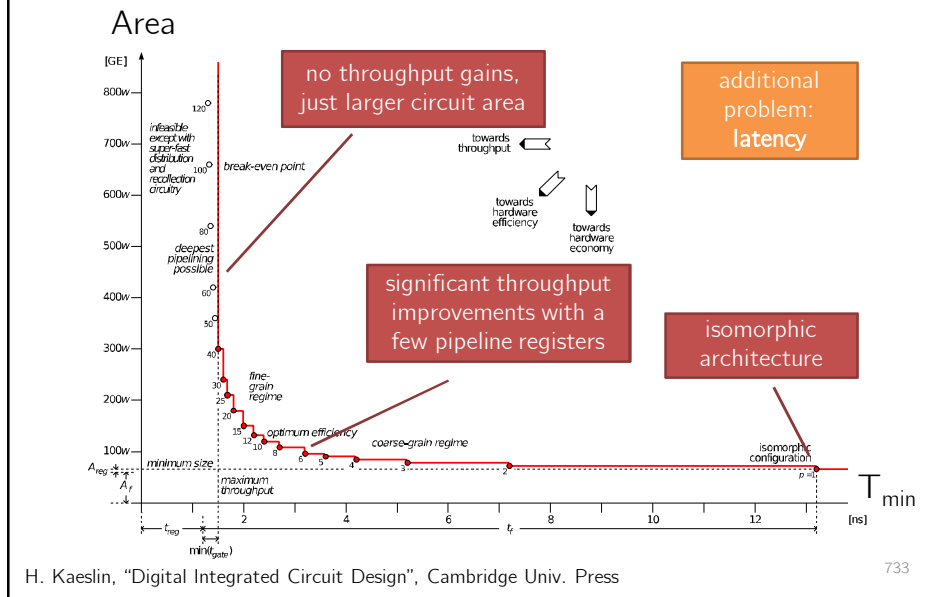


$$T_{\min} = t_{pd,ff} + t_{pd,multB} + t_{pd,add} + t_{su,ff} = 6.5ns$$

$f_{\max} = 153MHz$

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## Coarse grain vs. fine grain pipelining



## Pipelining/retiming summary

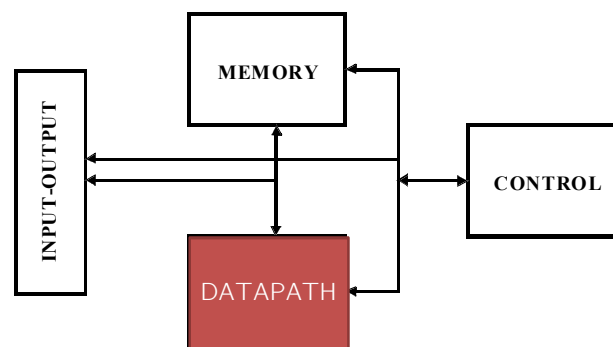
- Pipelining and retiming can significantly improve throughput of a VLSI design
- For coarse-grained pipelining, hardware overhead (area) is rather small
- Pipelining suppresses glitches!
- Problems:
  - Feedback causes data-dependency problems
  - Too many registers do, in general, not help much but increase area (fine-grained pipelining)

Very important basic building blocks

## Adder circuits

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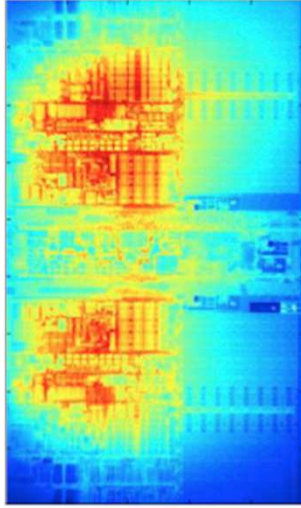
## Architecture of generic processor



- Datapath contains main arithmetic units: **adder**, multiplier, divider, shifter, etc.

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## Processor thermal map

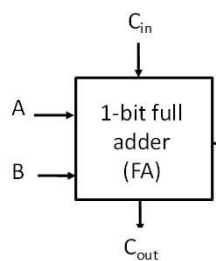


- AMD Athlon II dual core (45nm, 2.1GHz, SPEC CPU 2006)
- Highest power consumption in integer and floating-point arithmetic units

<http://scale.engin.brown.edu/tools/>

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## The full adder (FA)



A	B	C <sub>in</sub>	C <sub>out</sub>	S	carry status
0	0	0	0	0	kill (K)
0	0	1	0	1	kill (K)
0	1	0	0	1	propagate (P)
0	1	1	1	0	propagate (P)
1	0	0	0	1	propagate (P)
1	0	1	1	0	propagate (P)
1	1	0	1	0	generate (G)
1	1	1	1	1	generate (G)

- $G = A * B$
- $P = A \oplus B$
- $K = !A * !B$

- $S = A \oplus B \oplus C_{in} = P \oplus C_{in}$
- $C_{out} = A * B + A * C_{in} + B * C_{in}$   
 $= G + P * C_{in}$

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## The full adder (cont)

A	B	C <sub>in</sub>	C <sub>out</sub>	S	carry status
0	0	0	0	0	kill (K)
0	0	1	0	1	kill (K)
0	1	0	0	1	propagate (P)
0	1	1	1	0	propagate (P)
1	0	0	0	1	propagate (P)
1	0	1	1	0	propagate (P)
1	1	0	1	0	generate (G)
1	1	1	1	1	generate (G)

- $G = A * B$
- $P = A \oplus B$
- $K = !A * !B$

- $S = A \oplus B \oplus C_{in} = P \oplus C_{in}$
- $C_{out} = A * B + A * C_{in} + B * C_{in}$   
 $= G + P * C_{in}$

independent of C<sub>in</sub>

majority function

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## GPK: generate, propagate, kill

A	B	C <sub>in</sub>	C <sub>out</sub>	S	carry status
0	0	0	0	0	kill (K)
0	0	1	0	1	kill (K)
0	1	0	0	1	propagate (P)
0	1	1	1	0	propagate (P)
1	0	0	0	1	propagate (P)
1	0	1	1	0	propagate (P)
1	1	0	1	0	generate (G)
1	1	1	1	1	generate (G)

- $G = A * B$
- $P = A \oplus B$
- $K = !A * !B$

- **Generate**  $C_{out}=1$  independent of  $C_{in}$
- $C_{out} = G + P * C_{in}$

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## GPK: generate, propagate, kill

A	B	C <sub>in</sub>	C <sub>out</sub>	S	carry status
0	0	0	0	0	kill (K)
0	0	1	0	1	kill (K)
0	1	0	0	1	propagate (P)
0	1	1	1	0	propagate (P)
1	0	0	0	1	propagate (P)
1	0	1	1	0	propagate (P)
1	1	0	1	0	generate (G)
1	1	1	1	1	generate (G)

- $G = A*B$

- $P = A \oplus B$

- $K = !A*!B$

- Propagate  $C_{out} = C_{in}$

- $C_{out} = G + P * C_{in}$

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## GPK: generate, propagate, kill

A	B	C <sub>in</sub>	C <sub>out</sub>	S	carry status
0	0	0	0	0	kill (K)
0	0	1	0	1	kill (K)
0	1	0	0	1	propagate (P)
0	1	1	1	0	propagate (P)
1	0	0	0	1	propagate (P)
1	0	1	1	0	propagate (P)
1	1	0	1	0	generate (G)
1	1	1	1	1	generate (G)

- $G = A*B$

- $P = A \oplus B$

- $K = !A*!B$

- Kill  $C_{out}$ , i.e.,  $C_{out} = 0$  independent of  $C_{in}$

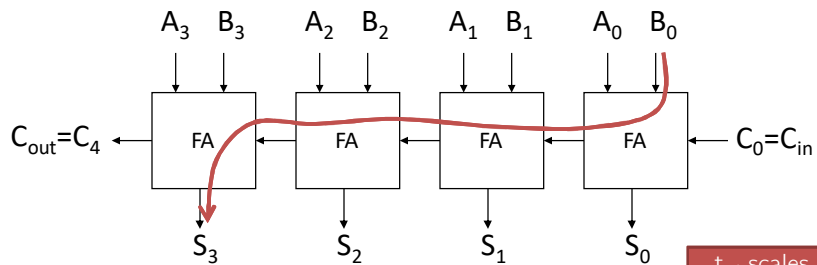
- $C_{out} = G + P * C_{in} = 0 + 0 * C_{in}$

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## How to build an N-bit adder?

- Simplest solution: ripple carry adder (RCA)

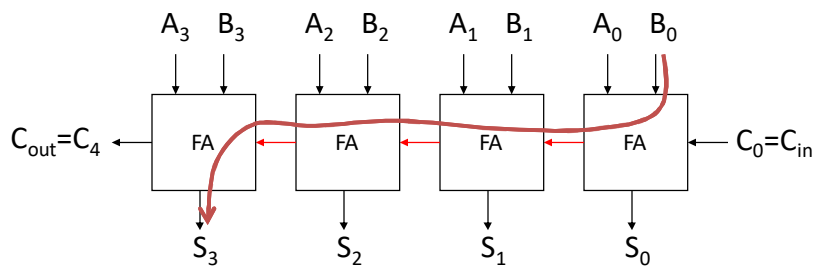


- $t_{pd,RCA} \approx (N-1)t_{pd,carry} + t_{pd,sum}$
- Too slow in most applications (e.g., in a processor where  $N=64$  or  $N=128$ )

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## Ripple carry adder (RCA)

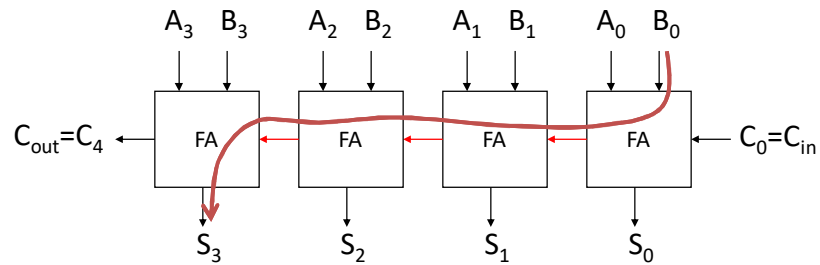
- Carry signal is critical!



- $t_{pd,RCA} \approx (N-1)t_{pd,carry} + t_{pd,sum}$
- Goal: make fastest possible carry path!

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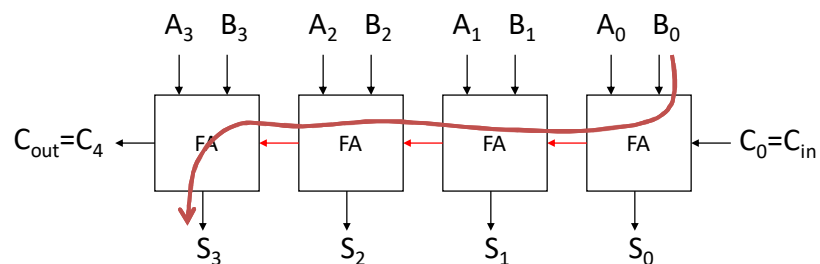
## Worst/best-case input patterns?



- What is a worst-case input pattern?
- What is a best-case input pattern?

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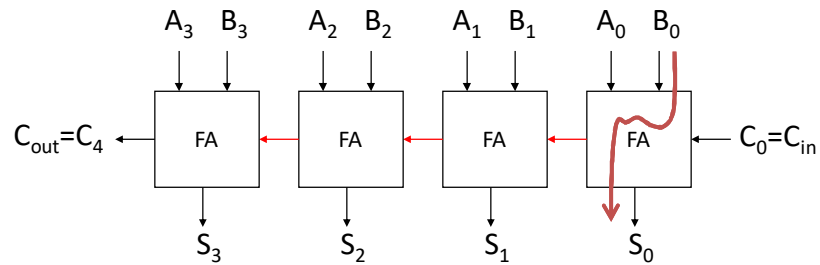
## Worst-case input patterns



- Not that easy... one needs to know FA design
- One "solution":  $A=[0111], B=[0000], C=0$   
 $A=[0000], B=[0001], C=0$

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## Best-case input patterns



- Not that easy... one needs to know FA design
- One "solution":  $A=[0000]$ ,  $B=[0000]$ ,  $C=0$   
 $A=[0000]$ ,  $B=[0000]$ ,  $C=1$

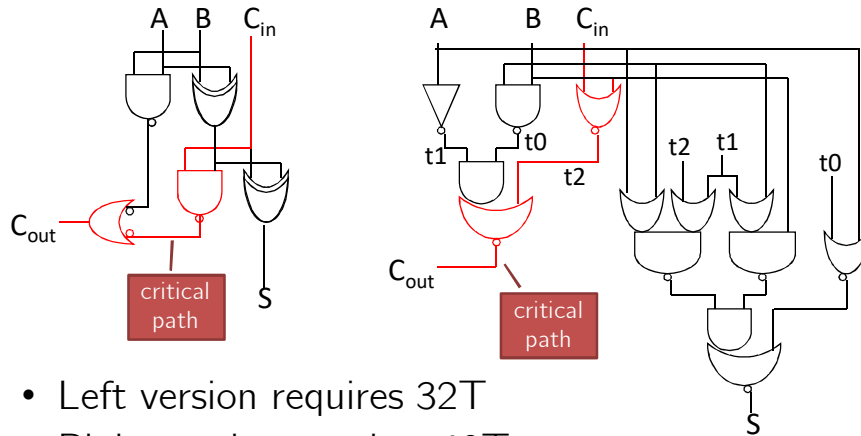
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Some basic tricks

## How to speed up adders

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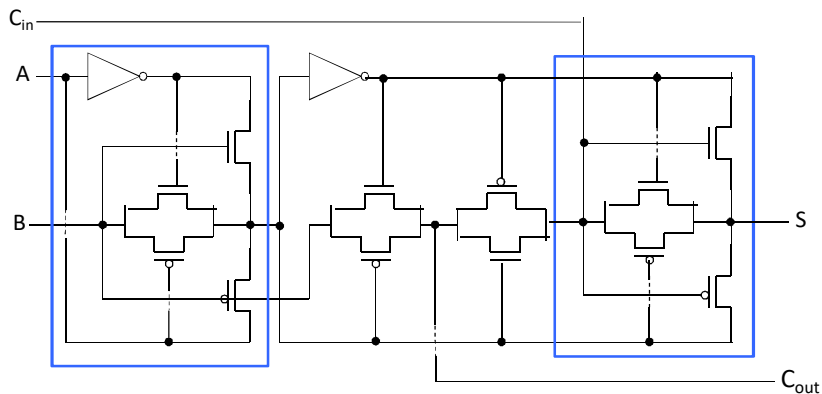
### (bad) FA gate-level implementations



- Left version requires 32T
- Right version requires 40T

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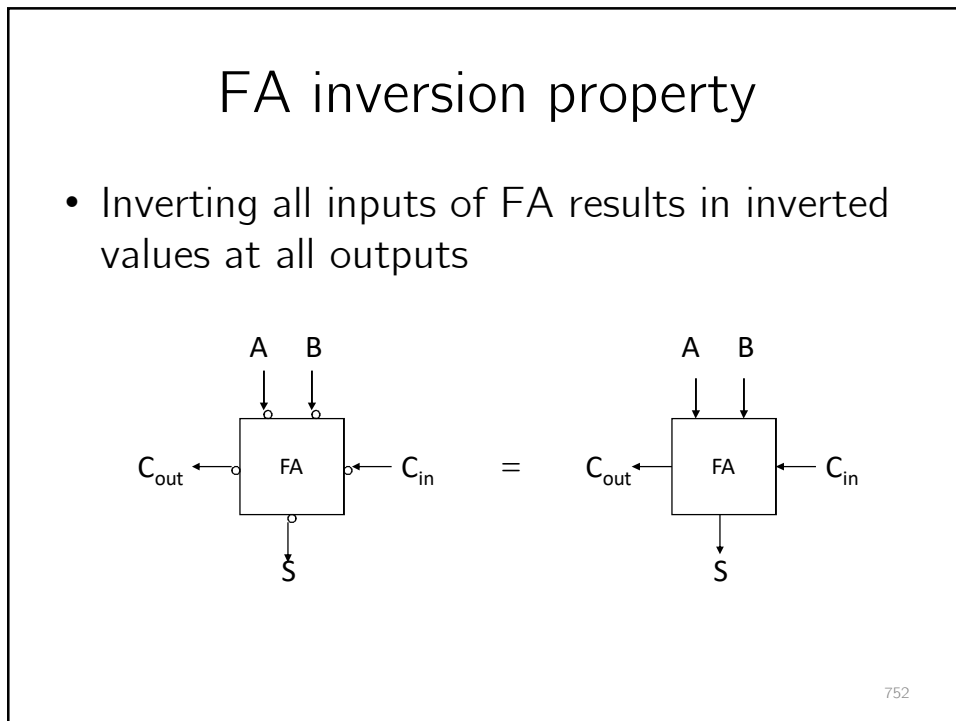
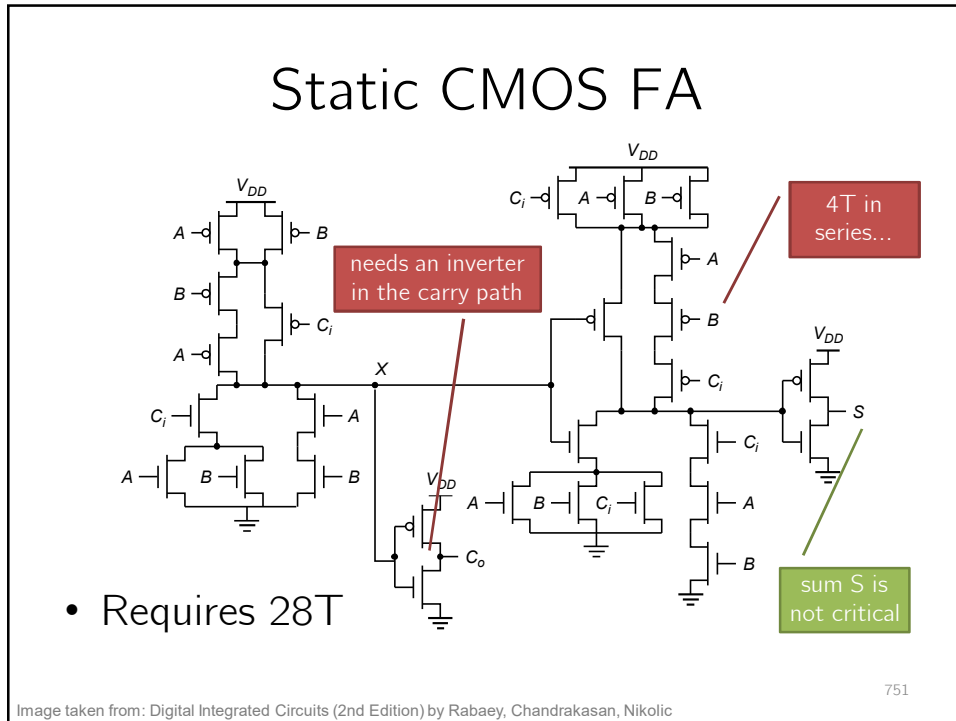
### Recall the TG XOR FA



- Requires only 16T
- Circuit is not very fast (TGs in series)

Vesterbacka, Mark. "A 14-transistor CMOS full adder with full voltage-swing nodes." *Signal Processing Systems, 1999. SIPS 99. 1999 IEEE Workshop on*. IEEE, 1999.

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## FA inversion property (cont'd)

- Inverting all inputs of FA results in inverted values at all outputs

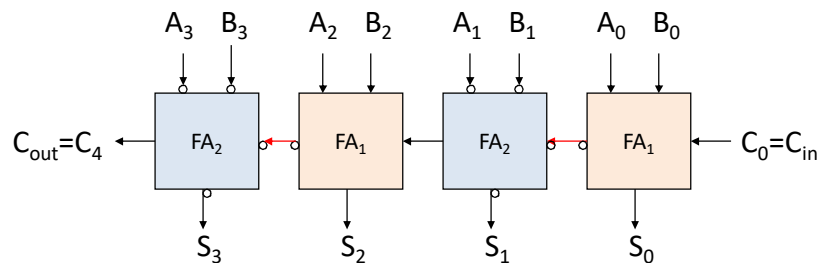
A	B	C <sub>in</sub>	C <sub>out</sub>	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

A	B	C <sub>in</sub>	C <sub>out</sub>	S
1	1	1	1	1
1	1	0	1	0
1	0	1	1	0
1	0	0	0	1
0	1	1	1	0
0	1	0	0	1
0	0	1	0	1
0	0	0	0	0

- $S = !A \oplus !B \oplus !C_{in}$
- $C_{out} = !A * !B + !A * !C_{in} + !B * !C_{in}$

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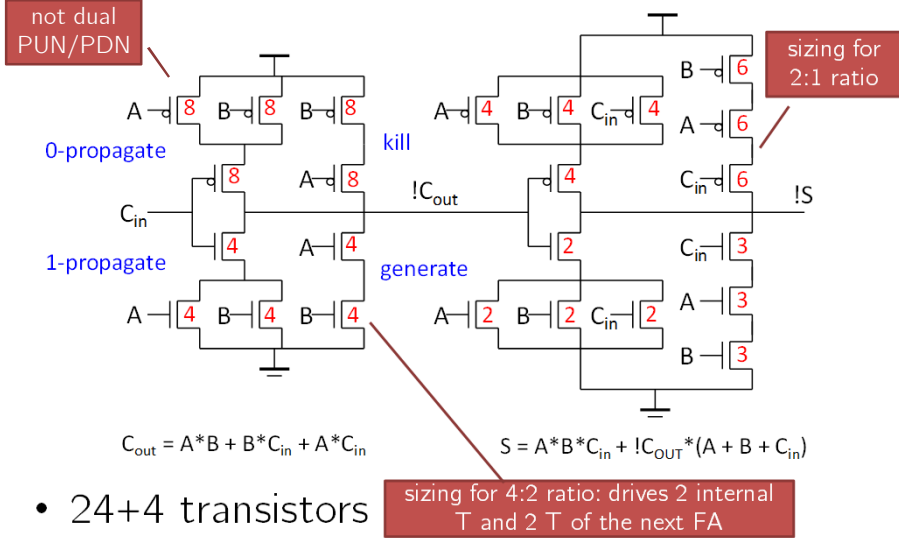
## Exploiting the inversion property



- Reduces critical path in carry chain by eliminating inverters between FAs
- Requires two different FA designs

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## A better FA structure: mirror adder



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## Stick diagram of mirror adder

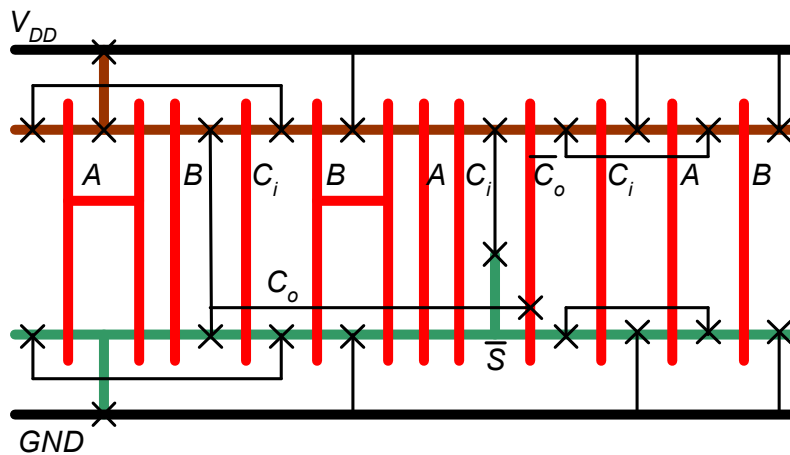


Image taken from: Digital Integrated Circuits (2nd Edition) by Rabaey, Chandrakasan, Nikolic

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## Features of the mirror adder

- PUN and PDN are completely symmetrical
- Proper sizing → symmetric rise & fall times
- Transistors connected to  $C_{in}$  are placed closest to output → faster
- For layout, critical to minimize capacitance at node ! $C_{out}$  → use shared diffusions
- In carry circuit, maximum of 2T in series
- Only transistors in carry stage need to be optimized for speed, others → minimum size!

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