Prelab Problem 5.1: Input and output sinusoids

Figure 1 shows the input and output sinusoidal voltage waveforms for a two-port linear network. Since the network is linear both sinusoids have the same frequency.

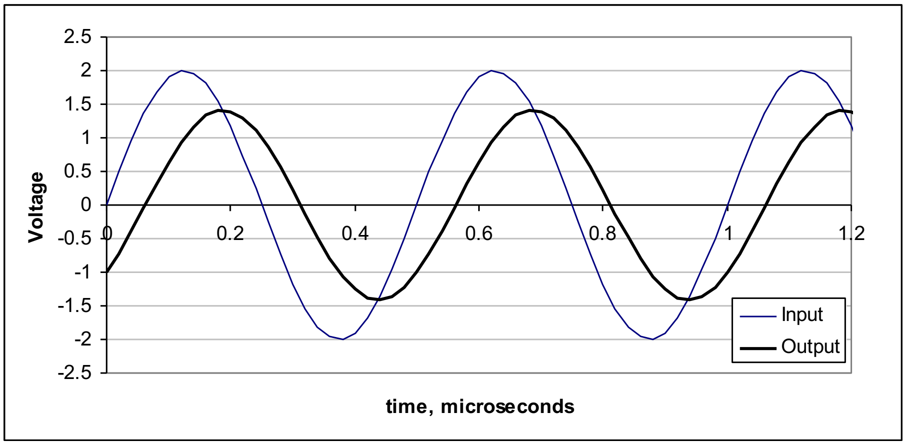

Fig. 1. Input and output sinusoidal waveforms for a two-port linear network.

From these two overlaid sinusoidal voltage waveforms, estimate:

(a) The frequency of the sinusoidal waveforms in Hz and in radians per second.

(b) The ratio of the amplitude of the output waveform to the amplitude of the input waveform.

(c) The phase of the output waveform relative to the phase of the input waveform. (Note that time delay corresponds to negative phase.)

(d) Express the input and output voltage waveforms as phasors (i.e., as $\mathrm{A}e^{j\phi}$, where A is the amplitude and ϕ is the phase; note you can arbitrarily choose the phase of the input voltage waveform as zero).

(e) Express the transformation $\widehat{\mathrm{H}}$ from input to output voltage as a phasor, i.e., determine $\widehat{\mathrm{H}} \equiv \widehat{\mathrm{V}}_{\mathrm{out}}/\widehat{\mathrm{V}}_{\mathrm{in}}$, where $\widehat{\mathrm{V}}_{\mathrm{out}}$ is the output voltage phasor and $\widehat{\mathrm{V}}_{\mathrm{in}}$ is the input voltage phasor. Also convert $\widehat{\mathrm{H}}$ into a complex number in Cartesian coordinates $\widehat{\mathrm{H}} = a + jb$, i.e., determine a and b. (*Hint: You can use Euler's Formula:* $e^{j\phi} = \cos\phi + j\sin\phi$).

Prelab Problem 5.2: Impedance and voltage-division characteristics of RLC networks

In lab 5, you will be analyzing the impedance and voltage-division characteristics of several two-port "black box" networks to reverse engineer them. To gain insights helpful for lab 5, here you will perform similar analysis on the four different two-port networks shown in Fig. 2. In Fig. 2, network A is purely resistive, network B has a capacitor in addition to two resistors, network C has an inductor in addition to two resistors, and network D has an inductor, a capacitor and a resistor.

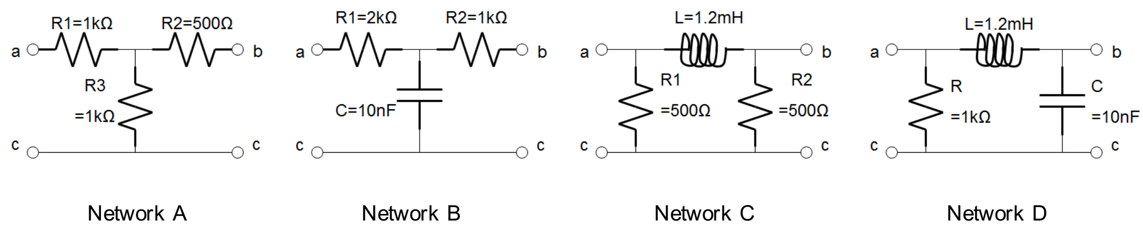

Fig. 2. Four example two-port linear networks.

For each network of Fig. 2, do the following:

(a) Determine the impedance across each pair of terminals, i.e., determine Z_{ac}, Z_{bc} and Z_{ab}? (Note that in general for an RLC network, impedance is a complex number and is a function of the applied sinusoid's frequency $\omega = 2\pi f$). Also compute the values of these three impedances at $f = 0$ Hz.

The two-port networks of Fig. 2 are driven by a sinusoidal voltage source $v_1(t) = 1\text{ V} \cdot \cos(\omega t)$ ($\hat{V}_1 = 1e^{j0}$) applied to port 1 (across terminals a and c), as shown in Fig. 3(a). For each network of Fig. 2, use voltage divider analysis to do the following:

(b) Determine the magnitude $|\hat{V}_2|$ and phase ϕ_2 of the voltage $\hat{V}_2$ at port 2 (across terminals b and c) as a functions of ω. Also compute the values of $|\hat{V}_2|$ and ϕ_2 at $f = 1$ kHz and $f = 100$ kHz. Also find the frequency in the range 1 kHz $< f <$ 100 kHz, where $|\hat{V}_2|$ is maximum.

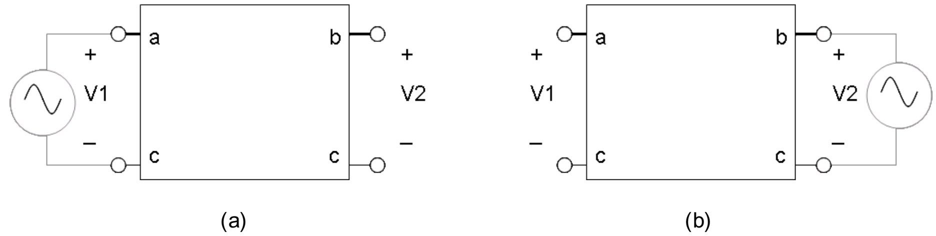

Fig. 3. Two-port network driven by a sinusoidal voltage source applied to: (a) port 1 and (b) port 2.

The two-port networks of Fig. 2 are driven by a sinusoidal voltage source $v_2(t) = 1\text{ V} \cdot \cos(\omega t)$ ($\hat{V}_2 = 1e^{j0}$) applied to port 2 (across terminals b and c), as shown in Fig. 3(b). For each network of Fig. 2, use voltage divider analysis to do the following:

(c) Determine the magnitude $|\hat{V}_1|$ and phase ϕ_1 of the voltage $\hat{V}_1$ at port 1 (across terminals a and c) as a functions of ω. Also compute the values of $|\hat{V}_1|$ and ϕ_1 at $f = 1$ kHz and $f = 100$ kHz. Also find the frequency in the range 1 kHz $< f <$ 100 kHz, where $|\hat{V}_1|$ is maximum.

(d) Based on the zero-frequency impedances and voltage divider results for network B, describe how you would extract the values of R_1 and R_2 in network B.

(e) Based on the zero-frequency impedances and voltage divider results for network C, describe how you would extract the values of R_1 and R_2 in network C.

(f) Based on the zero-frequency impedances and voltage divider results for networks D and C, describe how you would distinguish network D from network C, and how you would extract the values of each component in network D.