Poles and Zeros: s-Domain Analysis and Design

Goals:

1. Examine the frequency response of an inverting amplifier.
2. Determine frequency response of an RLC low pass filter modeling a low bandwidth channel.
3. Design and build an equalizer circuit to extend the bandwidth of the channel.
4. Explore effects of poles and zeros on frequency and step responses.

Preparation:

1. Carefully review this document.
2. Review lecture slides on Bode plots and Laplace Transform (s-domain) based circuit analysis.
3. Complete Prelab 6.

Experiments:

1. Frequency Response of an Inverting Amplifier

Build the inverting amplifier shown in Fig. 1 (same as the one built in Lab 3 part 1.2) using the UA741CP op-amp. Measure the values of both resistors of the amplifier using the DMM and record. Remember to connect the ± 6 volt power supplies. This inverting amplifier will now be driven with a sinusoidal signal and tested at a few different frequencies.

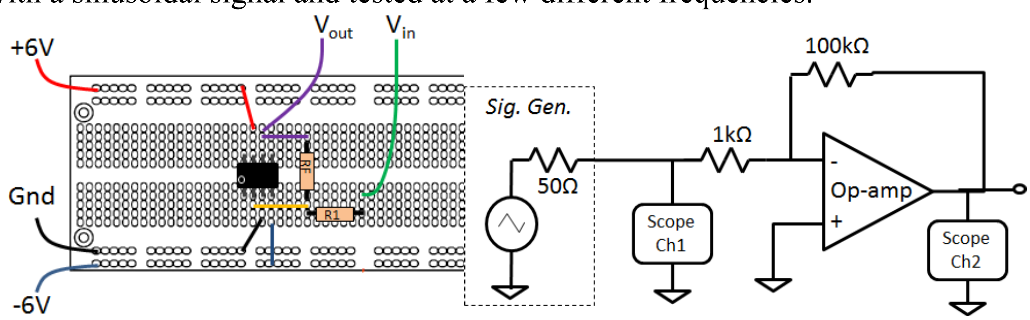

Fig. 1. Inverting amplifier circuit.

(a) **Setup function generator and scope, and test the amplifier**. Set up the signal generator to produce a sine-wave with amplitude of $50\ \mathrm{mV_{pp}}$, a zero offset, and a frequency of $1\ \mathrm{kHz}$. Connect the signal generator output and channel 1 (CH 1) of the oscilloscope to the input of the inverting amplifier, and channel 2 (CH 2) to the output of the amplifier. On the scope, the time base should be set to show a few cycles, and both channels scaled to fill the scope screen vertically. Remember we expect a big inverted voltage gain (of -100), so CH 1 must have a much finer scale than CH 2. Make sure each channel is still on DC coupling.

(b) **Determine the magnitude and phase of the input-to-output gain**. Enable the peak-to-peak voltage measurements (under "measure" menu) for each scope channel, as well as the phase (setup: source = CH 2, source2 = CH 1). Turn averaging on. Confirm that at 1 kHz the two waveforms are phase shifted by 180° with respect to each other, as this is an inverting amplifier. Record the input and output waveforms' peak-to-peak amplitudes and the approximate phase difference between the two waveforms with the function generator frequency set to the following different values: $1\ \mathrm{kHz}$, $2\ \mathrm{kHz}$, $5\ \mathrm{kHz}$, $10\ \mathrm{kHz}$, $20\ \mathrm{kHz}$, 50 kHz, 100 kHz and 200 kHz. Generate a magnitude and phase Bode plot of the measured results (you may use the provided file Bode.xls to generate the Bode plot).

(c) **Change the function generator output to a square wave.** For the square wave use $50\ \mathrm{mV_{pp}}$ and frequency of 1 kHz. Set the time scale of the oscilloscope to 25 μs/div. You should see an exponential response. ***Save your data for later analysis.***

(d) **Repeat parts b and c with the 100 kΩ feedback resistor replaced by a 10 kΩ resistor.** What impact does this have on the 3-dB bandwidth? What impact does this have on the time constant τ?

Next, you will design an equalizer circuit, intended to suppress the effects of an over-damped RLC circuit, which models an old-school telephone cable suitable for voice but not data transmission. ***You should leave the op-amp connected to its power supplies on the breadboard,*** as you will be using it again in parts 3 and 4.

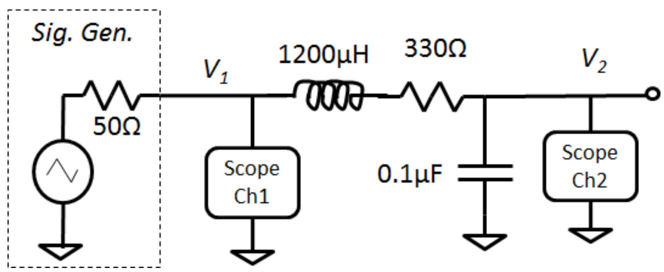

Fig. 2. RLC low-pass filter circuit serving the test platform for a low bandwidth channel.

2. RLC Low-Pass Filter Circuit

An RLC low-pass filter circuit will be used to emulate a low-bandwidth channel such as a telephone cable.

(a) **Build the RLC circuit.** Get the components (resistor, inductor and capacitor) from the bins, measure and record their values, and build the RLC low-pass filter shown in Fig. 2. Attach the signal source and CH 1 of the oscilloscope to V_1, and CH 2 of the oscilloscope to V_2.

(b) **Frequency response characterization of the RLC circuit.** Set the signal source to a $1\ \mathrm{V_{pp}}$ sinewave, and step frequency through: 1 kHz, 2 kHz, 5 kHz, 10 kHz, 20 kHz, 50 kHz, 100 kHz and 200 kHz, adjusting the oscilloscope's timescale to match. Use the scope to measure the peak-to-peak amplitude and phase of each waveform (same setup as part 1) and record the values. You need not save the actual waveforms. Generate a magnitude and phase Bode plot, and based on these results, estimate the location of the dominant (lowest frequency) pole of the circuit. Compare this to the calculation from Prelab problem 6.2(a). Note that at higher frequencies for the RLC, you may not be able to get accurate phase readings. That is fine, just note this fact in your comments. But also note that phase $= 140°$ is the same as $-220°$, so weird phases may just be wrap-around.

(c) **Response to a square-wave.** Now configure the signal source for a 20 kHz, $1\ \mathrm{V_{pp}}$ square wave and capture the input and output waveforms. ***Save the input and output waveforms for later analysis.***

Do not disassemble your RLC circuit.

3. Equalizer Circuit Design

Design an equalizer circuit using the topology shown in Fig. 3. Choose values of C_1, R_1 and R_F to generate an amplifier transfer function with a zero at the same location as the low frequency pole of the RLC low pass filter (modeling the telephone cable), so as to cancel the effect of this dominant pole. Choose your component values such that, for an equalizer circuit using an ideal op-amp $V_{eq}/V_{sig} = G(s) = (1 + s/z)/(1 + s/p)$, where the pole $p \sim 300$ krad/s (50 kHz), and the zero z cancels the dominant pole of the RLC channel model. One additional constraint you may want to impose is to make $R_F > 1$ kΩ, to ensure you don't overload the output current of the op-amp.

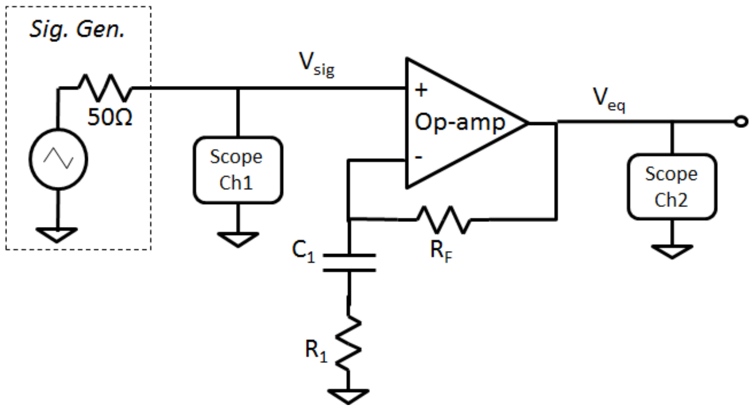

Fig. 3. Equalizer circuit topology.

(a) **Build the equalizer circuit.** Using the selected component values build the equalizer circuit of Fig. 3. Attach the signal source and CH 1 of the oscilloscope to V_{sig}, and CH 2 to V_{eq}.

(b) **Frequency response of the equalizer circuit.** Once again, configure the signal generator for a 100 mV_{pp} sinewave and step frequency through 1 kHz, 2 kHz, 5 kHz, 10 kHz, 20 kHz, 50 kHz, 100 kHz and 200 kHz. Measure and record the input and output waveforms' peak-to-peak amplitudes and the phase difference between the two waveforms at each frequency. You need not save the actual waveforms. Generate a magnitude and phase Bode plot and confirm the location of the zero.

(c) **Response to a square-wave.** Now configure the signal source for a 20 kHz, 100 mV_{pp} square wave and capture the input and output waveforms. ***Save this data for your report.***

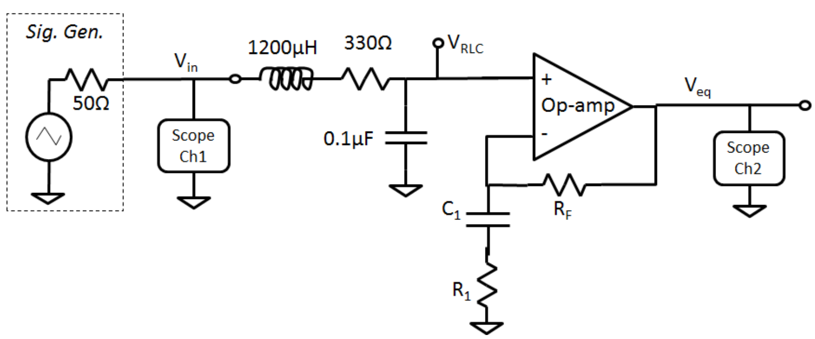

Fig. 4. RLC channel model and Equalizer circuit.

4. Combination of RLC Channel and Equalizer Circuit

Now combine the RLC channel and the equalizer circuit by driving the input of the equalizer from the output of the RLC circuit (by attaching V_2 of Fig. 2 to V_{sig} of Fig. 3), as shown in Fig. 4. Attach the signal source and CH 1 of the oscilloscope to V_{in}, and CH 2 to V_{eq}.

(a) **Frequency response.** Once again, for a 1 V_{pp} sinusoidal input, step frequency through 1 kHz, 2 kHz, 5 kHz, 10 kHz, 20 kHz, 50 kHz, 100 kHz and 200 kHz. Measure the input and output waveforms' peak-to-peak amplitudes and the phase difference between the two waveforms at each frequency. You need not save the actual waveforms. Generate a magnitude and phase Bode plot. What do you notice about the combined bandwidth?

(b) **Response to a square-wave.** Now configure the signal source for a 20 kHz, 1 V_{pp} square wave and capture the input and output waveforms. Compare this to your result from 2(c). Which output looks more like the input?

Wind down:

Clean up around your bench (including wire scraps) and return any components back to their storage bins. Be sure all data is collected and placed on your own storage media. Delete all files on your desktop or at least organize them in a folder. ECE makes no guarantee that these files left on your desktop will remain over time.

Analysis:

1. Generate a magnitude and phase Bode plot of the input-to-output gain of the inverting amplifier from part 1. Indicate the -3-dB frequency on the plot.
2. Plot the step response of the inverting amplifier, scaled so that the exponential decay is visible, and estimate its time constant τ from the plot.
3. Determine an approximate s-domain transfer function for the gain of the inverting amplifier.
4. Based on part 3, estimate the gain-bandwidth product of the op-amp used to build the inverting amplifier.
5. Generate a magnitude and phase Bode plot for the input-to-output gain of the RLC circuit. Indicate the -3-dB frequency on the plot, i.e., determine the 3-dB bandwidth of the channel. Identify the location of the poles, and compare to your theoretical prediction.
6. Plot the response of the RLC circuit to a square wave.
7. Draw your design of the equalizer circuit and explain your choice of component values, i.e., how you derived them.
8. Generate a magnitude and phase Bode Plot for the input-to-output gain of the equalizer circuit. Identify the location of zeros and poles.
9. Plot the response of the equalizer circuit to a square wave.
10. Generate a magnitude and phase Bode Plot for the input-to-output gain of the combined RLC channel and equalizer circuit. Identify the location of zeros and poles. What is the 3-dB bandwidth of the combined circuit?
11. Plot the response of the combined RLC channel and equalizer circuit to a square wave.