ECE/ENGRD 2100 – Spring 2019 Issued: March 8, 2019, 10 am
Homework 4 Due: March 22, 2019, 11:59 pm

Helpful readings for this homework: Nilsson and Riedel, Chapter 7 and Chapter 8.

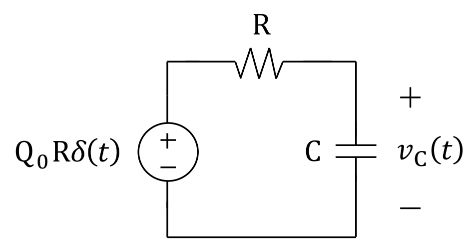
Grading Criteria

Show all work, as each problem will be graded using the grading criteria given below:

- 100% of maximum score if approach is correct and answer is also correct
- 80% of maximum score if approach is correct, but answer is incorrect due to algebraic or other math error
- 60% of maximum score if approach is mostly correct, but there is some conceptual error
- 40% of maximum score if problem has been seriously attempted, but approach is incorrect and/or there are major conceptual errors.
- 20% of maximum score if problem has been attempted, but is illegible.
- 0% of maximum score if there is no attempt to solve the problem.

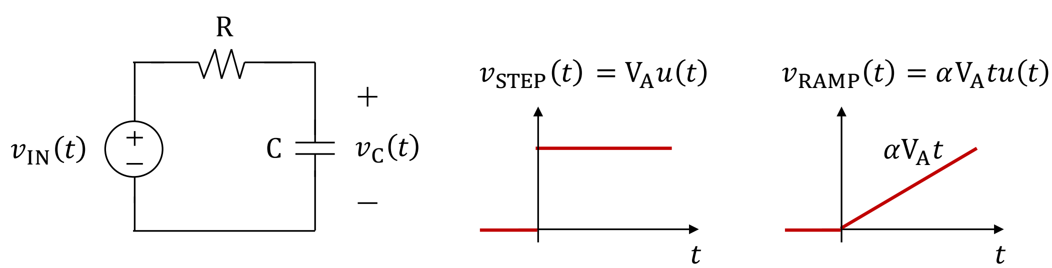
Problem 4.1: ($8\frac{1}{3}$ points)

In the network to the right, the voltage source delivers an impulse of area Q_0R volt-seconds at time $t = 0$. Determine an expression for the capacitor voltage for $t > 0$. *Hint: Converting the source and the resistor into a Norton equivalent may make the solution more intuitive.*



Problem 4.2: ($8\frac{1}{3}$ points)

This problem examines the relationship between the responses to different inputs in a linear circuit. The circuit shown below is driven first by a voltage step ($v_{IN}(t) = v_{STEP}(t)$) and then by a voltage ramp ($v_{IN}(t) = v_{RAMP}(t)$). In both cases the initial voltage across the capacitor at time $t = 0$ is zero.



(a) For the above circuit, determine the differential equation that describes the time evolution of the capacitor voltage $v_C(t)$.

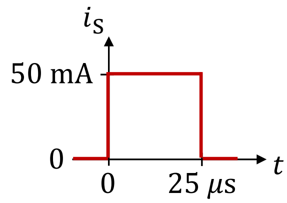
(b) By solving the differential equation, find the capacitor voltage $v_C(t)$ for $t \geq 0$ in response to the voltage step $v_{STEP}(t)$ shown above.

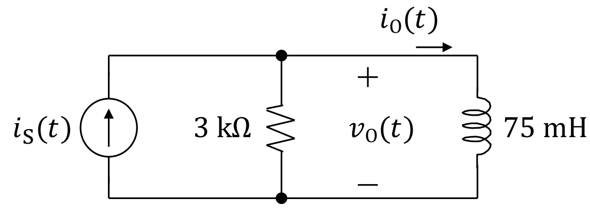
(c) By solving the differential equation, find the capacitor voltage $v_C(t)$ for $t \geq 0$ in response to the voltage ramp $v_{RAMP}(t)$ shown above.

(d) The step input can be constructed from the ramp input according to $v_{STEP}(t) = \frac{1}{\alpha}\frac{d}{dt}v_{RAMP}(t)$. Show that their respective responses are related in a similar manner.

Problem 4.3: [Problem 7.81 from Nilsson and Riedel] ($8\frac{1}{3}$ points)

The current source in the circuit to the right generates the current pulse shown next to it. There is no energy stored in the inductor at time $t = 0$.



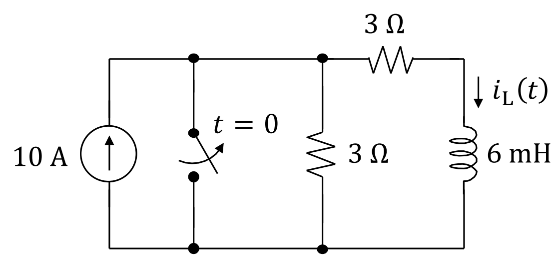


(a) Derive the numerical expressions for $v_O(t)$ for the time intervals: $t < 0$, $0 \leq t \leq 25\ \mu s$, and $25\ \mu s \leq t \leq \infty$.
(b) Calculate $v_O(25^-\ \mu s)$ and $v_O(25^+\ \mu s)$.
(c) Calculate $i_O(25^-\ \mu s)$ and $i_O(25^+\ \mu s)$.

Problem 4.4: ($8\frac{1}{3}$ points)

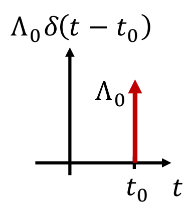
In the circuit to the right, the inductor current just before the switch opens (at time $t = 0$) is $i_L(0^-) = 2$ A.

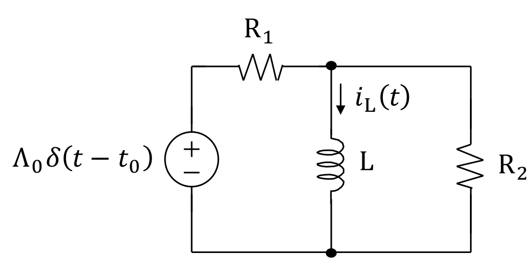
Determine the expression for $i_L(t)$ for $t \geq 0$.



Problem 4.5: ($8\frac{1}{3}$ points)

In the circuit to the right, the voltage source delivers an impulse of area Λ_0 volt-seconds at time $t = t_0$.

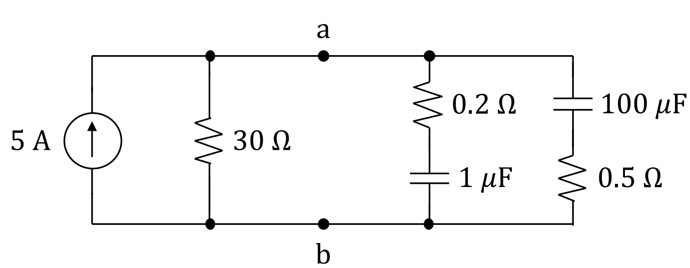




(a) Determine the inductor current at time $t = t_0^+$, i.e., $i_L(t_0^+)$.
(b) Determine an expression for the inductor current $i_L(t)$ for $t > t_0$.

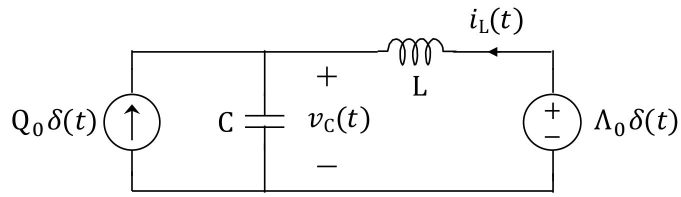
Problem 4.6: ($8\frac{1}{3}$ points)

After the circuit shown to the right has been in operation for a longtime, a screwdriver is inadvertently connected across the terminals a,b at time $t = 0$. Assume the resistance of the screwdriver is negligible.



(a) Determine the current that flows through the screwdriver at $t = 0^+$ and at $t = \infty$.
(b) Derive the expression for the current through the screwdriver for $t > 0$.

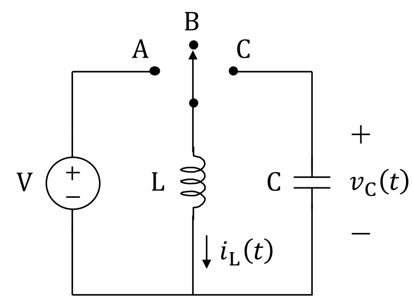
Problem 4.7: ($8\frac{1}{3}$ points)



In the circuit to the right, the current source delivers an impulse of area Q_0 coulombs at time $t = 0$, and the voltage source delivers an impulse of area Λ_0 volt-seconds at time $t = 0$.

(a) Determine the capacitor voltage and the inductor current at time $t = 0^+$, i.e., $v_C(0^+)$ and $i_L(0^+)$.
(b) Determine an expression for the capacitor voltage $v_C(t)$ for $t > 0$.

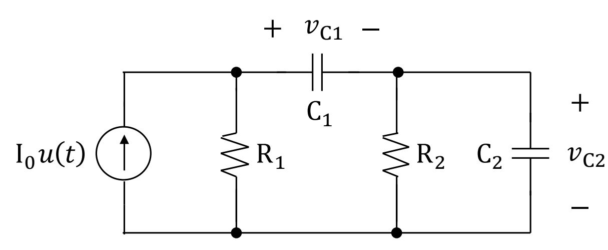
Problem 4.8: ($8\frac{1}{3}$ points)



The network to the right includes a switch with three positions: A, B, and C. Prior to $t = 0$, the switch is in position B and both the inductor current, $i_L(t)$, and the capacitor voltage, $v_C(t)$, are zero. The voltage source, V, is constant.

(a) At time $t = 0$, the switch moves to position A and remains there until $t = T_1$ seconds. Determine $i_L(t)$ and $v_C(t)$ for $0 \le t \le T_1$.
(b) At time $t = T_1$, the switch moves to position C without interrupting the current $i_L(t)$. It remains there until $i_L(t)$ goes to zero for the first time, at which time $t = T_2$ seconds the switch moves back to position B. Determine $i_L(t)$ and $v_C(t)$ for $T_1 \le t \le T_2$. Also determine T_2.
(c) The switch remains in position B until time $t = T_3$ seconds. Find $i_L(t)$ and $v_C(t)$ for $T_2 \le t \le T_3$.
(d) Determine the energy stored in the inductor at time $t = T_1$.
(e) The energy stored in the inductor at time $t = T_1$ is fully transferred to the capacitor at time $t = T_2$. Use this fact (and physical intuition regarding the polarity of v_C) to determine $v_C(t = T_2)$ from your answer in part (d). This should match your answer in part (b) when $v_C(t)$ is evaluated at $t = T_2$.

Problem 4.9: ($8\frac{1}{3}$ points)

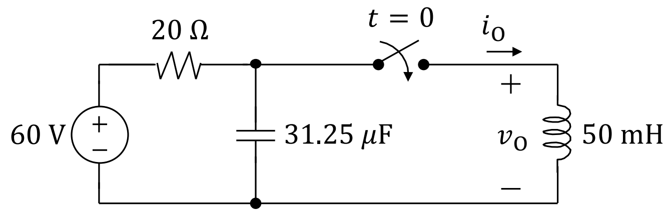


An n^{th}-order physical system has n independent energy storage elements. A swinging pendulum, for example, contains both kinetic and potential energy; an L-C circuit contains both electric and magnetic energy. These examples are second-order systems. A second-order system often (but not always) exhibits oscillatory behavior. In a circuit, the energy storage elements are independent if they do not share a voltage or a current. An example of such a circuit is shown above.

(a) Derive the differential equation for v_{C1}.
(b) If $R_1 = R_2 = 10\ k\Omega$ and $C_1 = C_2 = 100\ \mu F$, what are the natural frequencies and/or time constants of this circuit?
(c) If $v_{C1}(t = 0^+) = v_{C2}(t = 0^+) = 0$, determine $v_{C1}(t)$ for time $t > 0$.

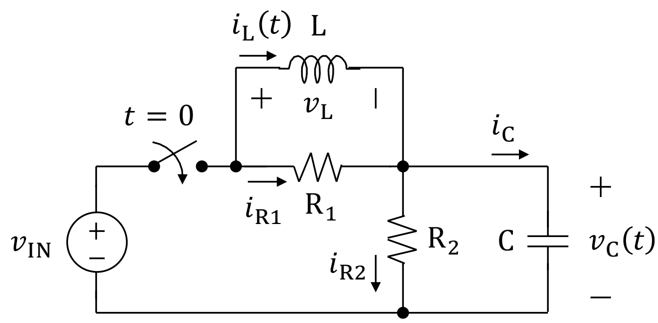
Problem 4.10: [Problem 8.31 from Nilsson and Riedel] ($8\frac{1}{3}$ points)

The switch in the circuit below has been open for a long time before closing at $t = 0$. Determine $i_O(t)$ for $t \geq 0$.



Problem 4.11: ($8\frac{1}{3}$ points)

Consider the second-order circuit shown below in which the switch is closed at time $t = 0$. Determine the differential equation that describes the time evolution of the capacitor voltage $v_C(t)$ for time $t > 0$.



Problem 4.12: [Problem 8.53 from Nilsson and Riedel] ($8\frac{1}{3}$ points)

The circuit shown below has been in operation for a long time. At $t = 0$, the source voltage suddenly drops to 150 V. Find $v_O(t)$ for time $t \geq 0$.