Helpful readings for this homework: Nilsson and Riedel, Chapter 4.

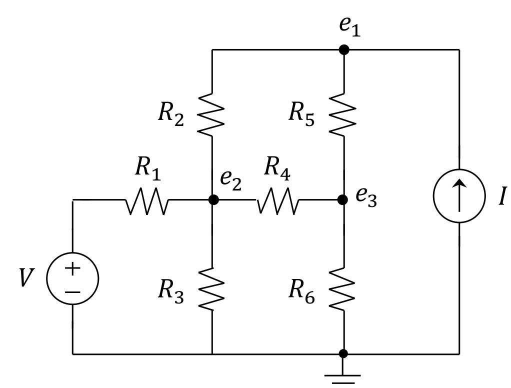
Grading Criteria

Show all work, as each problem will be graded using the grading criteria given below:

- 100% of maximum score if approach is correct and answer is also correct
- 80% of maximum score if approach is correct, but answer is incorrect due to algebraic or other math error
- 60% of maximum score if approach is mostly correct, but there is some conceptual error
- 40% of maximum score if problem has been seriously attempted, but approach is incorrect and/or there are major conceptual errors.
- 20% of maximum score if problem has been attempted, but is illegible.
- 0% of maximum score if there is no attempt to solve the problem.

Problem 2.1: ($8\frac{1}{3}$ points)

Following the node method, develop a set of simultaneous equations for the network shown on the right that can be used to solve for the three unknown node voltages. Express these equations in the form:

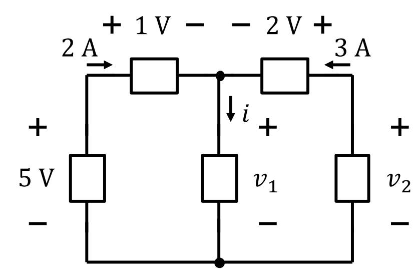


$$\boldsymbol{G}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \boldsymbol{S}$$

where $\boldsymbol{G}$ is a 3x3 matrix of conductance terms and $\boldsymbol{S}$ is a 3x1 vector of terms involving the sources. You do not have to solve the set of equations for the node voltages.

Problem 2.2: ($8\frac{1}{3}$ points)

Each box in the network to the right may represent either a voltage source, current source or a resistor. Write the KVL and KCL equations. Because the diagram specifies five voltages or currents (units of volts, amps) you have enough information to solve for the remaining i's and v's.

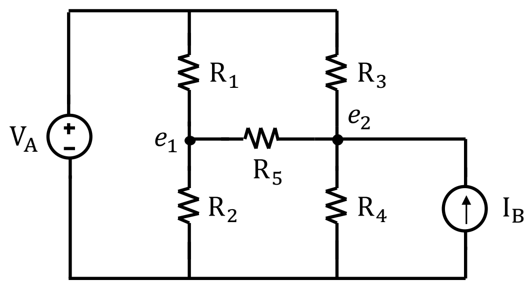


(a) Solve for i, v_1 and v_2.

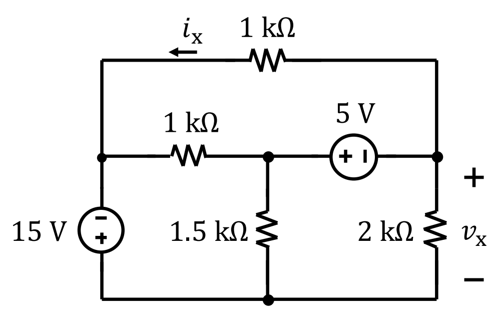
(b) Design the network to give the above voltages and currents by making a minimum number of boxes to be sources, and the remaining boxes to be resistors. Specify element values. There is no unique answer, but there are lots of impossible answers.

Problem 2.3: ($8\frac{1}{3}$ points)

(a) Use node method to determine expressions for the two unknown node voltages e_1 and e_2 in the circuit below.

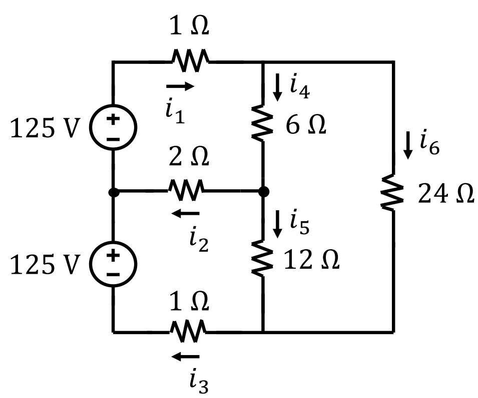


(b) Use node method to determine values of v_x and i_x in the circuit below. (*Hint: Use a supernode*)



Problem 2.4: [Problem 4.15 from Nilsson and Riedel] ($8\frac{1}{3}$ points)

The circuit shown below is a dc model of residential power distribution circuit.

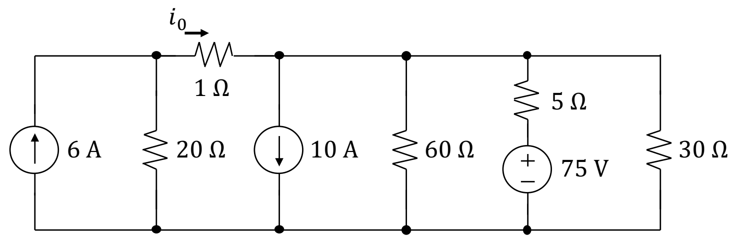


(a) Use the node-voltage method to find the branch currents i_1, i_2, i_3, i_4, i_5 and i_6. (*Hint*: *Define the node between the two 125 V sources as ground (0 V). You may use MATLAB to solve the system of equations.*)

(b) Test your solution for the branch currents by showing that the total power dissipated equals the total power developed.

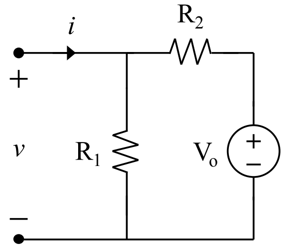
Problem 2.5: [Problem 4.95 from Nilsson and Riedel] ($8\frac{1}{3}$ points)

Use the principle of superposition to find the current i_0 in the circuit shown below.

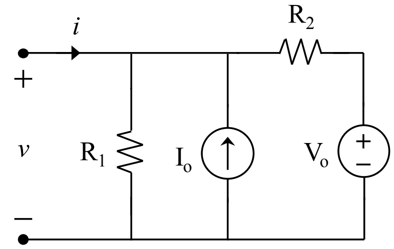


Problem 2.6: ($8\frac{1}{3}$ points)

Find the Thevenin and Norton equivalents of the following networks as viewed from their ports. Draw the equivalent circuits and provide expressions for the Thevenin voltage and resistance, and the Norton current in terms of the circuit parameters. (*Hint: Use superposition for Network B.*)

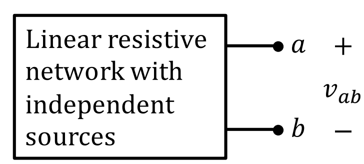


Network (A)



Network (B)

Problem 2.7: [Problem 4.69 from Nilsson and Riedel] ($8\frac{1}{3}$ points)



A Thevenin equivalent can also be determined from measurements made at the pair of terminals of interest. Assume the following measurements were made at the terminals a,b in the circuit shown:

- When a $20\ \Omega$ resistor is connected to the terminals a,b, the voltage v_{ab} is measured and found to be 100 V.
- When a $50\ \Omega$ resistor is connected to the terminals a,b, the voltage v_{ab} is measured and found to be 200 V.

Find the Thevenin equivalent of the network with respect to the terminals a,b.

Problem 2.8: ($8\frac{1}{3}$ points)

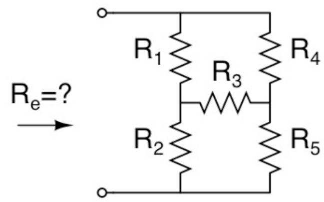
After graduating from Cornell, you decide you like Ithaca so much that you buy a chicken farm nearby, next to a gorge. You set up a hydro power generator that puts out an open-circuit voltage of 200 V, and a short-circuit current of 100 A.

(a) Find the Thevenin equivalent of your hydro power generator.

(b) What is the theoretical maximum power it can deliver?

(c) You use the hydro power generator to heat your chicken coop and hovel: your limited finances (organic free range chickens only make so much money) mean you only have 10 space heaters each of which looks like a 5 Ω resistor when on. What series/parallel arrangement of the heaters will provide the most total power to the coop?

Problem 2.9: ($8\frac{1}{3}$ points)



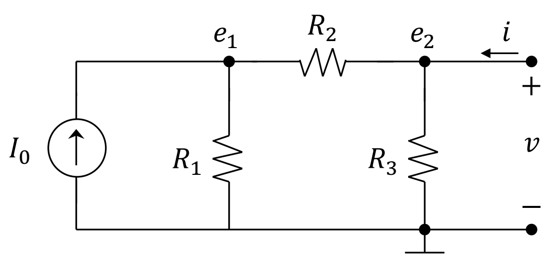
Determine the equivalent resistance, R_e, seen at the terminals of the circuit above for the following two conditions:

(a) $R_1 = R_2$ and $R_4 = R_5$. (*Hint: You should be able to do this almost by inspection*).

(b) All resistances are different and have the following values: $R_1 = 0.25\ \Omega$, $R_2 = 0.5\ \Omega$, $R_3 = 0.1\ \Omega$, $R_4 = 1\ \Omega$ and $R_5 = 0.2\ \Omega$. (*Hint: Use the test source method*).

Problem 2.10: ($8\frac{1}{3}$ points)

All four parts of this problem are concerned with Network 1 which has a single port, as shown below.



Network 1

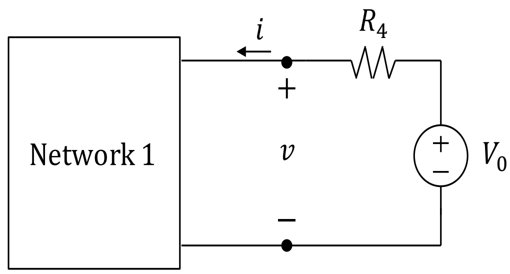
In parts (a), (b) and (c), Network 1 stands alone as shown above.

(a) Consider Network 1 alone so that the port current $i = 0$. Find the node voltages e_1 and e_2 with respect to ground in terms of the parameters I_o, R_1, R_2, and R_3.

(b) Find the Thevenin resistance R_{TH} and voltage V_{TH} for Network 1 when viewed from its port in terms of the parameters I_o, R_1, R_2, and R_3.

(c) Graph the i–v relation for Network 1 when viewed from its port assuming that $I_o > 0$. Clearly label all intercepts and slopes in terms of R_{TH} and V_{TH}.

In Part (d), Network 1 is connected to an external resistor R_4 and voltage source V_0, as shown below.

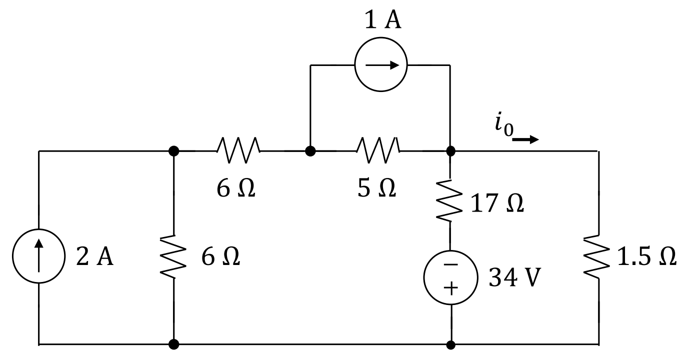


(d) Find v when Network 1 is connected to this external resistor and voltage source. Provide the answer in terms of R_4, R_{TH}, V_o and V_{TH}.

Problem 2.11: [Problem 4.62 from Nilsson and Riedel] ($8\frac{1}{3}$ points)

(a) Use a series of source transformations to find i_0 in the circuit shown to the right.

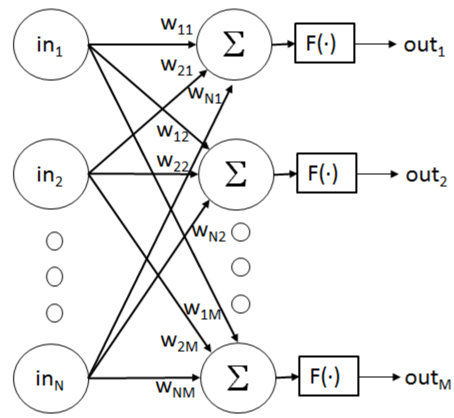
(b) Verify your solution by using the mesh-current method to find i_0.



Problem 2.12: Implementation of "synapses" in a neural network ($8\frac{1}{3}$ points)

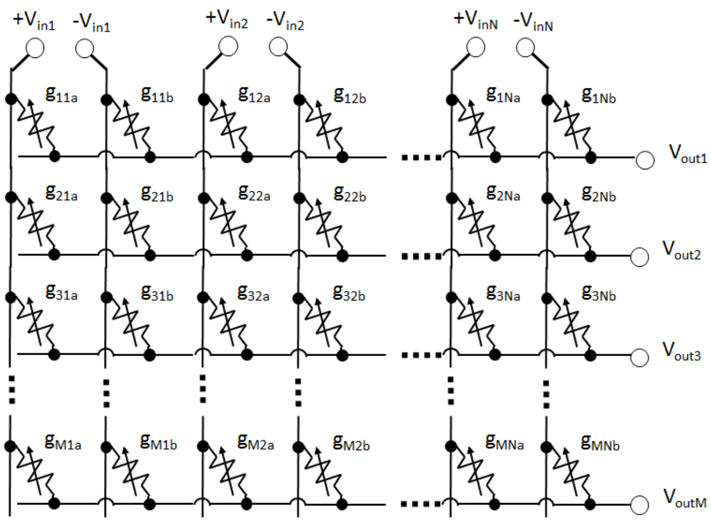
One of the proposed ways of accelerating neural networks while saving power is to implement parts of them in analog hardware. A typical layer in a deep neural network is made up of a set of M (M > 100) "neurons" each receiving inputs from N (N > 100) inputs (usually the neurons of the previous layer). In each neuron, the inputs are weighted and summed, and the result is passed through a nonlinear function, such that, for example, the m^{th} Neuron's output can be written as this nonlinear function applied to the weighted sum of the inputs, each weighted by a distinct weight:

$$out_{\text{m}} = F\left(\sum_{n=1}^{\text{N}} w_{\text{nm}} in_{\text{n}}\right)$$



This large weight-and-sum (equivalent to the dot product between an input vector and a neuron-specific weight vector) is by far the most computationally expensive aspect of running a trained neural network (training is orders-of-magnitude more expensive still, and is usually performed in gigantic data centers near industrial-scale hydro-power sources: ok I exaggerate, but only slightly).

One idea that is being actively explored for making this far more efficient is by using large arrays of RRAM connected in a "cross-bar array" as analog synapse arrays, as shown below. RRAM cells are basically tiny (10 nm x 10 nm x 10 nm) programmable resistors (conductors). For the array below, make the following assumptions: V_{in1} through V_{inN} can take both positive and negative values, and $g_{\text{nmb}} = g_0 - g_{\text{nma}}$, where $0 < g_{\text{nma}} < g_0$.



(a) Write a general equation for V_{outm} (as a sum over n from 1 to N) in terms of V_{inn} and g_{nma} and g_{nmb}.

(b) Substitute $g_{\text{nma}} = g_0/2 + \Delta g_{\text{nm}}$: what is g_{nmb} in terms of g_0 and Δg_{nm}. Rewrite the equation for V_{outm} in the same terms. Comparing to the equation above, with $in_{\text{n}} = \text{V}_{\text{inn}}$ and $out_{\text{m}} = \text{V}_{\text{outm}}$. What is w_{nm} (in terms of Δg_{nm} and g_0)?