Department of Electrical and Computer Engineering, Cornell University

ECE 5330: Semiconductor Optoelectronics

Fall 2014

Midterm **Due: October 29 by 5:00 PM**

RULES:

This is an open book exam. This means:

i) You can consult any course material (lectures, homeworks, handouts, etc) and all the recommended texts for this course.
ii) You cannot consult any other outside material (e.g. other texts, papers, and you cannot also "google" the answers).
iii) You cannot discuss the exam with anybody else.

Table of Parameter Values of III-V Semiconductors:

Parameters at 300K	GaAs	AlAs	InAs	InP	GaP
Eg (Γ-point) (eV)	1.424	3.03	0.354	1.344	2.78
m_e^*/m_o	0.067	0.15	0.023	0.077	0.25
m_{hh}^*/m_o	0.50	0.79	0.40	0.6	0.67
m_{lh}^*/m_o	0.087	0.15	0.026	0.12	0.17
Relative dielectric constant	13.0	10.0	15.0	12.5	11.0
Ep (eV)	25.7	21.1	22.2	20.7	22.2

Problem 1: 20 points

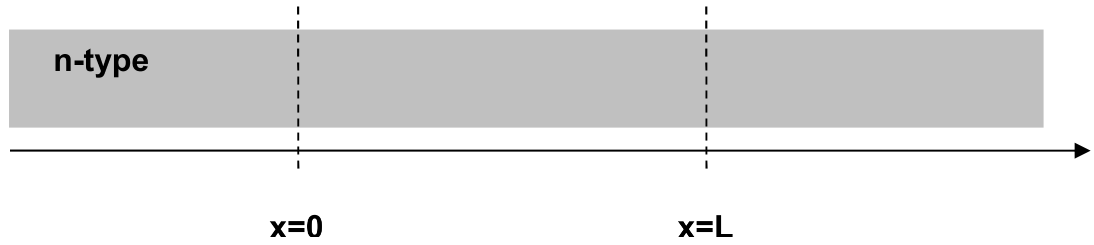
Problem 2: 25 points

Problem 3: 40 points

Problem 4: 15 points

Problem 1 (Semiconductor Physics)

a) Consider a piece of n-doped semiconductor shown below.

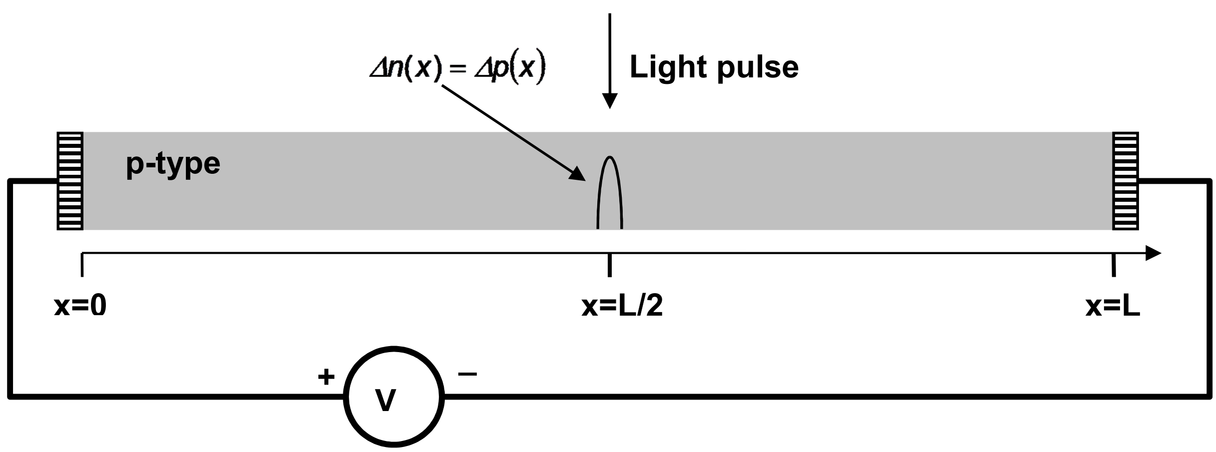


The n-doping in the material is position dependent in the region $0 \le x \le L$ as described in detail below:

$$N_d(x) = \begin{cases} N_1 & x \le 0 \\ N_1 \exp\left(\dfrac{x}{\lambda}\right) & 0 \le x \le L \\ N_1 \exp\left(\dfrac{L}{\lambda}\right) & x \ge L \end{cases}$$

Assume the doping is varying slowly (i.e. λ is much larger than the Debye length). Find an expression for the electric field in the entire structure as a function of position, and sketch the band diagram in equilibrium. **(10 points)**

b) Consider a long piece of p-doped semiconductor with ohmic contacts as shown below (L=100 μm).

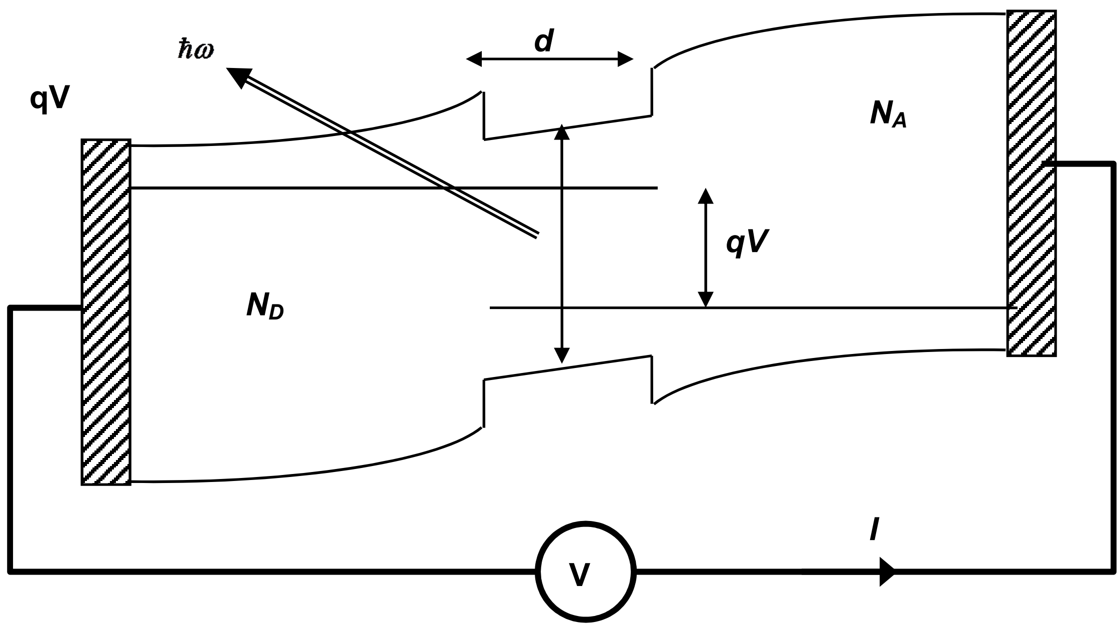


The sample is p-doped with $N_a = 10^{18}$ cm^{-3}. The electron and hole mobilities in the material are $\mu_e = 1000$ cm^2/V-sec and $\mu_h = 200$ cm^2/V-sec. You can ignore carrier recombination in this problem. The applied voltage V equals 100 volts.

At time t=0 a short optical pulse creates electron-hole pairs at the center position (at $x = L/2$). The resulting excess electron and hole densities just after the pulse are shown in the figure above. The integrated excess carrier sheet densities equal $10^{12}\ /\mathrm{cm}^2$ (i.e. $\int \Delta n(x)\, dx = 10^{12}\ /\mathrm{cm}^2$) and the width of the initial carrier distributions is approximately 1 μm. The relative dielectric constant of the semiconductor is 10. Sketch the excess electron and excess hole densities at time t=0.5 ns. Label the exact locations of the excess carrier densities. **(10 points)**

Problem 2 (LED Basics)

Consider the band diagram of a heterostructure p-i-n LED in forward bias, as shown below. Assume that the current injection efficiency of the LED is 100% (i.e. $\eta_i = 1$).



a) Suppose that the LED has 100% quantum efficiency, i.e. each electron-hole pair injected into the active region ends up producing a photon of energy $\hbar\omega$ (i.e. no non-radiative recombination). The light power output is then $\hbar\omega\, I/q$. The power provided by the battery is IV . If $\hbar\omega > qV$, how can one get more light power out from the device than the electrical power provided to the device by the battery? Explain. If this can never happen then explain why not? Points awarded will depend on the clarity of the explanation (**10 points**)

Now suppose that the LED is not ideal and non-radiative recombination does happen. Assume that the LED is moderately forward biased so that the statistics of the injected electrons and hole in the intrinsic region obey Maxwell-Boltzman distribution (not Fermi-Dirac). The cross-sectional area of the LED is A and the thickness of the intrinsic layer is d . Assume (as we always do) that the electron and hole densities in the narrow-gap intrinsic region are constant and not a function of position.

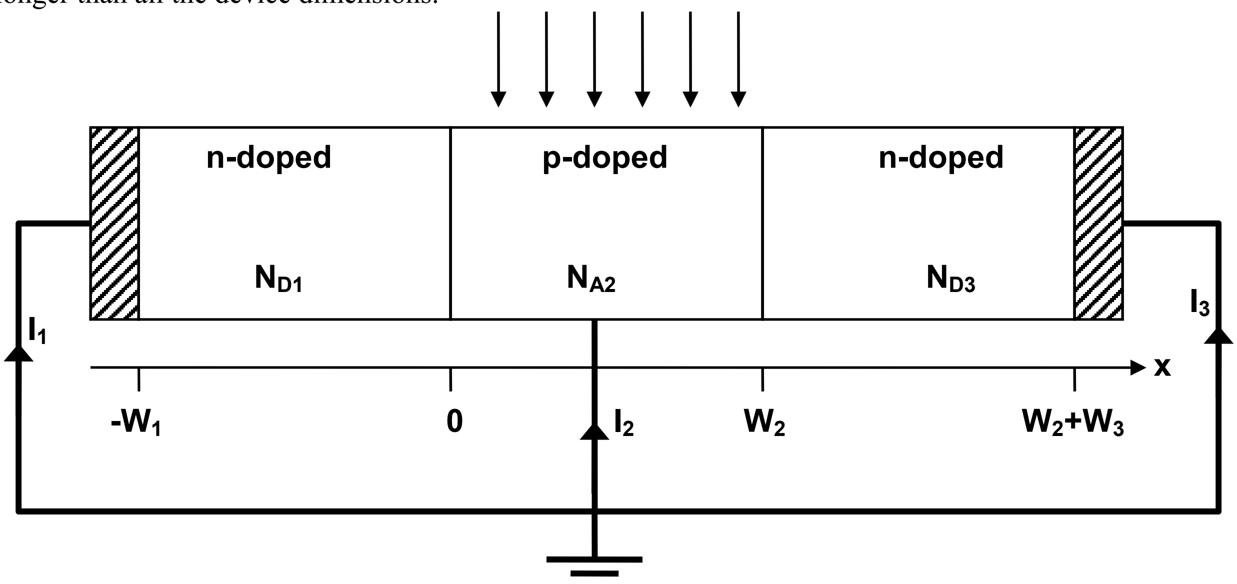
b) Find an expression that relates the injected electron and hole densities in the active region to the voltage V across the device. (**10 points**)

c) Find an expression that relates the current I to the voltage V across the device. (**5 points**)

Problem 3 (A Current Amplifying Photodetector)

It is usually necessary to get a strong current signal in response to a weak light incident on a photodetector. One way is to put a current amplifier right after a photodetector to amplify the current output from the photodetector. Another way is to build photodetectors that have internal current gain. An avalanche photodiode is an example of such a device. Another example is the subject of this problem. Assume electron diffusivity D_e, hole diffusivity D_h, and intrinsic concentration n_i.

Consider a npn structure shown in the figure below. Assume all depletion regions are of negligibly small widths and so their widths can be ignored in the analysis that follows. The middle p-region is also electrically connected to ground (although the contact is not shown in the figure). The cross-section area of each region can be assume to be A. Assume that the minority carrier diffusion lengths are much longer than all the device dimensions.



Suppose incident light uniformly illuminates the middle p-region so that the generation rate of electron-hole pairs in that region is M pairs per unit volume per second.

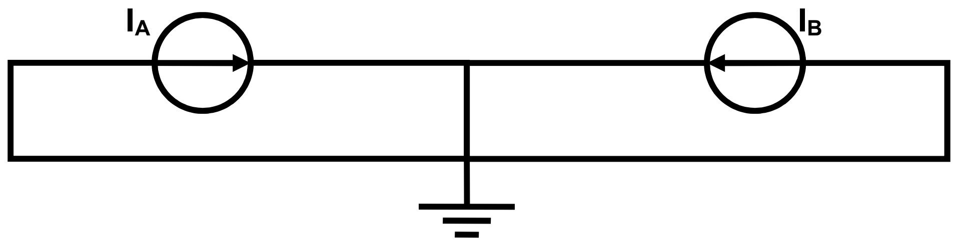
a) Find expressions for the minority carrier densities in all the three regions in terms of the given information and sketch your results. (**10 points**)

b) Find expressions for the electron current density J_e, the hole current density J_h, and the total current density J_T in the range $-W_1 \le x \le 0$ and sketch your results. (**5 points**)

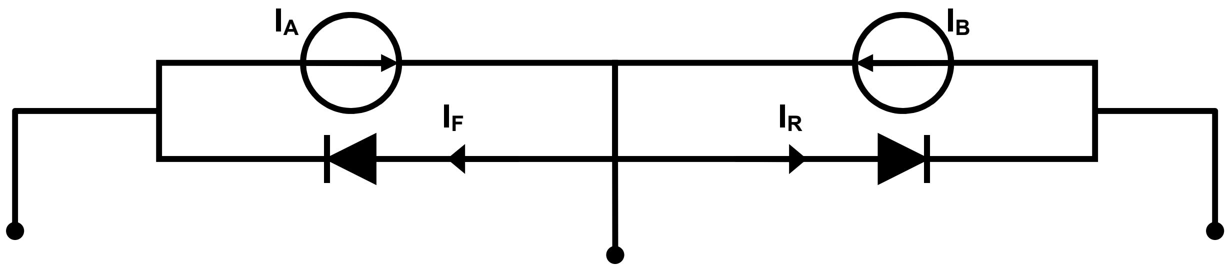
c) Find expressions for the electron current density J_e, the hole current density J_h, and the total current density J_T in the range $W_2 \le x \le W_2 + W_3$ and sketch your results. (**5 points**)

d) Find expressions for the short circuit currents I_1, I_2, and I_3 and make sure your signs are also correct. (**5 points**)

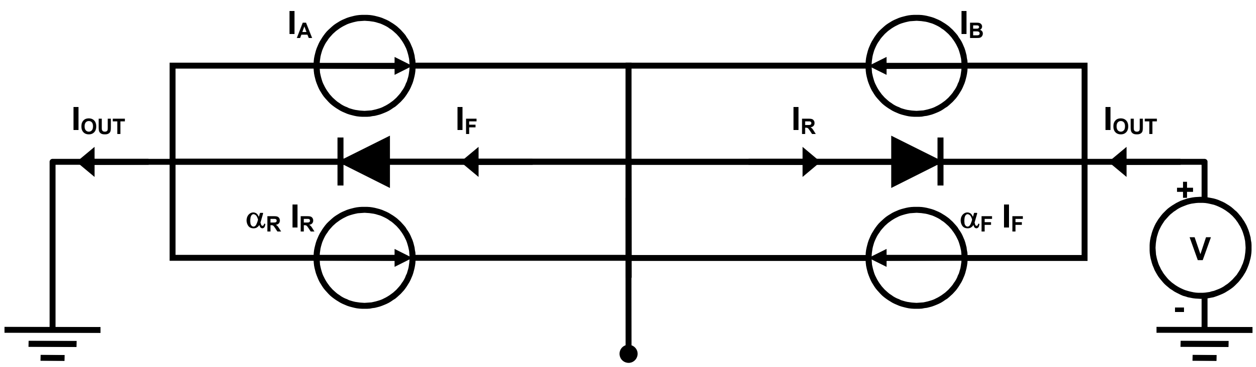
e) Now as far as the external circuit is concerned, under light illumination the above device can be replaced by the equivalent circuit model shown below where two current sources have been added. What are the magnitudes of I_A and I_B in terms of M ? (**5 points**)



The problem with the above circuit is that it is incomplete – it does not take into account what happens when either or both of the pn-junctions get forward or reversed biased. In lectures, when discussing photodetectors and solar cells, we took this into account in our circuit model by adding an ideal pn-diode in parallel with the current source which represented the effect of illumination. We can do the same here as well, as shown below.



There is still one thing missing. The npn structure resembles a transistor and in transistors whenever a current flows through one pn-diode, say the pn-diode on the left, a proportional current flows through the other pn-junction. We can now make the complete circuit model as shown below. In the figure below, we have also shorted the left end to the ground, have kept the middle region unconnected, and have attached a voltage source to the right. Here, α_F equals 0.99 and α_R equals 0.5.

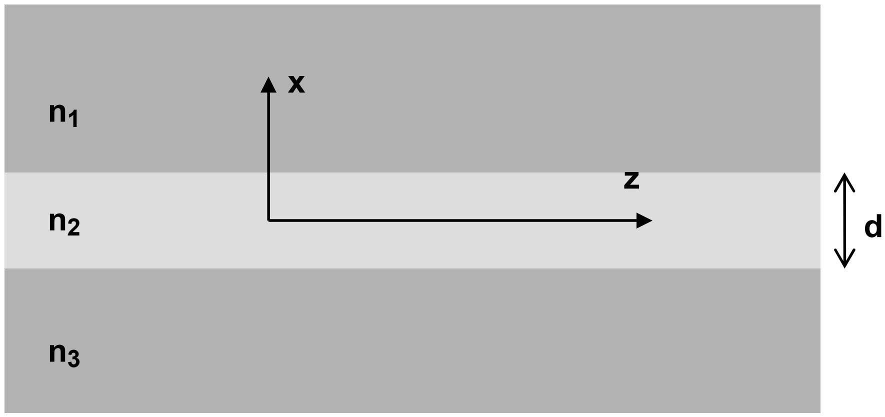


f) Assuming that the external voltage V is sufficiently large such that the right pn-junction is always reverse biased, find the output current I_{OUT} from this device and relate it to the electron-hole pair generation rate M. (**5 points**)

g) How does I_{OUT}/q compare with AW_2M (i.e. the total electron-hole pairs generated per second within the device)? If you see a current amplification compared to a regular photodiode, explain the physical origin of this amplification. Points awarded will depend on the clarity of the explanation. (**5 points**)

Problem 4: (Waveguides)

a) Consider asymmetric slab waveguide shown in the figure below. The mode is propagating in the +z-direction.

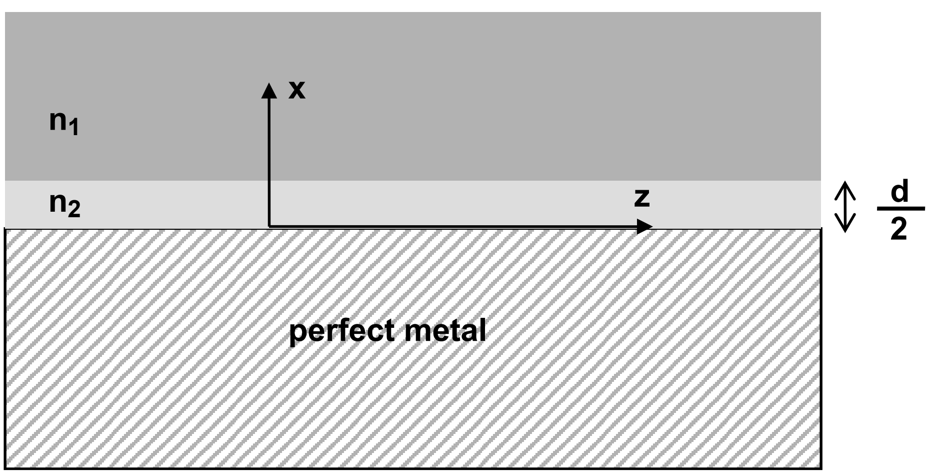


The indices of the layers are such that $n_2 > n_3 > n_1$. You will be looking at the TE modes of this waveguide. Consider the second TE mode. What are the maximum and minimum values that the mode effective index takes as the frequency ω is varied between the cut-off frequency of the mode and infinity. **(5 points)**

b) Prove the following expression in the course handout that relates the change in the propagation vector to the changes in the frequency and/or indices in a waveguide (**5 points**):

$$\Delta\beta = \frac{\iint \left[\Delta\left(\omega\varepsilon_o n^2\right)\vec{E}.\vec{E}^* + \Delta\left(\omega\mu_0\right)\vec{H}.\vec{H}^*\right] dxdy}{2\iint \mathrm{Re}\left[\vec{E}_t \times \vec{H}_t^*\right].\hat{z}\, dxdy}$$

b) The waveguide structure above is now changed as shown in the figure below.



The bottom half of the waveguide structcure in part (a) is replaced by a perfect ideal metal. A perfect metal has infinite conductivity. There can be no time dependent electric or magnetic fields inside a perfect metal. Also, electric fields parallel to the metal-core interface at x=0 must be zero as there can be no electric field parallel to the surface of an ideal metal. The effect of metal can thus be understood as a new boundary condition at x=0. Find out the maximum value of the thickness $d/2$ of the new waveguide structure shown above so that it supports only a single TE mode. **(5 points)**