ECE 5330: Homework 8 Solutions (Farhan Rana)

8.1

a) $L_1 = \frac{\lambda}{4n_1} = 121.78$ nm $\quad L_2 = \frac{\lambda}{4n_2} = 138.89$ nm.

b) See attached plots.

8.2

a). We must have: $\frac{\lambda}{2} = 3.49\,(2L_{SCH} + 16\text{nm}) + 3.65\,(24\text{ nm})$.

$\Rightarrow L_{SCH} = 40.34$ nm.

b) See the figure in the lecture notes $\lambda_o = 0.850677$ um.

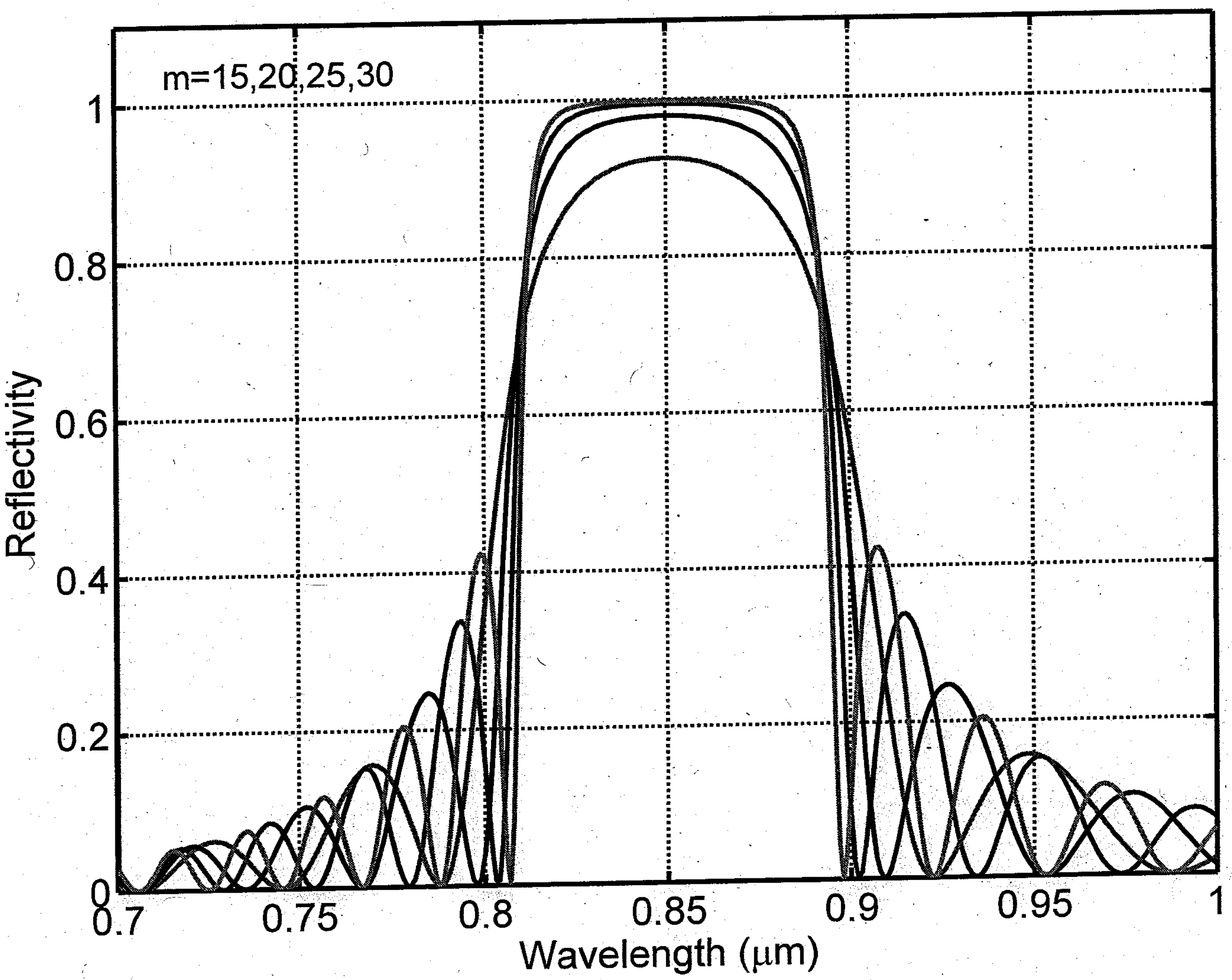
c) See the figure in the lecture notes $g_{th} = 803\ \text{cm}^{-1}$.

8.3

a) See the attached plot

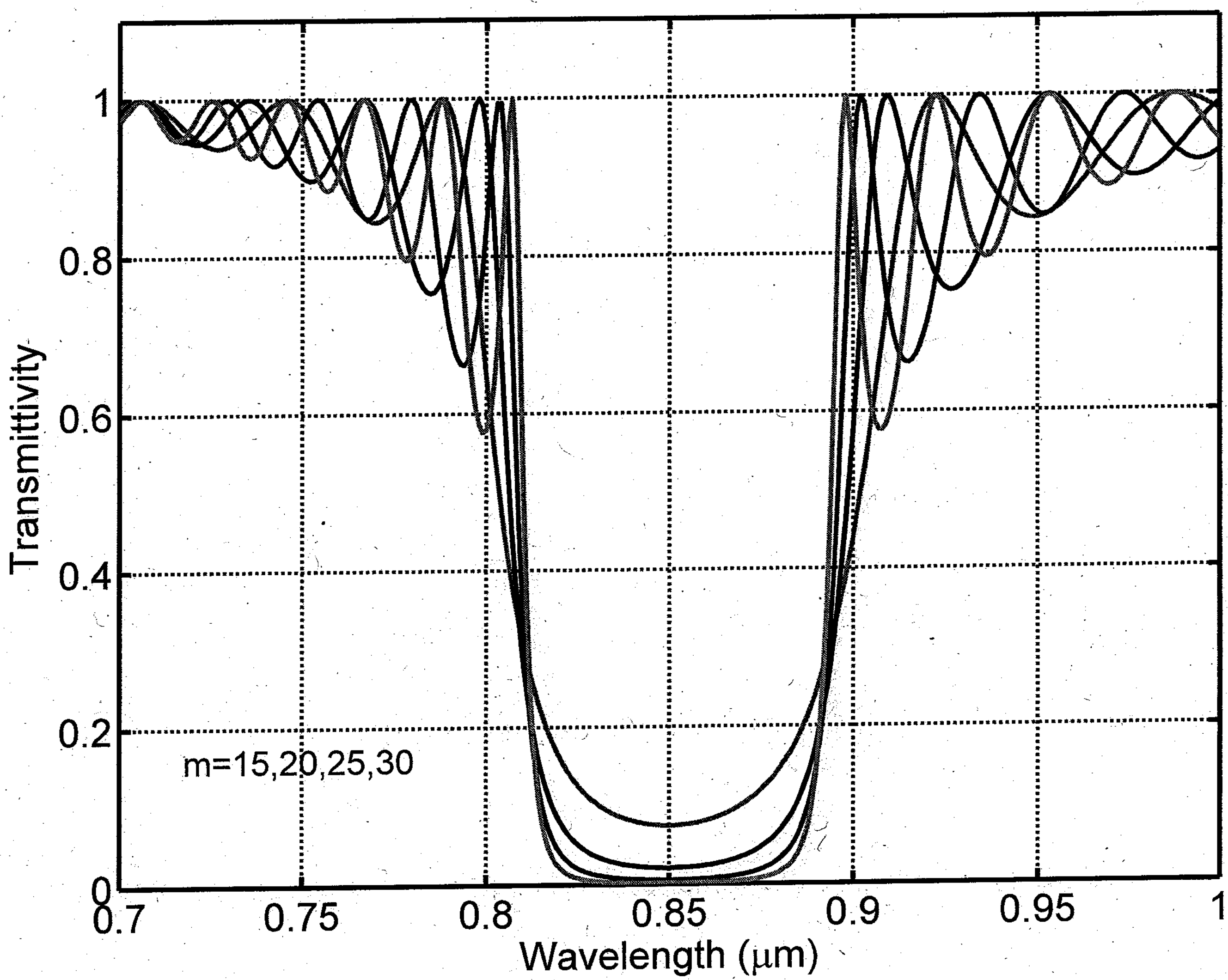
b) See the figure in the lecture notes

8.1 (b)

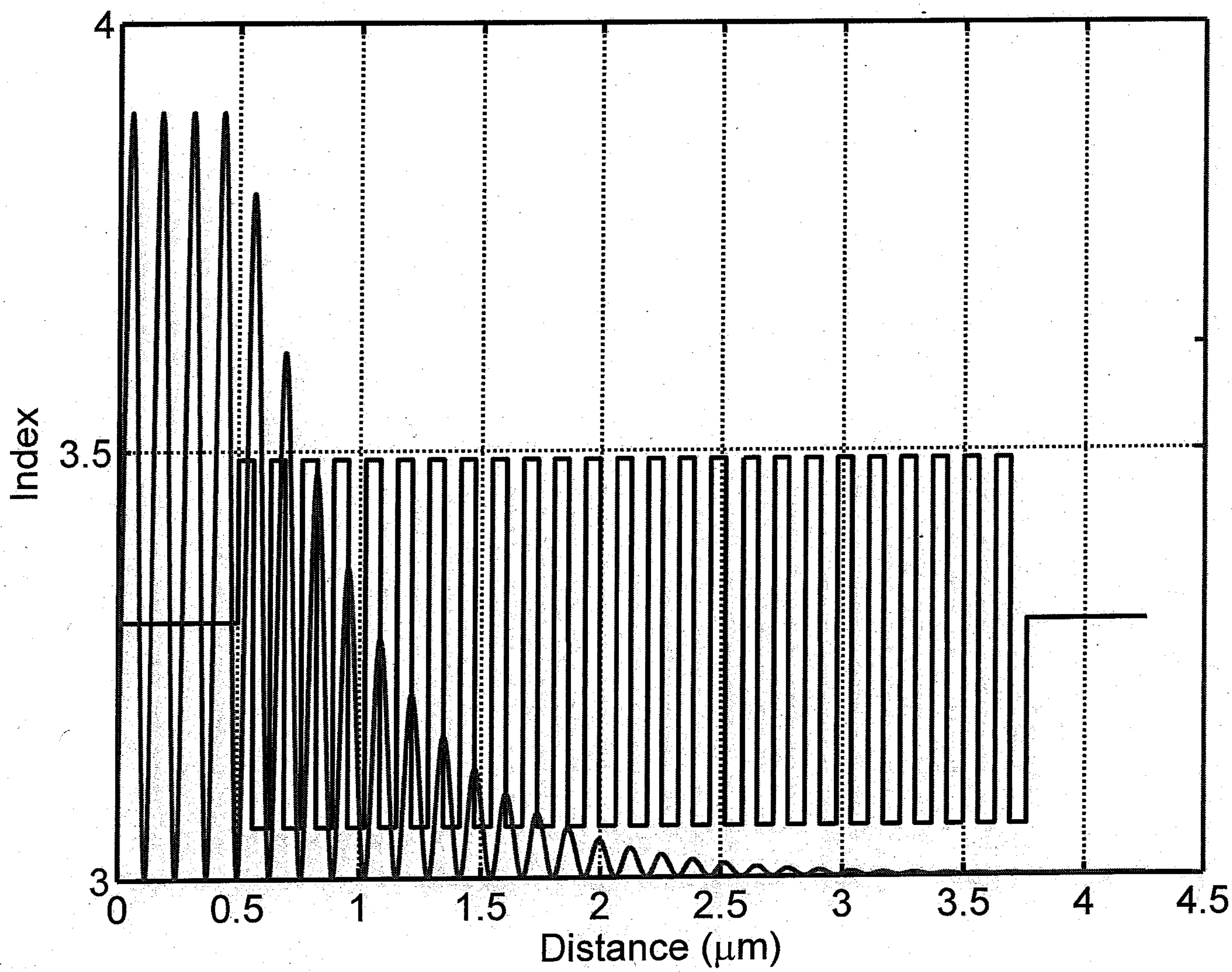


m=15,20,25,30
Reflectivity
Wavelength (μm)
1
0.8
0.6
0.4
0.2
0
0.7
0.75
0.8
0.85
0.9
0.95
1

8.1 (b)


Transmittivity
Wavelength (μm)
m=15,20,25,30
1
0.8
0.6
0.4
0.2
0
0.7
0.75
0.8
0.85
0.9
0.95
1

8.3 (a)



8.4

a) ε_{avg} in the grating region = $\frac{(3.17)^2+(3.386)^2}{2} = 10.757$

$\Rightarrow n_{avg} = \sqrt{\varepsilon_{avg}} = 3.28$.

b) We want $\frac{\omega}{c} n_{eff}(\omega) = \beta_0 = \frac{\pi}{\Lambda}$ where ω corresponds to 1550 nm.

n_{eff} at 1550 nm is found from the ece 533 solver to be = 3.207.

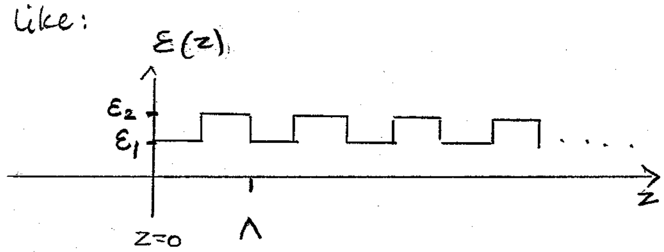
$\Rightarrow \Lambda = \frac{\lambda}{2n_{eff}} = 241.66$ nm $\Rightarrow d = \frac{\Lambda}{2} = 120.83$ nm

c)

$$n_g \approx \frac{1}{n_{eff} \sum_k \Gamma_k / n_k^2} = 3.363 \Rightarrow V_g = \frac{c}{n_g} = 8.92 \times 10^7 \text{ m/s}$$

d) $\Delta\varepsilon(x,y,z) = f(x,y) \sum_{\substack{p=-\infty \\ p\neq 0}}^{\infty} d_p \, e^{i\frac{2\pi p}{\Lambda}z}$ $\left\{ \varepsilon = \varepsilon_{avg}(x,y) + \Delta\varepsilon(x,y,z) \right.$

In the region $f(x,y)=1$ (i.e the grating region) ε as a function of z looks like:



$\varepsilon_1 = (3.17)^2$

$\varepsilon_2 = (3.386)^2$

$$\Rightarrow d_p = \frac{1}{\Lambda}\int_0^{\Lambda} \varepsilon \, e^{-i\frac{2\pi p}{\Lambda}z} dz.$$

$$= \frac{\varepsilon_1}{\Lambda}\int_0^{\Lambda/2} e^{-i\frac{2\pi p}{\Lambda}z} dz + \frac{\varepsilon_2}{\Lambda}\int_{\Lambda/2}^{\Lambda} e^{-i\frac{2\pi p}{\Lambda}z}$$

$$= \frac{(\varepsilon_2-\varepsilon_1)}{\pi p} e^{j\pi/2}$$ for p odd and zero for p even.

e)

$$\kappa = \frac{\omega d_1}{2\left(nn_g^M\right)_G v_g}\Gamma_G = \frac{(\varepsilon_2-\varepsilon_1)e^{i\pi/2}}{\lambda}\Gamma_G\left(\frac{n_g}{\left(n^2\right)_G}\right) = |\kappa|e^{i\pi/2}$$

where $|\kappa| = 78.9$ cm^{-1}. $\Rightarrow |\kappa|L = 3.945$

f). Need to solve the complex equation: $\Delta\beta L + iSL \coth(SL) = 0$

or, equivalently: $i\tan(qL) - \frac{qL}{\Delta\beta L} = 0$ where: $q = \sqrt{\Delta\beta^2 - |\kappa|^2}$ $\Delta\beta = (\beta-\beta_0) - i\Gamma_a \frac{\tilde{g}_{th}}{2}$

Note that $|\kappa|L = 3.945$. The solutions are plotted on the attached page. The two modes with the lowest value for $\tilde{g}_{th}$ have the solutions:

$$(\beta-\beta_0)L = \pm\ 4.9235.$$

$$\Gamma_a\ \tilde{g}_{th} L = 0.86.$$

From ece 533 solver Γ_a (for the gain region - i.e the quantum wells) is $= .0716 \Rightarrow \tilde{g}_{th}$ for the two lasing modes is $= 240\ cm^{-1}$

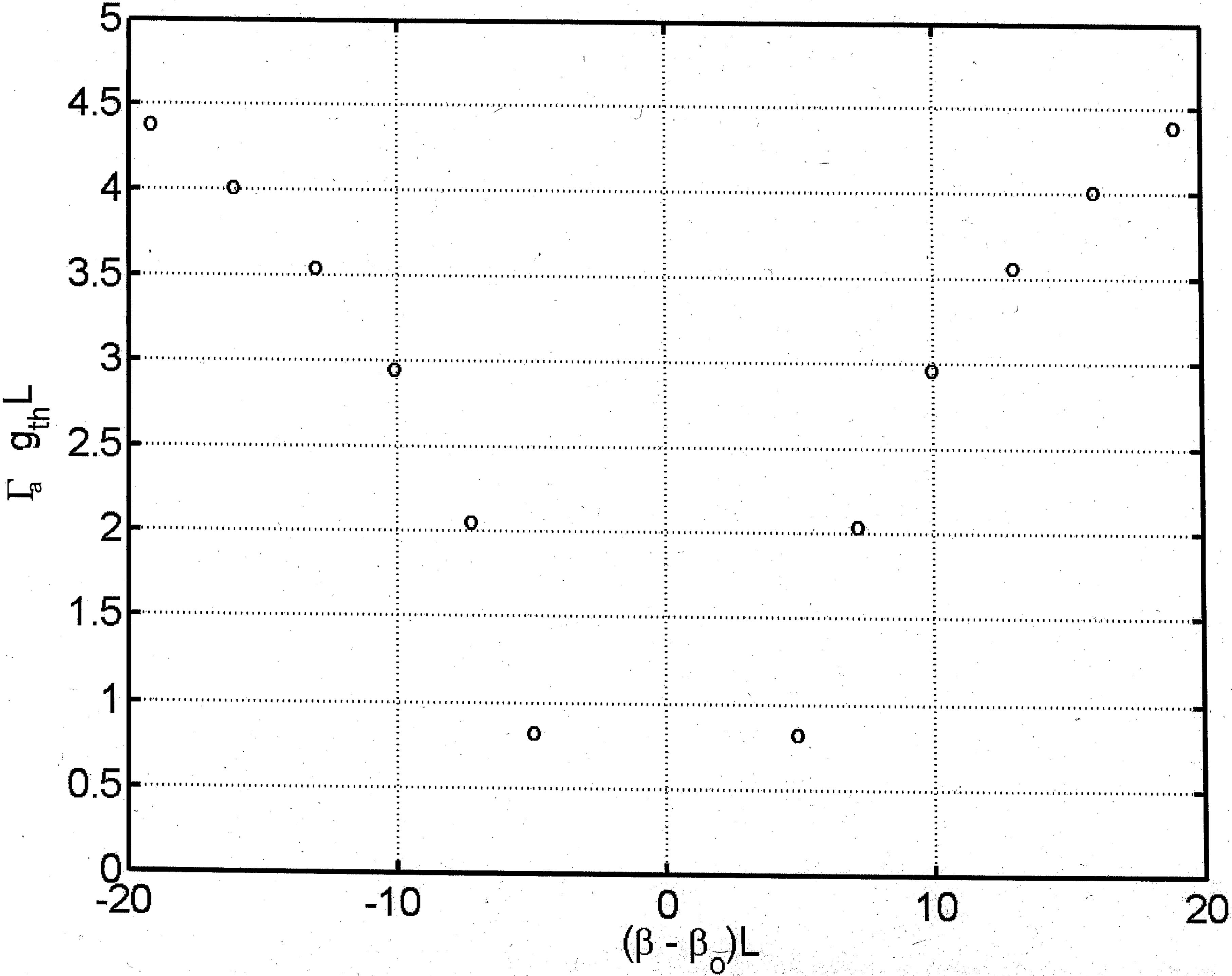
g) $\tilde{\alpha}_m = \Gamma_a\ \tilde{g}_{th} = 17.2\ cm^{-1}.$

h). $$\tilde{\alpha} = 1 + \sum_k \Gamma_k \tilde{\alpha}_k = 1 + \sum_k \Gamma_k \alpha_k \left(\frac{n_g}{n^M_{kg}} \right) = 1 + \sum_k \Gamma_k \alpha_k \left(\frac{n_g}{n_k} \right) = 6.31\ \text{cm}^{-1}$$

i) With loss $\Gamma_a\ \tilde{g}_{th} = \tilde{\alpha}_m + \tilde{\alpha} = 17.2 + 6.31 = 23.51\ cm^{-1}$

$\Rightarrow \tilde{g}_{th} = 329\ cm^{-1}$

j) $\frac{1}{\tau_p} = v_g(\tilde{\alpha}_m + \tilde{\alpha}) \Rightarrow \tau_p = 4.77\ ps$



Γa gth L
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-20
-10
0
10
20
(β - βo)L