

ECE 5330: Semiconductor Optoelectronics

Fall 2014

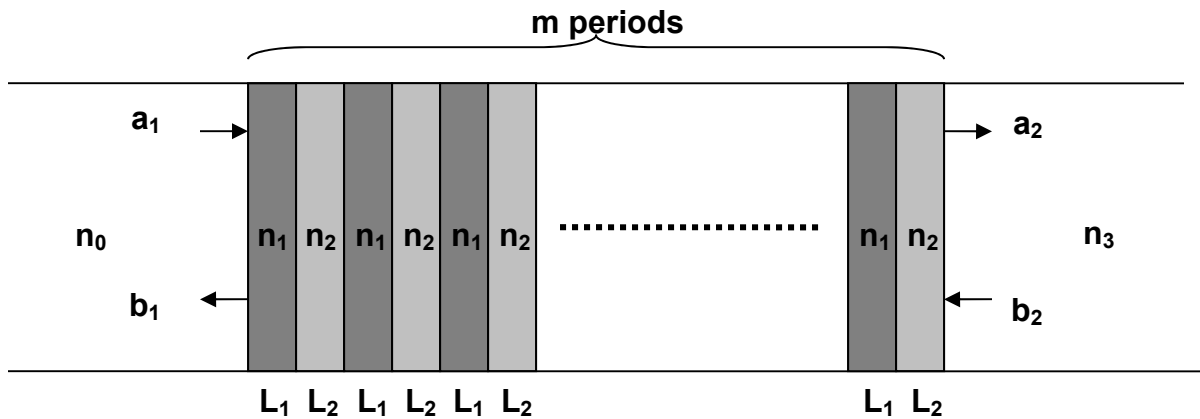
Homework 8

Due on Nov 20, 2014 by 5:00 PM

**This is a long 2-week homework (start early). It will count twice as much as any other homework.**

**Problem 8.1: (DBR mirrors – basic analysis)**

Consider an  $Al_{0.2}Ga_{0.8}As / Al_{0.9}Ga_{0.1}As$  DBR mirror as shown below.



**A DBR mirror**

The goal of this problem is to design a DBR mirror with peak reflectivity at the Bragg wavelength  $\lambda_B$  equal to 850 nm. The layer  $Al_{0.2}Ga_{0.8}As$  has index  $n_1$  at 850 nm equal to 3.49. The layer  $Al_{0.9}Ga_{0.1}As$  has index  $n_2$  at 850 nm equal to 3.06.

a) What must be the lengths  $L_1$  and  $L_2$  of the two layers in each period of the DBR stack so that the peak reflectivity of the DBR stack is at 850 nm.

The index  $n_0$  of the substrate is also equal to 3.49. The index  $n_3$  of the cap layer is also equal to 3.49. The scattering matrix for the entire structure is defined as:

$$\begin{bmatrix} b_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ b_2 \end{bmatrix}$$

b) In this part you will write an S-matrix/T-matrix program that will be heavily used in other problems in this problem set. Write a program that takes the following quantities as inputs:

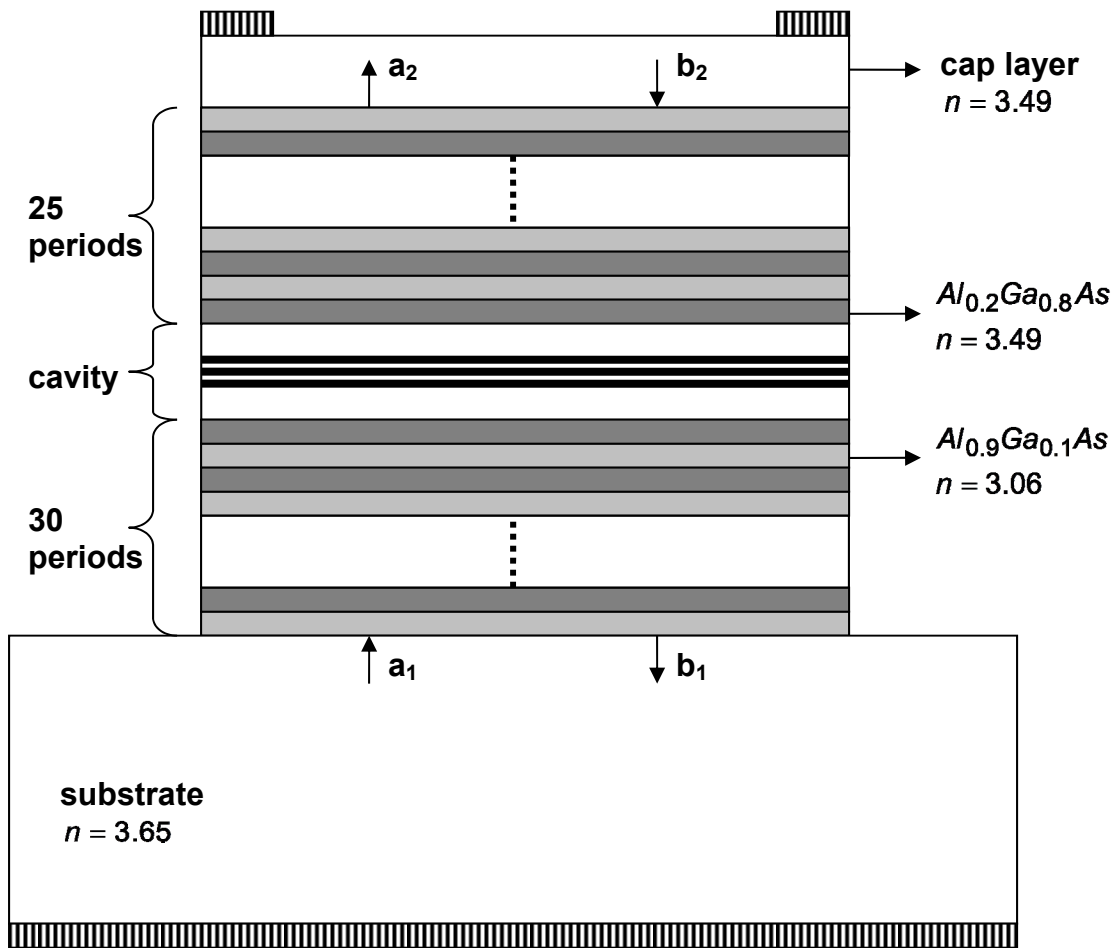
- i) An array containing the lengths of all the layers in the structure (minus the end layers). For example, for the DBR structure shown above the array would consist of  $(L_1, L_2, L_1, L_2, \dots, L_1, L_2)$ .
- ii) An array containing the complex indices of all the layers in the structure (minus the end layers). For example, for the DBR structure shown above the array would consist of  $(n_1, n_2, n_1, n_2, \dots, n_1, n_2)$ .

- iii) The wavelength  $\lambda_o$  for which the S-matrix/-matrix analysis is desired.
- iv) The complex indices of the end layers. For example, for the DBR structure shown above the indices of the end layers are  $n_0$  and  $n_3$ .

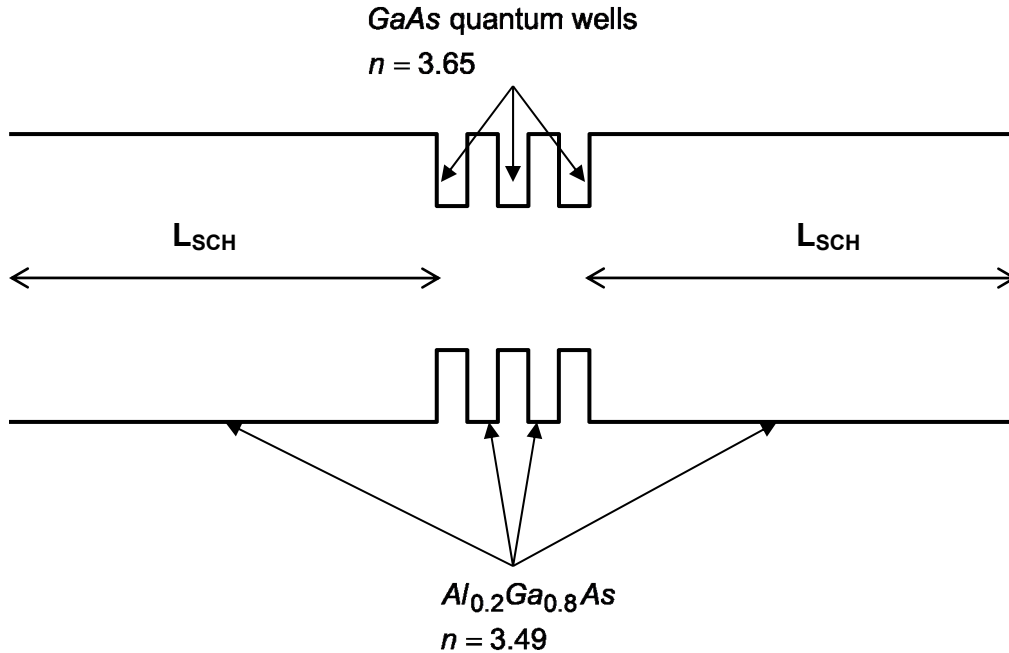
The program should be able to calculate the reflectivity  $R$  and the transmittivity  $T$  of the structure for the wavelength  $\lambda_o$ , where,  $R = |S_{11}(\lambda_o)|^2$  and  $T = |S_{21}(\lambda_o)|^2 \frac{n_3}{n_0}$

Use your program to calculate and plot the reflectivity of the  $Al_{0.2}Ga_{0.8}As / Al_{0.9}Ga_{0.1}As$  DBR mirror as a function of wavelength in the range 700 nm to 1000 nm when the number of periods “ $m$ ” in the mirror equals 15, 20, 25, and 30. The thicknesses of different layers are the same as those you calculated in part (a) of this problem for obtaining the peak reflectivity at 850 nm. Verify that the peak reflectivity you compute is the same as that obtained from the formula given in the notes.

**Problem 8.2: (Broad Area VCSELs – basic analysis )**



Consider the broad area 850 nm VCSEL structure shown above. The thicknesses of the layers in the DBR mirrors are those determined in problem 8.1 for achieving the peak reflectivity at 850 nm. The optical cavity has the band diagram shown below.



The three GaAs quantum wells are 8 nm thick. The two barriers and the two SCH layers are  $Al_{0.2}Ga_{0.8}As$ . The barriers are also 8 nm thick.

- a) Find the thickness  $L_{SCH}$  of the two SCH layers cavity such that the lowest order resonant mode of the optical cavity has wavelength equal to the Bragg wavelength of the DBR mirrors (i.e. 850 nm).
- b) The scattering matrix for the entire VCSEL structure is defined as:

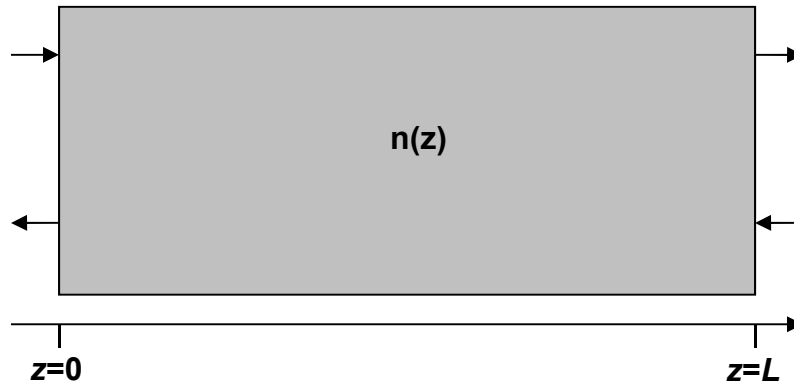
$$\begin{bmatrix} b_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ b_2 \end{bmatrix}$$

Find the **exact** wavelength  $\lambda_o$  of the cavity resonant mode by computing the transmittivity of the entire structure as a function of the wavelength. You will need to get the exact wavelength (say, in microns) to an accuracy of about six places after decimal. Make sure you are able to numerically resolve the narrow transmission peak that occurs within the reflection bandwidth of the DBR mirrors. You will want to use your program of problem 8.1(b).

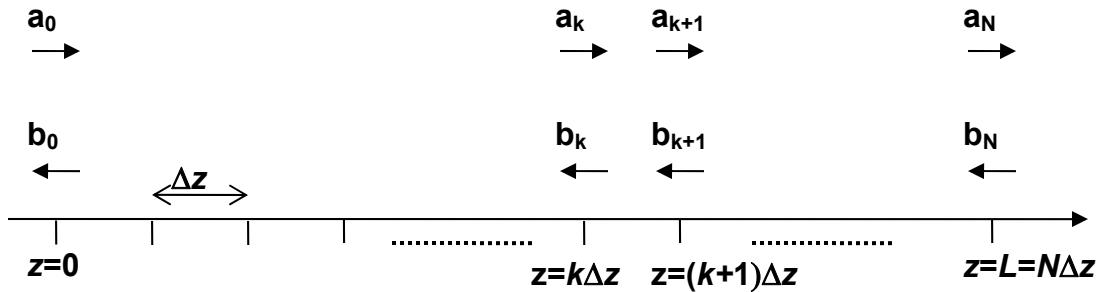
- c) In this part you will calculate the threshold gain of the VCSEL (assuming no material losses in the cavity or in the DBR mirrors). Fix the wavelength at the value  $\lambda_o$  computed in part (b), and find the threshold gain  $g_{th}$  for the VCSEL by plotting the transmittivity of the entire structure as a function of the gain  $g$  in the quantum wells.

### Problem 8.3: (Using T-matrix to compute fields)

In this problem you will use the T-matrix method to calculate the field intensity at every point in space in any device structure. Consider the dielectric structure shown below. The index  $n(z)$  is an arbitrary function of position for  $0 \leq z \leq L$ .



One can take the entire structure between  $z=0$  and  $z=L$  and divide it up into  $N$  intervals of length  $\Delta z$  each as shown below. One has essentially discretized space.



The refractive index  $n_k$  at each location is  $n_k = n(z = k\Delta z)$ . And the propagation constant in the region between  $z = (k - 1/2)\Delta z$  and  $z = (k + 1/2)\Delta z$  is  $\beta_k$  where,

$$\beta_k = \frac{\omega}{c} n_k = \frac{2\pi}{\lambda_0} n_k$$

The complex electric field at each location is given by:  $E(z = k\Delta z) = a_k + b_k$  (the actual electric field is of course just the real part of this). Suppose the quantities  $a_N$  and  $b_N$  are known. Then one can use T-matrices to calculate  $a_k$  and  $b_k$  for all  $k$ . The algorithm for calculating  $a_k$  and  $b_k$  given  $a_{k+1}$  and  $b_{k+1}$  is as follows.

i) if  $n_k = n_{k+1}$  then:

$$\begin{bmatrix} a_k \\ b_k \end{bmatrix} = \begin{bmatrix} \exp(-i\beta_k \Delta z) & 0 \\ 0 & \exp(i\beta_k \Delta z) \end{bmatrix} \begin{bmatrix} a_{k+1} \\ b_{k+1} \end{bmatrix}$$

ii) else if  $n_k \neq n_{k+1}$  then:

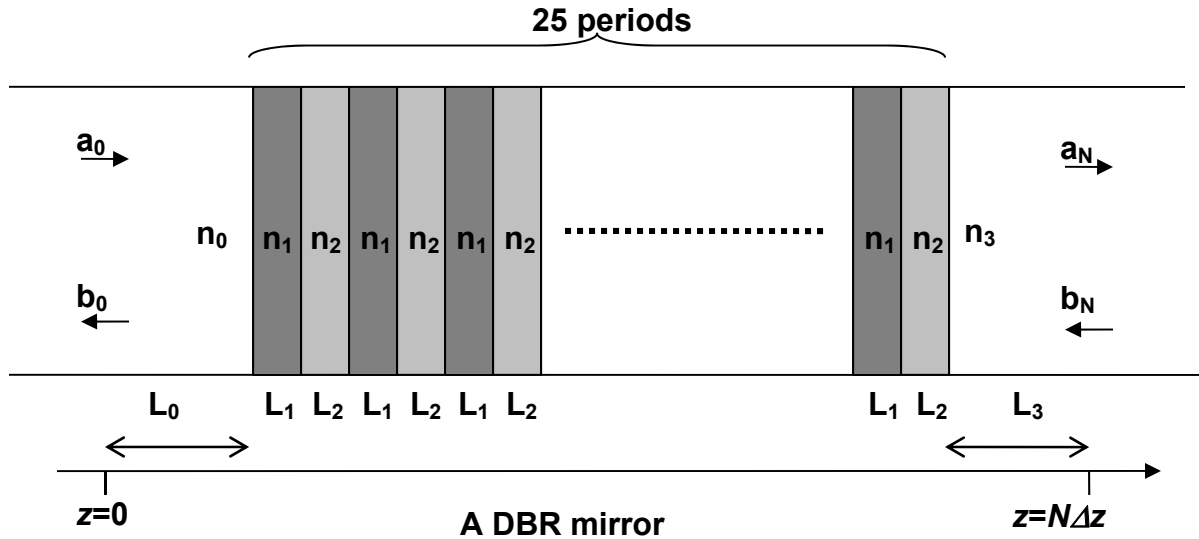
$$\begin{bmatrix} a_k \\ b_k \end{bmatrix} = \begin{bmatrix} \exp(-i\beta_k \Delta z/2) & 0 \\ 0 & \exp(i\beta_k \Delta z/2) \end{bmatrix} \begin{bmatrix} 1/t & r/t \\ r/t & 1/t \end{bmatrix} \begin{bmatrix} \exp(-i\beta_{k+1} \Delta z/2) & 0 \\ 0 & \exp(i\beta_{k+1} \Delta z/2) \end{bmatrix} \begin{bmatrix} a_{k+1} \\ b_{k+1} \end{bmatrix}$$

where:

$$r = \frac{n_k - n_{k+1}}{n_k + n_{k+1}} \text{ and } t = \frac{2n_k}{n_k + n_{k+1}}$$

In this way, given the field at one boundary, one can calculate the field at every point.

a) Consider the DBR mirror shown below.



The indices and dimensions of various layers are as follows:

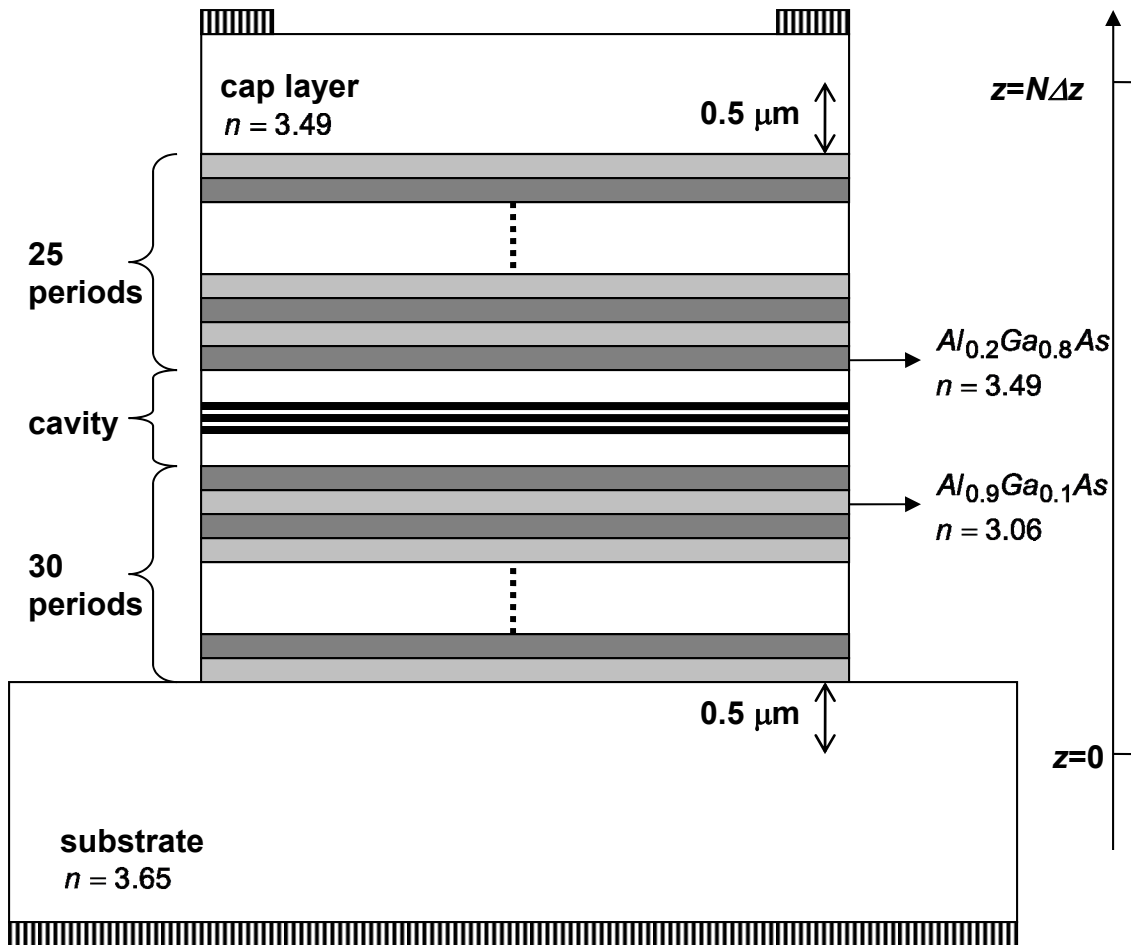
$$n_0 = n_3 = 3.3, \quad n_1 = 3.49, \quad n_2 = 3.06, \quad L_1 = 60.888 \text{ nm}, \quad L_2 = 69.444 \text{ nm}, \quad L_0 = L_3 = 500 \text{ nm}$$

$$\lambda_0 = 850 \text{ nm}$$

First thing you will need to do is set up an array “z” of size (N+1) that has the position of all the grid points. Choose a large value of N (e.g., N=10,000). Also, you will need to set up an array “n” (of the same length as “z”) that has the index values at all the grid points.

You will calculate the field intensity (magnitude square of electric field) at every point given that there is an input wave incident from the left side. In this case,  $b_N = 0$  (no input from the right side). You can assume that  $a_N = 1$  (without losing generality). Work your way backwards and calculate  $a_k$  and  $b_k$  for all k. On the same figure, plot the index and the magnitude square of the electric field as a function of position for  $0 \leq z \leq N \Delta z$ . The matlab function “plotyy” will be useful here. You will see that the incident field decays exponentially inside the DBR mirror.

b) In this problem you will use the technique developed above to calculate the field inside the VCSEL structure of problem 10.2. Make sure you use the correct value of the wavelength  $\lambda_0$  (calculated in problem 10.2(b)). The first and the last grid points should be 500 nm in the substrate and the cap layer, respectively. Choose N=20,000.



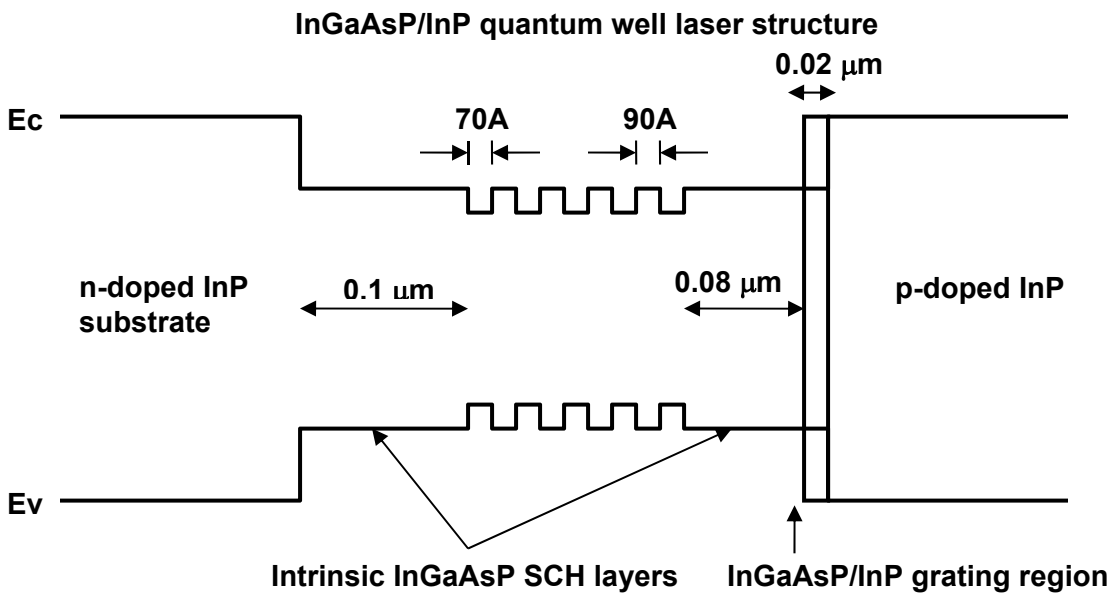
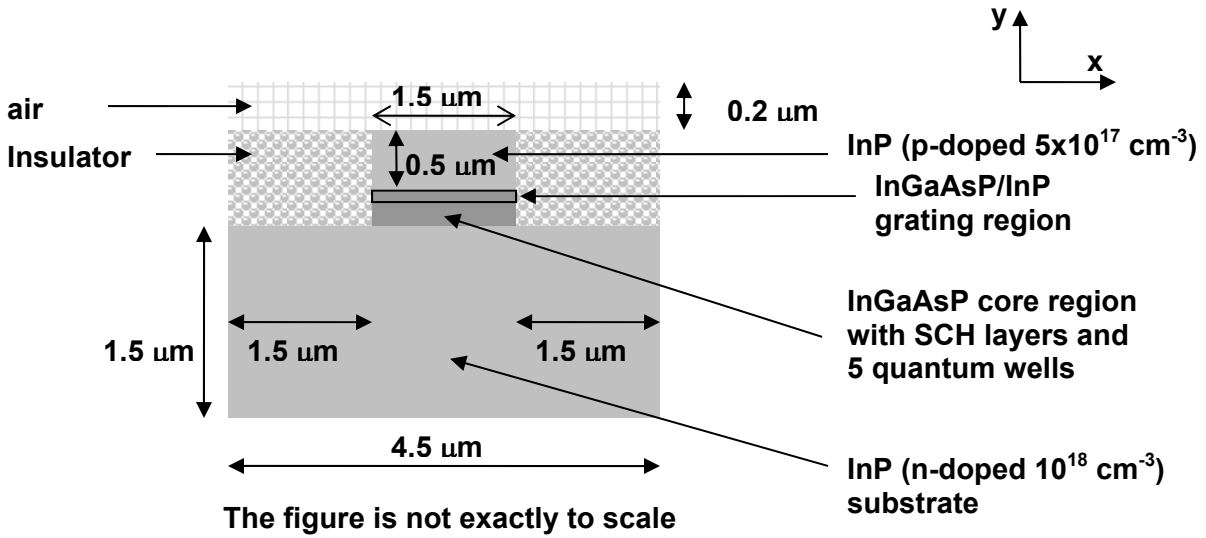
On the same figure, plot the index and the magnitude square of the electric field as a function of position for  $0 \leq z \leq N\Delta z$ . The matlab function “plotyy” will be useful here. You will see that the cavity field decays exponentially inside both the DBR mirrors.

### Problem 8.4: (DFB lasers)

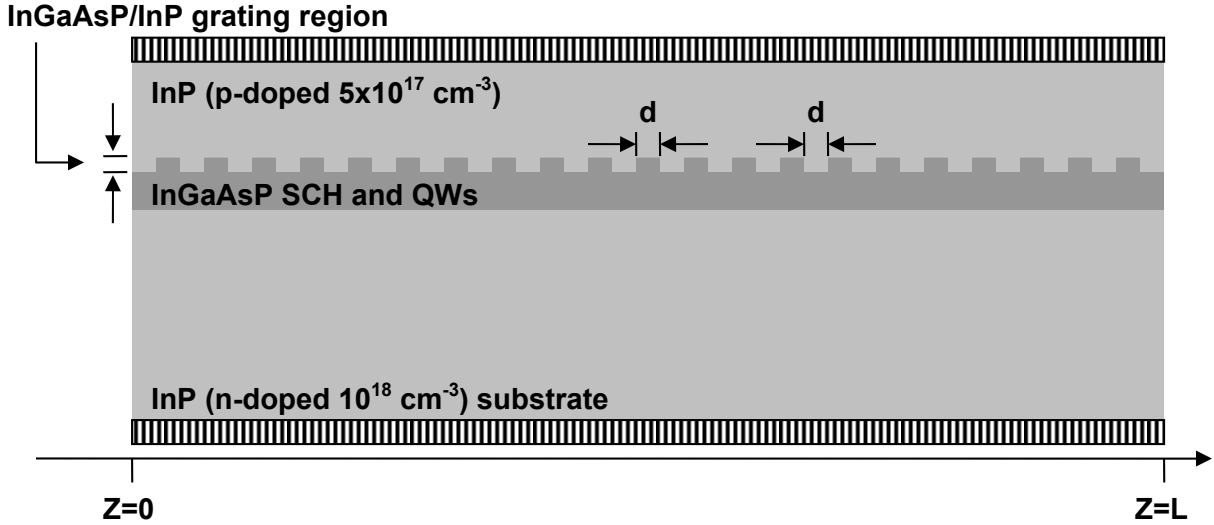
In this problem you will analyze a semiconductor DFB diode laser operating at wavelengths close to 1550 nm. The waveguide for the laser is shown below.

The thicknesses of various layers in the core region (SCH layers plus quantum wells) are shown in the figure below. The quantum well barriers and SCH layers have the same composition. The core region is undoped, and shown in detail below. The refractive indices of InP, InGaAs SCH layer, InGaAsP quantum well barrier, InGaAsP quantum well, and the insulator are 3.170, 3.386, 3.386, 3.550, and 1.70, respectively, at wavelengths close to 1550 nm.

The intrinsic material losses are  $40 \text{ cm}^{-1}$  per  $10^{18} \text{ cm}^{-3}$  doping in the p-doped regions, and  $5 \text{ cm}^{-1}$  per  $10^{18} \text{ cm}^{-3}$  doping in the n-doped regions (due to free carrier absorption, intervalence band absorption, etc). The waveguide also has an additional modal loss of  $1 \text{ cm}^{-1}$  due to the photons scattering out of the waveguide as a result of waveguide surface roughness, material inhomogeneities, etc.



The length of the laser cavity be  $L = 500 \mu\text{m}$ . The gratings are etched in the top InGaAsP SCH layer which was originally  $0.1 \mu\text{m}$  thick. The gratings get filled with InP when the top p-doped InP layer is grown. The grating depth is  $20 \text{ nm}$ . The side view of the device is as shown below (in the  $y$ - $z$  plane going through the center of the device).



In this problem use the ece533solver as needed.

- What is the value of  $n_{avg}^2(x, y) (= \epsilon_{avg}(x, y))$  in the grating region shown in the figure above?
- What should be the period  $\Lambda (= 2d)$  of the grating so that the center of the grating reflectivity band is at 1550 nm?
- What is the group velocity  $v_g$  of the optical mode?
- What is the expression for  $\Delta\epsilon(x, y, z)$ ? Your answer should not contain any unknowns or unevaluated constants. **Note carefully the phase of the grating from the figure.**
- What is the value (magnitude and phase) of the grating coupling constant  $\kappa$  (with proper units)?

For parts (f)-(g) assume no intrinsic modal losses (i.e.  $\tilde{\alpha} = 0$ ). Also, assume that both the facets of the cavity have perfect AR coating.

- Generate a scatter plot of  $\Gamma_a \tilde{g}_{th} L$  versus  $(\beta - \beta_o) L$  for at least 5 longitudinal modes on either side of  $\beta_o$  (like the one in the lecture notes) for the value of  $|\kappa| L$  calculated in part (e). Calculate the threshold gain  $\tilde{g}_{th}$  for the lasing mode (i.e. the longitudinal mode with the lowest threshold gain).
- Calculate the mirror loss  $\tilde{\alpha}_m$  for the lasing mode?

For parts (h)-(j) assume that the intrinsic loss  $\tilde{\alpha}$  is not zero.

- Calculate the value of  $\tilde{\alpha}$ .
- Calculate the actual threshold gain  $\tilde{g}_{th}$  for the lasing mode in the presence of intrinsic waveguide loss.
- What is the photon lifetime  $\tau_p$  in the cavity?