ECE5330: Homework #7 Solutions

7.1

a)

i) $n_{eff} \approx 3.21$

ii) In the slab waveguide approximation,

$$n_g \; n_{eff} = \frac{\iint n \, n_g^M \; |\phi|^2 \; dxdy}{\iint |\phi|^2 \; dxdy} \approx \frac{\iint n^2 \; |\phi|^2 \; dxdy}{\iint |\phi|^2 \; dxdy}$$

assuming dispersionless media,

$$n_g^M \approx n$$

But there is no easy way for you to figure out the integral on the right hand side. I discovered this just when writing the solutions. The more adventurous among you can, in principle, load the mode stored in the output file and calculate the integral directly. The answer is, $n_g \approx 3.38$. The waveguide modal loss is then (including the loss from surface roughness scattering),

$$\tilde{\alpha} = \sum_k \Gamma_k \left(\frac{n_g}{n_{kg}^M} \right) \alpha_k \approx \sum_k \Gamma_k \left(\frac{n_g}{n_k} \right) \alpha_k + 1 = 6.38 \, 1/\text{cm}$$

iii) $v_g = \dfrac{c}{n_g}$

iv) $\Gamma_a \approx .08 \quad \Rightarrow \quad A_{eff} = \dfrac{A_a}{\Gamma_a} = 0.66 \; \mu\text{m}^2$

v) $V_p = A_{eff} L = 330 \; \mu\text{m}^3$

b)

$$\frac{1}{\tau_p} = v_g \left[\tilde{\alpha}_m + \tilde{\alpha} \right] = \frac{v_g}{L} \log\left(\frac{1}{\sqrt{R_1 R_2}} \right) + v_g \tilde{\alpha}$$

$\Rightarrow \tau_p = 3.7 \text{ ps}$

$$\eta_o = \frac{\tilde{\alpha}_m}{\tilde{\alpha}_m + \tilde{\alpha}} = 0.79$$

c)

$\Gamma_a \tilde{g}_{th} = \tilde{\alpha}_m + \tilde{\alpha}$

$\Rightarrow \tilde{g}_{th} = 381 \text{ cm}^{-1}$

$$\tilde{g}_{th} = \tilde{g}_o \ln\left(\frac{n_{th}}{n_{tr}} \right)$$

$\Rightarrow \tilde{n}_{th} = 1.98 \times 10^{18} \; \text{cm}^3$

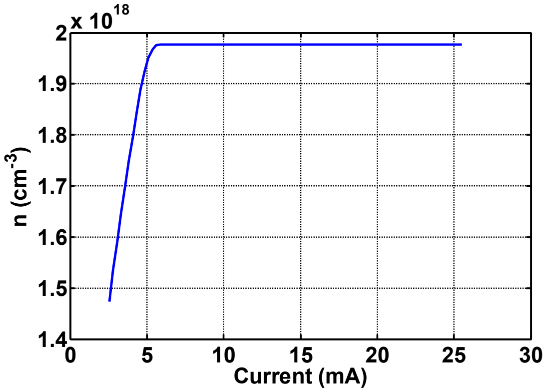
d)

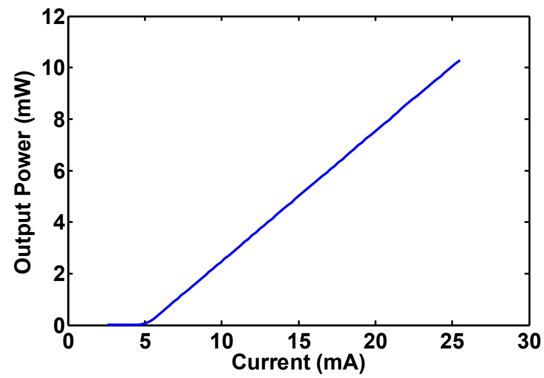
$$\frac{\eta_i I_{th}}{qV_a} = An_{th} + Bn_{th}^2 + Cn_{th}^3$$

$$\Rightarrow I_{th} = 5.1 \text{ mA}$$

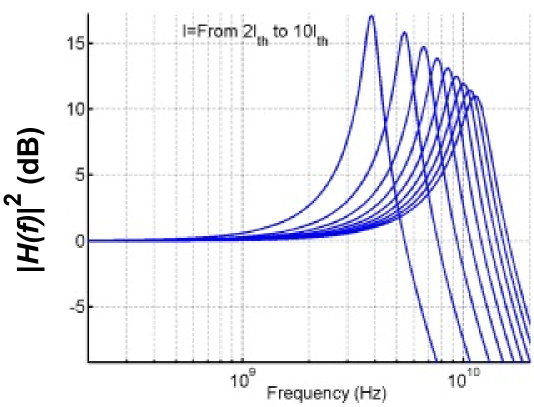
e) and f)

Notice how close the exact solutions look to the approximate model in the lecture notes.



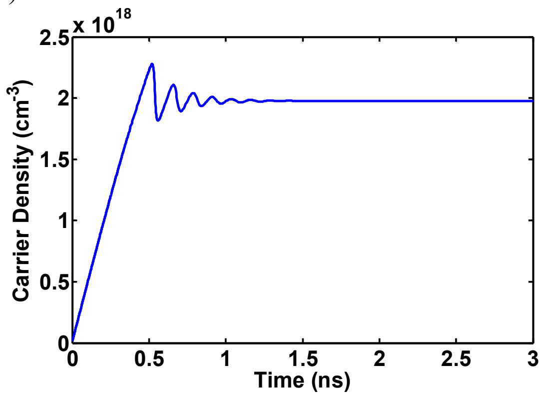


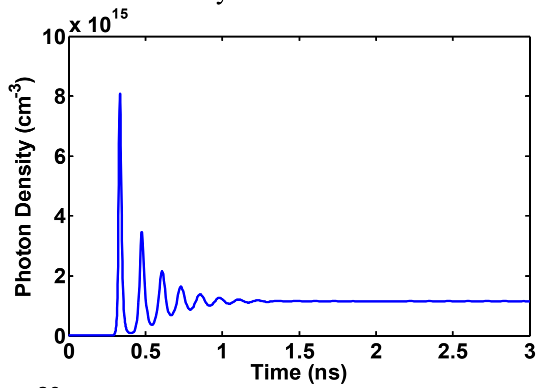
g)

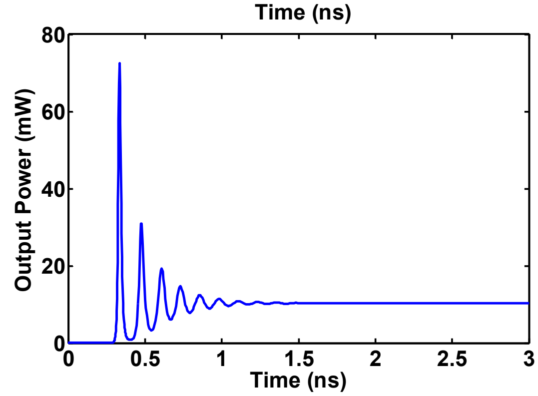


7.2

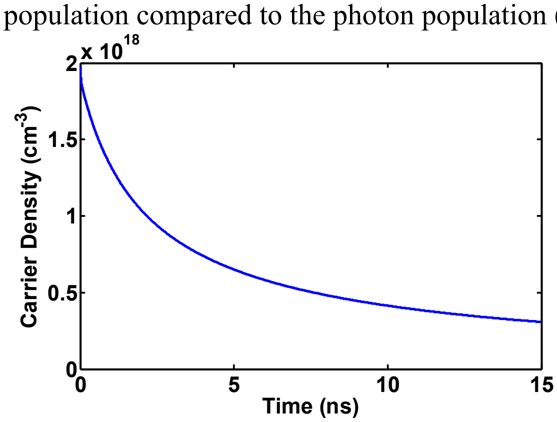
a) Notice the relaxation oscillations before the values settle to their steady state values.

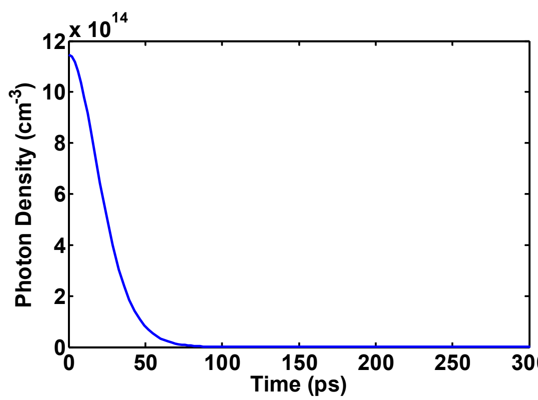


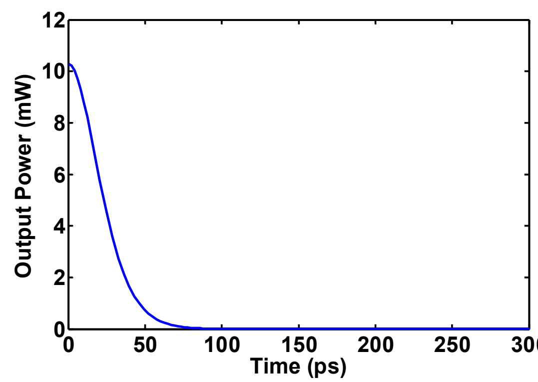




b) Notice the absence of relaxation oscillations (why?). Notice also the slow decay of the carrier population compared to the photon population (why?).







c) It takes the output power around ~1 ns to settle down in the turn-on transient. So the maximum data rate cannot be much faster than ~1 Gbits/s. Note also the spikes in the output power; these are also undesirable. For these reasons, it is better to never turn-off a laser completely for the logical 0 case. Then turn-on transients are much better when the laser is turned on hard for the logical 1 case.

7.3

a) We have:

$$n = \frac{m_e}{\pi\hbar^2} KT \log\left[1 + e^{\left(E_{fe} - E_c - E_1^c\right)/KT}\right] = p = \frac{m_{hh}}{\pi\hbar^2} KT \log\left[1 + e^{\left(E_v - E_1^v - E_{fh}\right)/KT}\right]$$

$$E_{fe} - E_{fh} = qV$$

$$\Rightarrow dE_{fe} - dE_{fh} = qdV \qquad (1)$$

$$\Rightarrow dn = \frac{dn}{dE_{fe}} dE_{fe} = dp = \frac{dp}{dE_{fh}} dE_{fh} \qquad (2)$$

Solving (1) and (2) gives:

$$dE_{fe} = qdV \frac{\left(-\frac{dp}{dE_{fh}}\right)}{\left(\frac{dn}{dE_{fe}} - \frac{dp}{dE_{fh}}\right)} \Rightarrow qV_a dn = \frac{\left(-\frac{dn}{dE_{fe}}\frac{dp}{dE_{fh}}\right)}{\left(\frac{dn}{dE_{fe}} - \frac{dp}{dE_{fh}}\right)} q^2 V_a dV$$

$$\Rightarrow C_a = q^2 V_a \frac{\left(-\frac{dn}{dE_{fe}}\frac{dp}{dE_{fh}}\right)}{\left(\frac{dn}{dE_{fe}} - \frac{dp}{dE_{fh}}\right)}$$

The value of C_a at room temperature comes out to be around 82 pF.

b) $Z(f) = \frac{-i2\pi f}{\omega_R^2} H(f)\, \eta_i \frac{1}{qV_a} \frac{1}{\partial n/\partial V|_{ss}}$

For small frequencies,

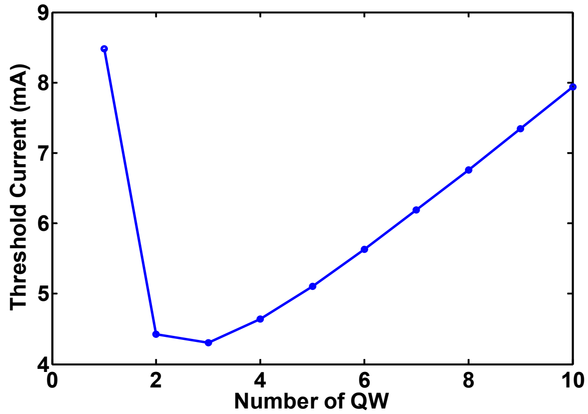
$$Z(f) \approx \frac{-i2\pi f}{\omega_R^2} \eta_i \frac{1}{qV_a} \frac{1}{\partial n/\partial V|_{ss}} = -i2\pi f L_a$$

$$\Rightarrow L_a = \frac{\eta_i}{\omega_R^2} \frac{1}{C_a}$$

I think I missed the injection efficiency term in the homework set.

7.4

a) See the plot below. The optimal number of quantum wells is 3.



7.5

a) In a laser operating above threshold one must have,

$$qV = E_{fe} - E_{fh} > \hbar\omega$$

otherwise there will be no gain at the lasing frequency. Therefore, even for an ideal laser,

$$\frac{P}{IV} < \frac{q}{\hbar\omega}\frac{P}{I} = 1 - \frac{I_{th}}{I} < 1$$