School of Electrical and Computer Engineering, Cornell University

ECE 5330: Semiconductor Optoelectronics

Fall 2014

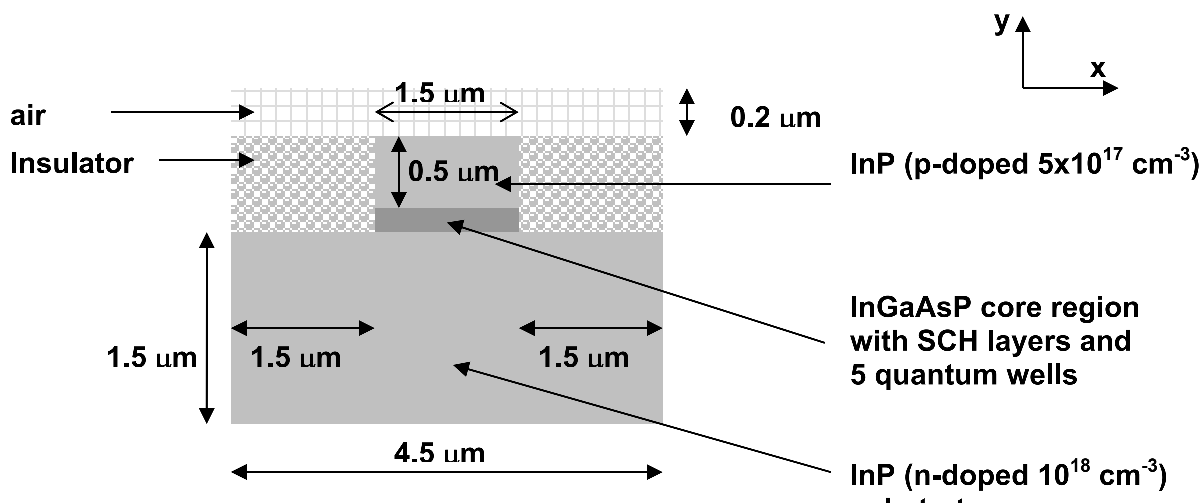
Homework 7 **Due on Nov. 06, 2014**

Suggested Readings:

i) Study lecture notes.
ii) Study Coldren and Corzine for more details on semiconductor lasers.

Problem 7.1: (An InGaAsP/InP semiconductor lasers – basic analysis)

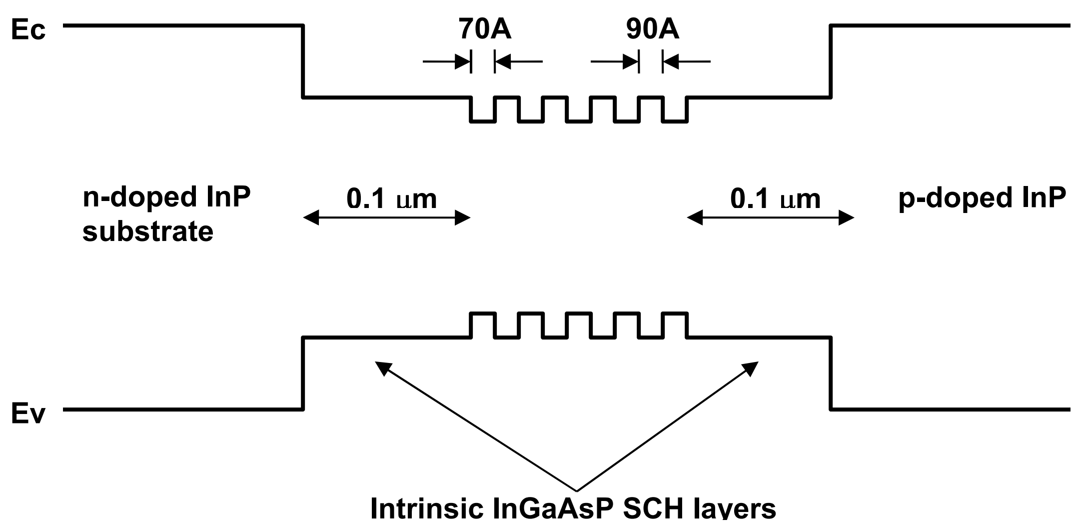
In this problem you will obtain the characteristics of a semiconductor laser. The waveguide for the laser is shown below. Same device will be used for all problems of this homework.



The figure is not exactly to scale

The thicknesses of various layers in the core region (SCH layers plus quantum wells) are shown in the figure below. The quantum well barriers and SCH layers have the same composition. The core region is undoped.

InGaAsP/InP quantum well laser structure



The refractive indices of InP, InGaAs SCH layer, InGaAsP quantum well barrier, InGaAsP quantum well, and the insulator are 3.170, 3.386, 3.386, 3.550, and 1.70, respectively, at wavelengths close to 1550 nm.

The intrinsic material losses are 40 cm^{-1} per 10^{18} cm^{-3} doping in the p-doped regions, and 5 cm^{-1} per 10^{18} cm^{-3} doping in the n-doped regions (due to free carrier absorption, intervalence band absorption, etc). The waveguide also has an additional modal loss of 1 cm^{-1} due to the photons scattering out of the waveguide as a result of waveguide surface roughness, material inhomogeneities, etc.

The length of the laser cavity is $L = 500$ µm. The mirror/facet reflectivities are $R_1 = R_2 = 0.3$. The current injection efficiency η_i is 0.8. The gain of each quantum well is given by:

$$\tilde{g} = \tilde{g}_o \ln\left(\frac{n}{n_{tr}}\right)$$

where $g_o = 1800$ cm^{-1} and $n_{tr} = 1.6x10^{18}$ cm^{-3}. The recombination constants are:

$$A = 0$$

$$B = 1.5x10^{-10} \text{ cm}^3 / s$$

$$C = 5x10^{-29} \text{ cm}^6 / s$$

In modeling the optical mode, we can safely assume that the metal contacts are not present (since the mode intensity at the metal contacts is zero). Since the core (SCH layers and the quantum wells) is thin in the y-direction and wide in the x-direction, the lowest order mode is expected to have the polarization predominantly in the x-direction. We model the x-component of the modal field by the scalar function $\phi(x, y)$. The scalar field satisfies the equation:

$$\left(-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\omega^2 n^2(x,y)}{c^2}\right)\phi(x,y) = -\frac{\omega^2 n_{eff}^2}{c^2}\phi(x,y)$$

One can solve this equation numerically using the finite element method. You will use the "ece533solver" to solve the equation above.

a) Make an input file for the ece533solver for the laser structure described above. Don't try to replace the quantum well and the barrier layers with a single layer with some averaged index. Enter the exact structure. Using the ece533solver do the following:

i) Calculate the effective index of the lowest mode.
ii) Calculate the modal waveguide loss $\tilde{\alpha}$.
iii) Calculate the group velocity v_g of the mode.
iv) Calculate the mode overlap (or the mode confinement factor) Γ_a for the active region (which consists of five quantum wells) and estimate the effective area A_{eff} of the mode.
v) Calculate the effective volume V_p of the mode.

b) Find the photon lifetime τ_p and the photon output coupling efficiency η_o.

c) Find the threshold gain $\tilde{g}_{th}$ and the threshold carrier density n_{th} for the laser.

d) Find the threshold current I_{th} of the laser.

e) Calculate and plot the steady state carrier density n and the photon density n_p for values of the bias current I between $0.5 I_{th} \le I \le 5 I_{th}$. Do not use the analytic approximations used in the lecture notes. You need to solve the two coupled nonlinear equations for the carrier and photon densities **exactly**. The product $\tilde{g} n_{sp}$, which appears in the spontaneous emission term, has complicated carrier density dependence but for this problem it can be approximated by the functional form $\tilde{g} n_{sp} = b\,n$, where $b = 5x10^{-16}$ cm^2. In the plot for the carrier density be sure to label and show the threshold carrier density n_{th} as a reference value.

f) Using your results in part (e), calculate and plot the output optical power P for values of the bias current I between $0.5 I_{th} \le I \le 5 I_{th}$.

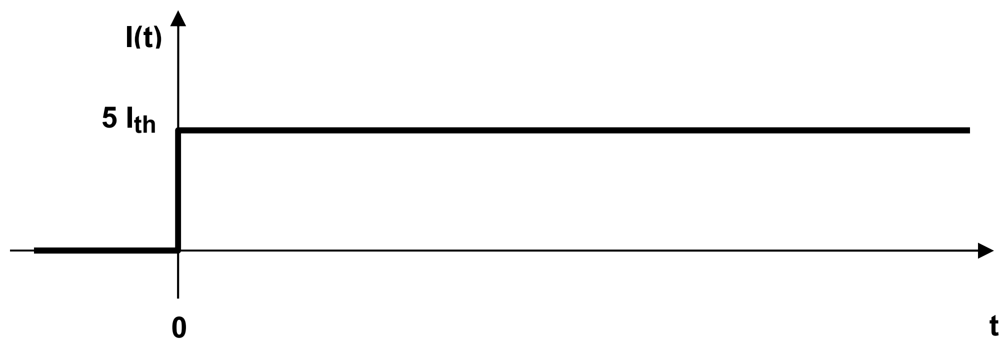
g) On the same plot calculate and plot the laser modulation response function $|H(f)|^2$ as a function of the modulation frequency f for the following values of the bias current I: $2I_{th},\ 3I_{th},\ 4I_{th},\ 5I_{th}$. Use the log scale for both the vertical and the horizontal axis.

Problem 7.2: (Semiconductor lasers – turn-on and turn-off transients)

This is a continuation of the previous problem. But instead of looking at the **small signal** modulation response of the laser diode – as we did in the previous problem – we will look at the turn on and turn off transients of the laser in time. So you will be doing a **large signal** analysis of the laser rate equations. You will be solving the laser rate equations for the carrier and photon densities in the time domain. The idea is to see how the semiconductor laser would perform in a digital optical communication system.

Turn-on transient:

a) The input current to the laser has the form shown below. It is a step function given by: $I(t) = 5\, I_{th}\, \theta(t)$.



Calculate and plot the carrier density, the photon density, and the output power, $n(t), n_p(t),$ and $P(t)$, respectively, as a function of time t for $t \geq 0$. You will have to solve the non-linear differential equations by direct numerical integration in time. Matlab **ode** solvers (e.g: ode45) will prove useful. Choose your time axis to be sufficiently long so that the dynamic variables get reasonably settled to their steady state values corresponding to an input current equal to $5 I_{th}$.

Turn-off transient:

b) Now suppose that the current equal to $5 I_{th}$ has been on for a long long time. At time $t = 0$, the current is suddenly switched off.

Calculate and plot the carrier density, the photon density, and the output power, $n(t), n_p(t),$ and $P(t)$, respectively, as a function of time t for $t \geq 0$.

c) Based on your results from parts (a) and (b), what is the maximum data rate (in GBits/sec) at which this laser can be reliably used in NRZ format fiber optical communications systems in which a logical "1" corresponds to light present and a logical "0" corresponds to no light present.

Problem 7.3: (Semiconductor lasers – quantum capacitance of the active region)

This is also a continuation of the last two problems. Suppose each quantum well of the laser has **only one** electron subband in the conduction band and **only one** heavy-hole subband in the valence band. The energies of these subbands are given by:

$$E_c\left(n=1,\vec{k}_{||}\right)=E_c+E_{n=1}+\frac{\hbar^2 k_{||}^2}{2m_e}$$

$$E_{hh}\left(m=1,\vec{k}_{||}\right)=E_v-E_{m=1}-\frac{\hbar^2 k_{||}^2}{2m_{hh}}$$

The electron and heavy-hole effective masses are: $m_e = 0.05 m_o$ and $m_{hh} = 0.5 m_o$. Above threshold, assume that the steady state carrier density is equal to the threshold carrier density n_{th} (i.e. $n = p = n_{th}$).

a) Suppose the laser is biased above threshold. If the voltage across the active region is changed by a small amount dV (by some means) then the total electron charge inside the quantum wells changes by $qV_a\,dn$, where V_a is the total volume of the quantum well layers, and dn is the change in the electron density in a single quantum well, and (to preserve charge neutrality) $dn = dp$. One can write,

$$qV_a\;dn = qV_a\;dp = C_a\;dV \qquad \text{or} \qquad \left.\frac{dn}{dV}\right|_{ss}=\frac{C_a}{qV_a}$$

where C_a is the "differential capacitance" of the active region (sometimes also called the quantum capacitance). The quantum capacitance is needed in the expression for the impedance of the active region (see the lecture notes). Find an expression for C_a and calculate the numerical value of C_a for the laser structure under consideration.

b) At small frequencies, $2\pi f \ll \omega_R$, the impedance of the active region is mostly inductive. Suppose the value of this inductance is L_a. Show that,

$$L_a C_a = \frac{1}{\omega_R^2}$$

Problem 7.4: (Semiconductor lasers – optimal number of quantum wells)

Device heating is a critical problem in most semiconductor lasers. Heating reduces the material gain and increases the nonradiative recombination rates and, thereby, increases the laser threshold current which in turn causes more heating creating a vicious cycle. This vicious cycle can be detrimental enough to not even allow lasing to occur no matter how much the current is increased. Reducing the threshold current is therefore extremely important in semiconductor lasers. Consider the semiconductor laser from the last three problems. In this problem we will choose the number of quantum wells that will minimize the laser threshold current. If the number of quantum wells is increased from one, the active region confinement factor will increase and the material threshold gain will decrease and so will the threshold carrier density and the threshold current. However, if the number of quantum wells is increased too much, the threshold current will increase because the current will have to supply carriers to all the quantum wells. There is therefore an optimal number of quantum wells that minimizes the threshold current.

Take the active region confinement factor Γ_a calculated in problem 7.1 and divide it by 5 to get the confinement factor per quantum well and call it Γ_{1a}. Now assume, for simplicity, that the total active region confinement factor for an N quantum well laser, goes as,

$$\Gamma_a = N\,\Gamma_{1a}$$

This linear scaling is not a bad assumption since the quantum wells are almost always located near the maximum intensity of the mode where the mode spatial variation is small.

a) Using the above linear scaling of the active region confinement factor find the optimal number of quantum wells that will give the lowest threshold current.

Problem 7.5: (Semiconductor lasers – power efficiency and thermodynamics)

Consider the laser to be ideal in the sense that the current injection efficiency is 100% (i.e. $\eta_i = 1$), and the output coupling efficiency is 100% (i.e. $\eta_o = 1$). The differential quantum efficiency of the laser above threshold is then unity (or 100%). Above threshold, the output power can be written as,

$$P = \frac{\hbar\omega}{q}\left(I - I_{th}\right)$$

The power efficiency of a laser above threshold is,

$$\eta_p = \frac{P}{(IV)}$$

a) Show that the power efficiency of an ideal laser diode is always less than unity above threshold.

Compare this to the problem from your midterm where you showed that the power efficiency of an ideal LED is greater than unity.