ECE 5330: Homework #6 Solutions

Problem 6.1

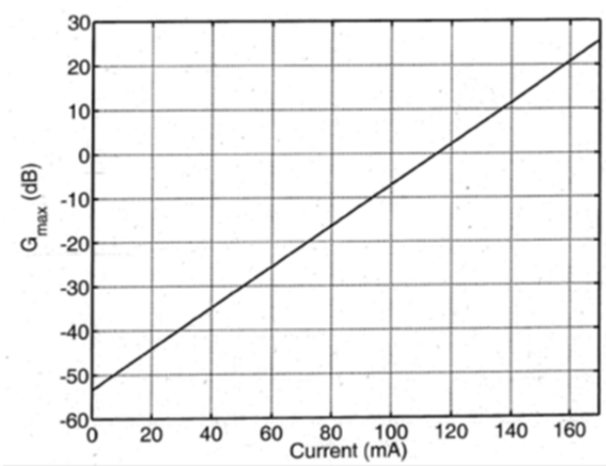
a) Use the slab waveguide approximation:

$$n_g \; n_{eff} = \frac{\iint n \, n_g^M \, |\phi|^2 \, dxdy}{\iint |\phi|^2 \, dxdy} \qquad \Rightarrow \qquad n_g = 3.2355$$

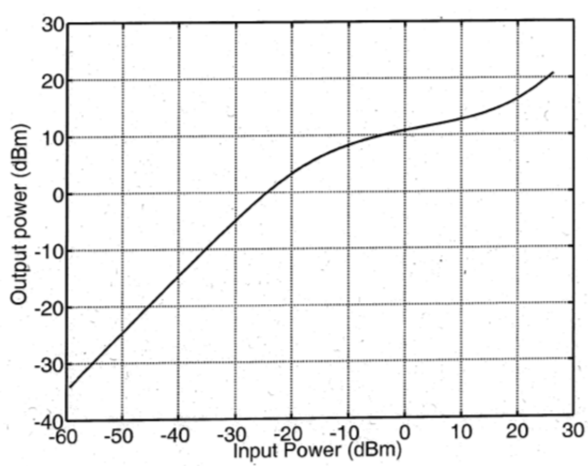
b) $A_{eff} = \dfrac{A_a}{\Gamma_a} = 1.5 \, (\mu\text{m})^2$

c) $P_{sat} = \dfrac{\hbar\omega A_{eff}}{\tilde{a}_o \tau_r} = 6 \, \text{mW}$

d) $G^* = e^{\left(\Gamma_a \tilde{g}^* - \tilde{\alpha}\right)L}$ and $\Gamma_a \tilde{g}^* = \Gamma_a \, \tilde{a}_o \left(n(z) - n_{tr}\right) \big|_{P(z) << P_{sat}} = \Gamma_a \, \tilde{a}_o \left(\dfrac{\eta_i \, I \tau_r}{q V_a} - n_{tr} \right)$. See the plot below.



e) $\Gamma_a \tilde{g}^* / \tilde{\alpha} = 4.88$ and $G^* = e^{\left(\Gamma_a \tilde{g}^* - \tilde{\alpha}\right)L} \approx 341.$

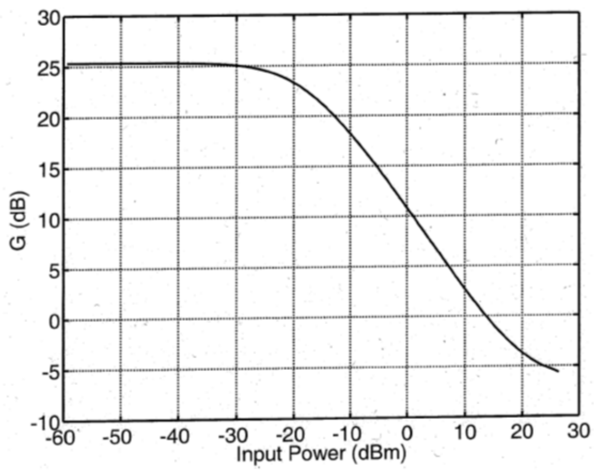


I have used

$$\frac{P(0)}{P_{sat}}=\left(\frac{\Gamma_a\tilde{g}^*}{\tilde{\alpha}}-1\right)\left(\frac{e^{\frac{(\Gamma_a\tilde{g}^*-\tilde{\alpha})L-\ln G}{\Gamma_a\tilde{g}^*/\tilde{\alpha}}}-1}{G\,e^{\frac{(\Gamma_a\tilde{g}^*-\tilde{\alpha})L-\ln G}{\Gamma_a\tilde{g}^*/\tilde{\alpha}}}-1}\right)=\left(\frac{\Gamma_a\tilde{g}^*}{\tilde{\alpha}}-1\right)\left(\frac{\left(\frac{G^*}{G}\right)^{\tilde{\alpha}/\Gamma_a\tilde{g}^*}-1}{G\left(\frac{G^*}{G}\right)^{\tilde{\alpha}/\Gamma_a\tilde{g}^*}-1}\right)$$

The easiest way to do this in matlab is to assume values for G and then calculate $P(0)$ using the above expression, and then plot the graph.

f)



g) Use,

$$P_{ASE}=\left(\frac{\Delta\omega_f}{2\pi}\right)\hbar\omega_f\;\frac{\Gamma_a\tilde{g}(\omega_f)}{\Gamma_a\tilde{g}(\omega_f)-\tilde{\alpha}}\,n_{sp}(\omega_f)\left(e^{(\Gamma_a\tilde{g}(\omega_f)-\tilde{\alpha})L}-1\right)\approx 51\,\mu W$$

Problem 6.2

a) Integrate,

$$\frac{dP(z)}{dz}=\left[\frac{\Gamma_a\tilde{g}^*}{1+\frac{P(z)}{P_{sat}}}-\tilde{\alpha}\right]P(z)$$

from 0 to z to get,

$$\frac{P(0)}{P_{sat}}=\left(\frac{\Gamma_a\tilde{g}^*}{\tilde{\alpha}}-1\right)\left(\frac{e^{\frac{(\Gamma_a\tilde{g}^*-\tilde{\alpha})L-\ln G(z)}{\Gamma_a\tilde{g}^*/\tilde{\alpha}}}-1}{G(z)\,e^{\frac{(\Gamma_a\tilde{g}^*-\tilde{\alpha})L-\ln G(z)}{\Gamma_a\tilde{g}^*/\tilde{\alpha}}}-1}\right)=\left(\frac{\Gamma_a\tilde{g}^*}{\tilde{\alpha}}-1\right)\left(\frac{\left(\frac{G^*}{G(z)}\right)^{\tilde{\alpha}/\Gamma_a\tilde{g}^*}-1}{G(z)\left(\frac{G^*}{G(z)}\right)^{\tilde{\alpha}/\Gamma_a\tilde{g}^*}-1}\right)$$

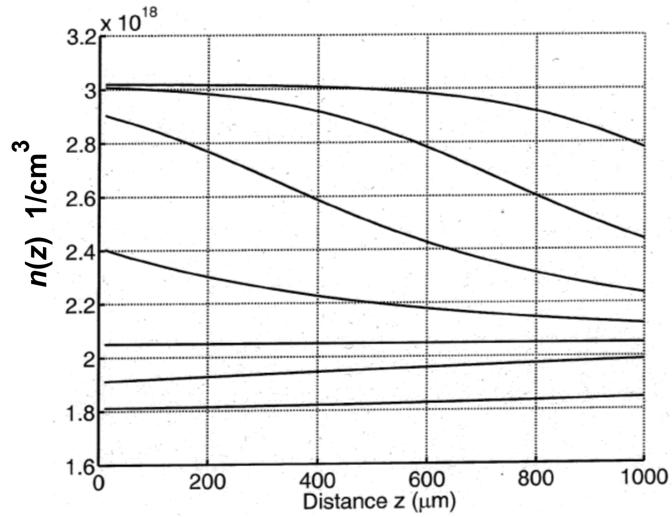
where,

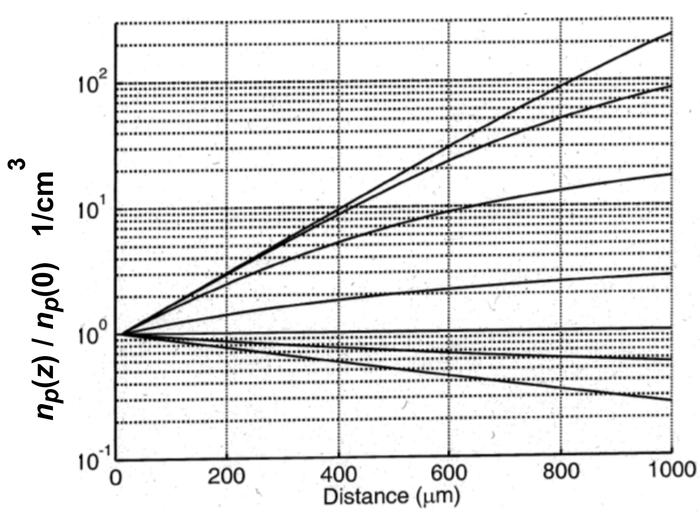
$$G(z)=\frac{P(z)}{P_{sat}}$$

The above equation can be used to calculate $P(z)$ given $P(0)$ and then the carrier density can be obtained using,

$$n(z) - n_{tr} = \frac{\dfrac{\eta_i I \tau_r}{qV_a} - n_{tr}}{1 + \dfrac{P(z)}{P_{sat}}}$$

The results are plotted below.





b) When $P(0) \to \infty$, $n(z) \to n_{tr}$ and the amplifier is completely saturated.

c) When the input power is zero,

$$n(z) = \frac{\dfrac{\eta_i I \tau_r}{qV_a} - n_{tr}}{1 + \dfrac{P(z)}{P_{sat}}} + n_{tr} = \frac{\eta_i I \tau_r}{qV_a} = 3 \times 10^{18}\ 1/\mathrm{cm}^3$$

d) Perhaps a poorly worded question. The question is really asking, "What input power level will make the amplifier gain G unity and, therefore, the modal gain at every location in the amplifier equal to the modal loss at that location?" . If the modal gain is equal to the modal loss at every location, then this means that the carrier density $n(z)$ and the optical power $P(z)$ are both independent of position. This also means that, $P(L) = P(z) = P(0)$. If $G = 1$, we get,

$$\frac{P(0)}{P_{sat}} = \left(\frac{\Gamma_a \tilde{g}^*}{\tilde{\alpha}} - 1 \right)$$

e) This is the same situation as in part (d). When the gain of the amplifier is unity,

$$\Gamma_a \, \tilde{a}_o \left(n(z) - n_{tr} \right) = \tilde{\alpha}$$

$$\Delta n(z) = 2.05 \times 10^{18} \ 1/\mathrm{cm}^3$$