## ECE 5330: Semiconductor Optoelectronics

Fall 2014
Homework 6
Due on Oct. 23, 2014

## Suggested Readings:

i) Study lecture notes.

## Table of Parameter Values of III-V Semiconductors:

| Parameters at 300K | GaAs | AlAs | InAs | $\operatorname{InP}$ | GaP |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Lattice constant $(\mathrm{A})$ | 5.6533 | 5.6600 | 6.0584 | 5.8688 | 5.4505 |
| Eg( $\Gamma$-point) $(\mathrm{eV})$ | 1.424 | 3.03 | 0.354 | 1.344 | 2.78 |
| $m_{e}^{*} / m_{O}$ | 0.067 | 0.15 | 0.023 | 0.077 | 0.25 |
| $m_{h h}^{*} / m_{O}$ | 0.50 | 0.79 | 0.40 | 0.6 | 0.67 |
| $m_{l h}^{*} / m_{O}$ | 0.087 | 0.15 | 0.026 | 0.12 | 0.17 |
| $E p(\mathrm{eV})$ | 25.7 | 21.1 | 22.2 | 20.7 | 22.2 |

## Problem 6.1: (Semiconductor Optical Amplifiers)

Consider the following waveguide structure for a bulk $1.55 \mu \mathrm{~m} \operatorname{InP} / \mathrm{InGaAsP}$ semiconductor optical amplifier (the cross-section of the waveguide is shown):


The refractive indices of InP and InGaAsP layers are 3.17 and 3.55 , respectively, at a wavelength of 1.55 $\mu \mathrm{m}$. The refractive index of the insulator is 1.7. The optical confinement factors ( $\Gamma$ ) for each region are shown in the figure above. The effective index of the fundamental mode is 3.18 at $1.55 \mu \mathrm{~m}$. The waveguide modal loss $\tilde{\alpha}$ is $15 \mathrm{~cm}^{-1}$ and is assumed (for simplicity) to be fixed and independent of the carrier density. All materials are non-dispersive.

The gain in the active region is given by the relation,
$\tilde{g}(\lambda=1.55 \mu m)=\tilde{a}_{o}\left(n-n_{t r}\right)$
where,
$\tilde{a}_{0}=4 \times 10^{-16} \mathrm{~cm}^{2}$
$n_{t r}=1.8 \times 10^{18} \mathrm{~cm}^{-3}$

In the InGaAsP active layer, the non-radiative recombination rate and the radiative recombination rate due to spontaneous emission into all the modes (guided as well as unguided) is described by the lifetime $\tau_{r}$, where $\tau_{r}=0.8 \times 10^{-9} \mathrm{sec}$. The length of the amplifier is $1000 \mu \mathrm{~m}$. The current injection efficiency $\eta_{i}$ is 0.80 .
a) From the information given, estimate the group index of the fundamental mode at $1.55 \mu \mathrm{~m}$ ?
b) What is effective area $A_{\text {eff }}$ of the optical mode?
c) What is $P_{\text {sat }}$ ?
d) Calculate and plot the maximum (unsaturated) amplifier gain $G^{*}$ as a function of the current $I$ for values of $I$ between 0 mA and 170 mA .
e) Suppose the amplifier is biased with a 170 mA current. Calculate and plot the output optical power vs the input optical power for values of the input optical power between $10^{-6} P_{\text {sat }}$ and $100 P_{\text {sat }}$. The scale for both the axis should be dBm . The dBm scale is commonly used to specify optical powers in optoelectronics. Optical power in dBm is calculated as:
$P($ in dBm$)=10 \log _{10}\left[\frac{P(\text { in Watts })}{10^{-3}}\right]=10 \log _{10}[P($ in mWatts $)]$
The dBm scale tells (in dB scale) the ratio of the actual optical power to 1 mWatt of optical power.
f) Suppose the amplifier is biased with a 170 mA current. Calculate and plot the amplifier gain $G$ (in dB ) vs the input optical power (in dBm ) for values of the input optical power between $10^{-6} P_{\text {sat }}$ and 100 $P_{\text {sat }}$.
g) Calculate the amplified spontaneous emission (ASE) power coming out from one end of the amplifier for current $I$ equal to 170 mA assuming that an optical filter with a 5 nm bandwidth is placed in front of the SOA. The center frequency of the filter is $1.55 \mu \mathrm{~m}$. You are only supposed to calculate the ASE that goes through the filter. Assume no input optical signal and also assume that the carrier density is not
affected by the ASE. Assume that $n_{s p}(\lambda=1.55 \mu m)=1.5$ for the value of the carrier density that corresponds to the given value of the current $/$ equal to 170 mA .


## Problem 6.2: (Semiconductor Optical Amplifiers-II)

This can be a hard problem or a very easy problem. If you approach it right you will realize that you don't need to do any extra calculation or computation other than what is already in the lecture notes. Perhaps, numerical computation may require 5-6 lines of matlab code. But you will have to understand the stuff in the lecture notes really well.

Consider the same structure as in the previous problem. In this problem you will calculate and plot the photon density and the carrier density inside the waveguide as a function of distance for different values of the input optical power. Suppose the amplifier is biased with a 170 mA current in all parts of this problem.
a) Calculate and plot the carrier density $n(z)$ and the photon density $n_{p}(z)$ inside the amplifier (for $0 \leq z \leq L)$ in steady state when the input optical power $P(z=0)$ has the following values:
$P(z=0)=10^{-3} P_{\text {sat }}$
$P(z=0)=10^{-2} P_{\text {sat }}$
$P(z=0)=10^{-1} P_{\text {sat }}$
$P(z=0)=10^{0} P_{\text {sat }}$
$P(z=0)=(s-1) P_{\text {sat }}$ where $s=\frac{\Gamma_{a} \tilde{g}^{*}}{\tilde{\alpha}}$
$P(z=0)=10^{1} P_{\text {sat }}$
$P(z=0)=10^{2} P_{\text {sat }}$

## Notes:

1) All plots for the carrier density should be in the same figure.
2) All plots for the photon density should be in the same figure. Instead of plotting $n_{p}(z)$, plot the normalized quantity $n_{p}(z) / n_{p}(z=0)$, and choose a log scale for the vertical axis since the photon density will change exponentially as a function of distance.
3) Choose about at least 20 points along the length of the amplifier to generate your plots.

## Hint:

In solving for the input-output characteristics of an SOA in the lecture notes, I integrated from 0 to $L$ at one point. You can integrate from 0 to $z$. So instead of obtaining an equation for $P(L) / P(z=0)$, you will get an equation for $P(z) / P(z=0)$.
b) What is the carrier density $n(z)$ (for all $z$ ) when the input power is infinite? What is the gain $G$ of the amplifier in this case (in dB )?
c) What is the carrier density $n(z)$ (for all $z$ ) when the input power is zero? What is the gain $G$ of the amplifier in this case?
d) What optical power level $P(z)$ is needed to make the modal gain per unit distance, i.e. $\Gamma_{a} \tilde{a}_{o}\left[n(z)-n_{t r}\right]$, equal to the waveguide intrinsic loss $\tilde{\alpha}$ at the location $z$ ?
e) What is the carrier density $n(z)$ (for all $z$ ) when the input power is such that the gain $G$ of the amplifier is unity?

## Problem 6.3: (optical mode solver - scalar field approximation)

In this problem you will solve for the fundamental mode of a 2 D optical waveguide in the scalar field approximation. The waveguide is based on the SOA device structure of problem 5.1, and is shown below.


The thicknesses of various layers are also shown in the figure below.


## InGaAsP/InP quantum well structure for optical gain

The refractive indices of $\operatorname{InP}$, InGaAs SCH layer, InGaAsP quantum well, and the insulator are 3.170, $3.386,3.550$, and 1.70 , respectively, at wavelengths close to 1550 nm . All materials are non-dispersive. In modeling the optical mode, we can safely assume that the metal contacts are not present (since the mode intensity at the metal contacts is near zero). Since the core (SCH layers and the quantum well) is thin in the $y$-direction and wide in the $x$-direction, the lowest order mode is expected to have the polarization predominantly in the $x$-direction. We model the $x$-component of the modal field by the scalar function $\phi(x, y)$. The scalar field satisfies the equation:

$$
\left(-\frac{\partial^{2}}{\partial x^{2}}-\frac{\partial^{2}}{\partial y^{2}}-\frac{\omega^{2} n^{2}(x, y)}{c^{2}}\right) \phi(x, y)=-\frac{\omega^{2} n_{e f f}^{2}}{c^{2}} \phi(x, y)
$$

One can solve this equation numerically using the finite element method. You will use the "ece533solver" to solve the equation above. Since the quantum well is very thin compared to the wavelength, one can safely assume that the core is a 2060 Angstrom thick layer of refractive index equal to:
$\sqrt{\frac{(3.386)^{2} 2000+(3.55)^{2} 60}{2060}}=3.39$
Note that the index-squared is averaged.
The goals of this problem are to:
a) Calculate the effective index of the lowest mode
b) Calculate the confinement factor (also called the overlap factor) for the quantum well
c) Learn to use a mode solver

## STEP 1

Download the following files from the web and save them in a directory.
preprocess.exe
ece533solver.exe
postprocess.exe
optical_amp.dat
plotindex.m
plotmode.m

## STEP 2

The file optical_amp.dat is the input file that describes the structure of the SOA waveguide. Open the file and see what it contains.

```
XNUM 151
YNUM 151
WAVELENGTH 1.55
NUMREGION }
NUMOVERLAP 1
XMESH 1 151 150 1.0 0.0 4.500
YMESH 1 151 150 1.0 0.0 2.506
REGION 0.0 4.5 0.000 1.500 3.17
REGION 1.5 3.0 1.500 1.600 3.17
REGION 1.5 3.0 1.600 1.806 3.39
REGION 1.5 3.0 1.806 2.306 3.17
REGION 0.0 1.5 1.500 2.306 1.70
REGION 3.0 4.5 1.500 2.306 1.70
REGION 0.0 4.5 2.306 2.506 1.00
OVERLAPREGION 1.500 3.000 1.700 1.706
```

The commands XNUM and YNUM say that there are 151 mesh points in the x -direction and 151 mesh points in the y-direction. The $(0,0)$ of the co-ordinate system is the lower left corner of the device structure. The WAVELENGTH command specifies the wavelength in microns. The NUMREGION command says that there are 7 REGION commands in the input file. The NUMOVERLAP command says there is 1 OVERLAPREGION command in the input file. The commands XMESH and YMESH make the mesh. For the purpose of this course the XMESH and YMESH commands should have the form:

XMESH 1 <xnum> <xnum-1> 1.00 .0 <device dimension in the $x$-direction in microns> YMESH 1 <ynum> <ynum-1> 1.00 .0 <device dimension in the $y$-direction in microns>

The entire device is composed of several adjoining rectangular "regions". Each region is specified by the co-ordinates of its two corners as shown below. Each region can have its own value of refractive index.


```
REGION \(<\mathrm{x}_{\text {start }}><\mathrm{x}_{\text {end }}><\mathrm{y}_{\text {start }}><\mathrm{y}_{\text {end }}><\) index \(>\)
```

You can have as many REGION commands as you want in the input file (as long as you do not forget to change the NUMREGION value). You must have enough REGION commands to cover the entire device. Uncovered areas of the device may end up with random index values. REGION command is used to assign index values to different parts of the device.

And finally the OVERLAPREGION command tells the postprocessor to calculate the confinement factors (also called mode overlap factors in the literature) for the region co-ordinates that follow the OVERLAPREGION command. You can have as many overlap commands as you want in the input file (as long as you do not forget to change the NUMOVERLAP value).

Now you are ready to start. All command prompt directives need to be issued while in the directory that has the executables.

## STEP 3

At the command prompt type:

```
preprocess optical_amp.dat op1
```

This will create a directory "op1" that will have your device structure files. Copy the following matlab script files into the directory "opl":
plotindex.m
plotmode.m
Type plotindex on the matlab prompt while in the "op1" directory in matlab. You should see the crosssection of the amplifier. Make sure it looks ok and is what you expected.

## STEP 4

At the command prompt type:
ece533solver op1
This will calculate the effective index of the fundamental mode and display the result and it will also store the mode in a file in the "op1" directory. Note the effective index of the mode.

Type plotmode on the matlab prompt while in the "op1" directory in matlab. You should see the crosssection of the amplifier and the contour plots of the mode intensity. The inner most contour is where the mode intensity is half its maximum value. Each subsequent contour corresponds to a field intensity reduction by a factor of half.

## STEP 5

At the command prompt type:

This will use the files generated in STEP 4 to calculate the mode confinement factors for as many different regions as specified by the number of OVERLAPREGION commands in the input file. The overlaps are displayed for different regions in the same order as the corresponding OVERLAPREGION commands in the input file. If you decide to calculate overlaps for a different regions, just change the input file and run the postprocessor again without having to run the ece533solver again.

For this problem you need to hand in the plots generated by the scripts plotmode.m and plotindex.m, the calculated effective index of the mode, the confinement factor of the mode in the quantum well region, the confinement factor of the mode in the p-doped region, the confinement factor of the mode in the $\mathbf{n}$-doped region, the confinement factor of the mode in the two SCH regions, and the effective area of the mode (as seen by the quantum well).

