# ECE 5330: Homework #5 Solutions

## 5.1

Suppose the function $P(\theta, \phi)$ describes the probability of light being reflected at an angle $(\theta, \phi)$. Then $\iint P(\theta,\phi) \sin\theta \, d\phi \, d\phi = 1$

Since the dependence on $\phi$ is uniform we may only consider the probability function $P(\theta)$ normalized such that $\int_0^{\pi/2} P(\theta) \sin\theta \, d\theta = 1$

Also, since the reflector is Lambertian, $P(\theta) \propto \cos\theta \Rightarrow P(\theta) = 2\cos\theta$

a) $f_{out} = \int_0^{\theta_c} P(\theta) \sin\theta \, d\theta = \sin^2\theta_c = \left(\frac{n_a}{n_s}\right)^2$.

b) $f_{in} = \int_{\theta_c}^{\pi/2} P(\theta) \sin\theta \, d\theta = \cos^2\theta_c = \left(\sqrt{1-\left(\frac{n_a}{n_s}\right)^2}\right)^2 = 1 - \left(\frac{n_a}{n_s}\right)^2$

c) Answer is: $\frac{1}{f_{in}} \int_{\theta_c}^{\pi/2} P(\theta) \cdot \frac{2W}{\cos\theta} \cdot \sin\theta \, d\theta = 4W \frac{\cos\theta_c}{f_{in}} = \frac{4W\left[1 - \frac{1}{2}\left(\frac{n_a}{n_s}\right)^2\right]}{\left[1 - \left(\frac{n_a}{n_s}\right)^2\right]}$

$$\approx 4W\left[1 + \frac{1}{2}\left(\frac{n_a}{n_s}\right)^2\right] \approx 4W$$

d) Answer is: $\frac{1}{f_{out}} \int_0^{\theta_c} P(\theta) \cdot \frac{2W}{\cos\theta} \sin\theta \, d\theta = \frac{4W\left[1-\cos\theta_c\right]}{\left(\frac{n_a}{n_s}\right)^2} \approx \frac{4W \, \frac{1}{2}\left(\frac{n_a}{n_s}\right)^2}{\left(\frac{n_a}{n_s}\right)^2}$

$$\approx 2W.$$

e) The average distance travelled can be calculated by summing over all possible paths and weighing each distance by its probability:

$$= 2W f_{out} + 6W f_{in} f_{out} + 10W (f_{in})^2 f_{out} + \cdots$$

$$= 2W f_{out} \sum_{n=0}^{\infty} (1+2n)(f_{in})^n = 2W f_{out}\left\{\frac{1}{1-f_{in}} + 2\frac{f_{in}}{(1-f_{in})^2}\right\} = 2W f_{out} \frac{(1+f_{in})}{(1-f_{in})^2}$$

$$= 2W \frac{\left(\frac{n_a}{n_s}\right)^2 \left(1 + 1 - \left(\frac{n_a}{n_s}\right)^2\right)}{\left(\frac{n_a}{n_s}\right)^4} \approx \frac{4W}{\left(\frac{n_a}{n_s}\right)^2} = W \cdot \frac{4 n_s^2}{n_a^2}$$

5.2

a) Power delivered to $R = P = -I(V+IR_s)$ where $I = I_0(e^{qV/kT}-1) - I_L$.

$$\frac{dP}{dV} = -\frac{dI}{dV}(V+IR_s) - I\left(1+\frac{dI}{dV}R_s\right) = 0 \Rightarrow V+IR_s = -I(R_d+R_s) \quad \left\{\frac{1}{R_d} = \left|\frac{dI}{dV}\right|\right\}$$

$$\Rightarrow P = -I(V+IR_s) = I^2(R_d+R_s) \quad \text{but } P \text{ also equals } I^2R \Rightarrow R = R_d+R_s$$

b) The voltage across each junction is $\frac{V}{m}$.

$P = -IV$ where $I = I_0\left(e^{\frac{qV}{mkT}}-1\right) - I_L$

Note that $\frac{1}{R_d} = \frac{qI_0}{kT}e^{\frac{qV}{kT}}$ is how $R_d$ is defined for each individual junction. $\frac{dP}{dV} = -\frac{dI}{dV}V - I = 0 \Rightarrow \frac{-V}{mR_d} = I$

$\Rightarrow P = -IV = I^2 mR_d$. But $P = I^2R \Rightarrow R = mR_d$

5.3

a) See the attached plot.

b) At zero temperature, spontaneous emission that result in a loss of energy, is completely shut off. At 2.7K, it is not completely shut off but so small that it can be neglected. So the efficiency is essentially given by. $\eta = \frac{qA\sin^2\theta_s F_N(T_s, 0, E_g/k)V}{A\sin^2\theta_s F_E(T_s, 0, E_g/k)}$

where for small $T_a$, $qV$ is very close to $E_g$.

c) See the attached plot,

The reason for an optimal voltage is as follows. If the voltage is too small, then the output power will be small. If the voltage is too large then the spontaneous emission part will become large (remember that the Fermi level splitting is equal to $qV$) and energy will be lost due to radiative recombination (even if, as we have, all other recombination mechanisms are ignored). But at low temperatures spontaneous emission part is very small as long as $qV < Eg$. So the optimal voltage is always equal to $\frac{Eg}{q}$.

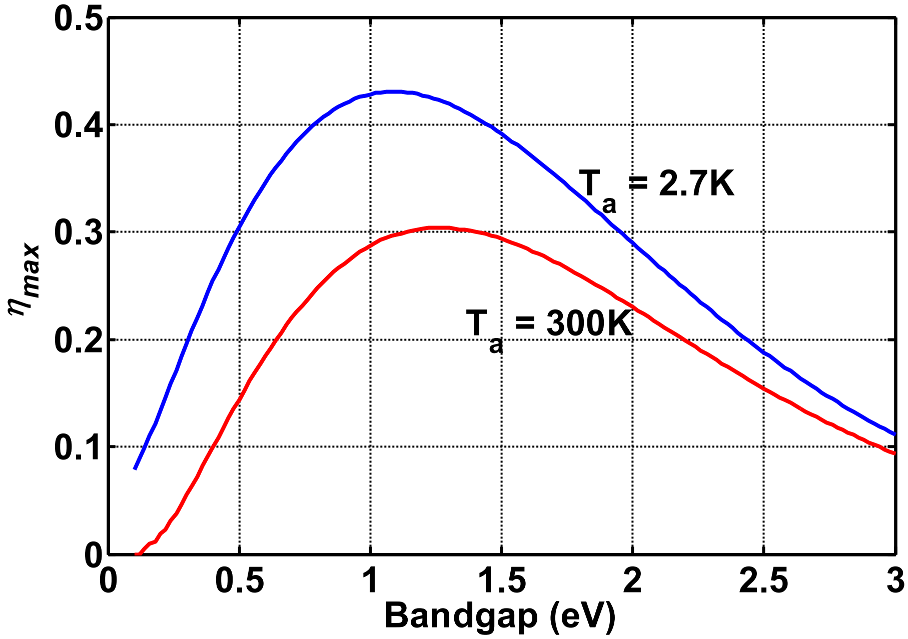
d) As noted in part (c), at low temperatures the spontaneous emission part can be neglected and the efficiency is given by:

$$\eta \cong \frac{qA \sin^2\theta_S \, F_N(T_S, 0, Eg/k_T)\, V}{A \sin^2\theta_S \, F_E(T_S, 0, Eg/k_T)} = \frac{qA\, F_N(T_S, 0, Eg/k_T)\, Eg}{A\, F_E(T_S, 0, Eg/k_T)}.$$

$=$ independent of $\sin^2\theta_S$.

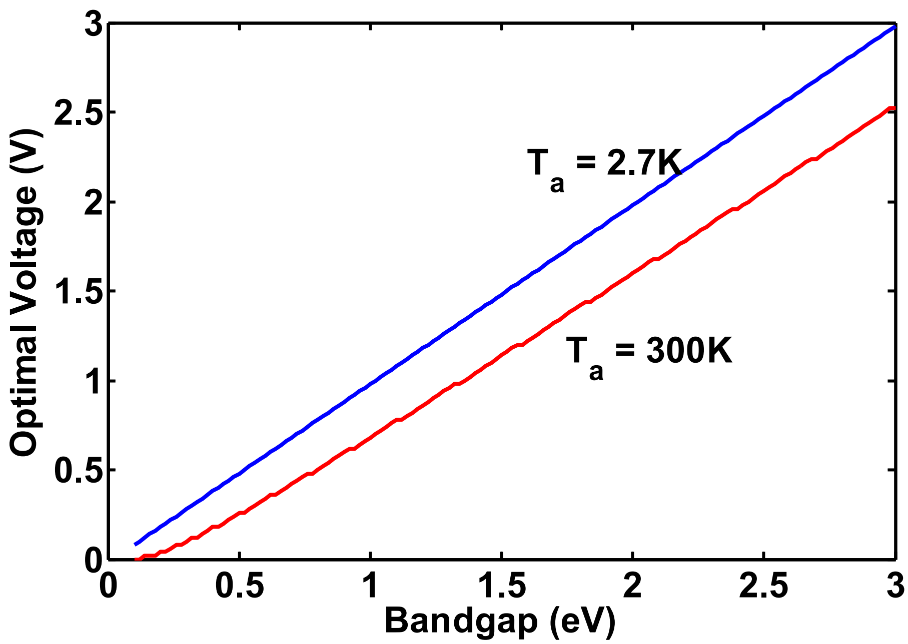
$\Rightarrow$ Concentration will not change the results in (a).

a)



0.5
0.4
0.3
0.2
0.1
0
$\eta_{max}$
$T_a$ = 2.7K
$T_a$ = 300K
0
0.5
1
1.5
2
2.5
3
Bandgap (eV)


c)



3
2.5
2
1.5
1
0.5
0
Optimal Voltage (V)
$T_a$ = 2.7K
$T_a$ = 300K
0
0.5
1
1.5
2
2.5
3
Bandgap (eV)


d) See (a)