Department of Electrical and Computer Engineering, Cornell University

ECE 5330: Semiconductor Optoelectronics

Fall 2014

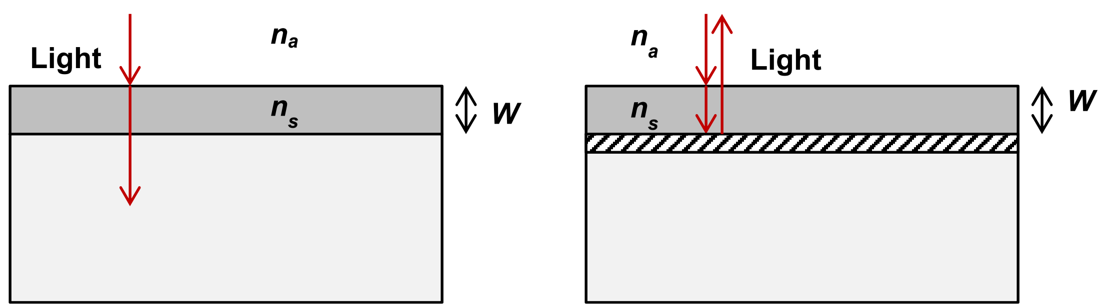
Homework 5 **Due on Thursday Oct. 09, 2014**

Suggested Readings:

i) Lecture Notes
iii) Review electrodynamics and dielectric optical slab waveguides from your favorite book for the coming weeks.

Problem 5.1: (Light trapping in solar cells)

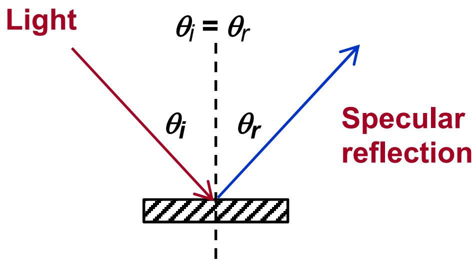
In this problem you will explore the benefits of trapping light in solar cells. Trapping light becomes important when either the light absorption layer is very thin (as in thin-film solar cells) or when the light absorption coefficient is not too large (as in Silicon and organic solar cells). Consider the following thin-film solar cell. Assume that the top semiconductor/ambient interface is suitably coated so that there is no reflection (other than total internal reflection).

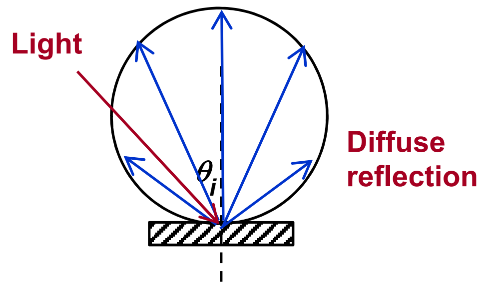


In the first case (top left), the optical path length through the light absorbing semiconductor is just W. If the absorption coefficient of the semiconductor is small, most of the light will go unabsorbed. In the second case (top right), a perfect (metal) reflector is placed underneath the semiconductor to reflect light back into the semiconductor. In this case the optical path length through the light absorbing semiconductor is $2W$. Now consider what happens when the perfect back reflector is replaced by a diffuse (Lambertian) reflector. A Lambertian reflector is one for which the reflected light power per unit solid angle falls off as the cosine of the angle from the normal to the surface of the reflector irrespective of the angle of the incident light. If $I(\theta,\phi)$ is the power reflected per unit solid angle then,

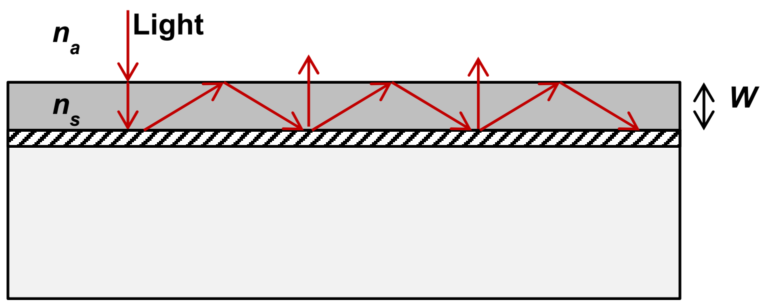
$$I(\theta,\phi) \propto \cos\theta$$

Light reflection from rough surface, also called diffuse reflection, has approximately a Lambertian pattern. The difference between specular reflection and diffuse reflection is shown in the Figure below.





Now, coming back to the solar cell, light reflected at angles larger than the semiconductor/ambient interface critical angle will get trapped and might bounce back and forth between the top interface and the back reflector multiple times before it eventually escapes. Your goal in this problem is to figure out the average increase in the optical path length because of the Lambertian back reflector. The parts below will help you through the problem.



a) What is the fraction of the incident power that is reflected from the back reflector at angles such that the reflected light does not get totally internally reflected when it reaches the top interface? Call this fraction f_{out}. Assume that,

$$n_s^2 >> n_a^2$$

$$\Rightarrow \sqrt{1-\left(\frac{n_a}{n_s}\right)^2} \approx 1-\frac{1}{2}\left(\frac{n_a}{n_s}\right)^2$$

b) What is the fraction of the incident power that is reflected from the back reflector at angles such that the reflected light gets totally internally reflected when it reaches the top interface? Call this fraction f_{in}. Assume that,

$$n_s^2 >> n_a^2$$

$$\Rightarrow \sqrt{1-\left(\frac{n_a}{n_s}\right)^2} \approx 1-\frac{1}{2}\left(\frac{n_a}{n_s}\right)^2$$

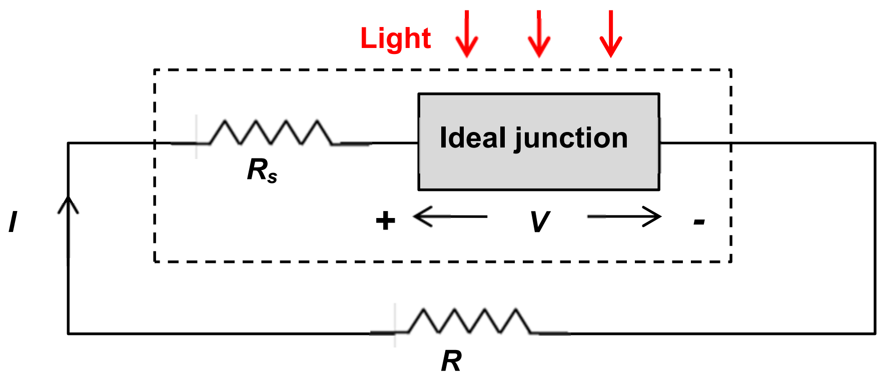
c) Consider photons that are reflected from the back reflector at an angle θ greater than the critical angle of the semiconductor/ambient interface. Such photons travel a distance $2W/\cos\theta$ inside the semiconductor after the reflection before they hit the back reflector again. Find the average value of this distance given the characteristics of a Lambertian reflector. Hint: answer should be approximately $4W$.

d) Consider photons that are reflected from the back reflector at an angle θ less than the critical angle of the semiconductor/ambient interface. Such photons travel a distance $W/\cos\theta$ inside the semiconductor after the reflection before they emerge from the top surface. Find the average value of this distance given the characteristics of a Lambertian reflector. Hint: answer should be approximately W.

d) Now we try to put everything we have learned so far and get the final answer. Consider light incident normally on the photodetector. A fraction f_{out} of the incident light will cover an average distance of $2W$ before it escapes from the top interface. A fraction $(f_{in})f_{out}$ will cover an average distance of $6W$ before it escapes from the top interface. A fraction $(f_{in})^2 f_{out}$ will cover an average distance of $10W$ before it escapes. A fraction $(f_{in})^3 f_{out}$ will cover an average distance of $14W$, and the list keeps going…..Find the average distance covered by the incident light in the semiconductor and compare it to W, the distance covered or the optical path length in the absence of a back reflector and find out the enhancement of the optical path length.

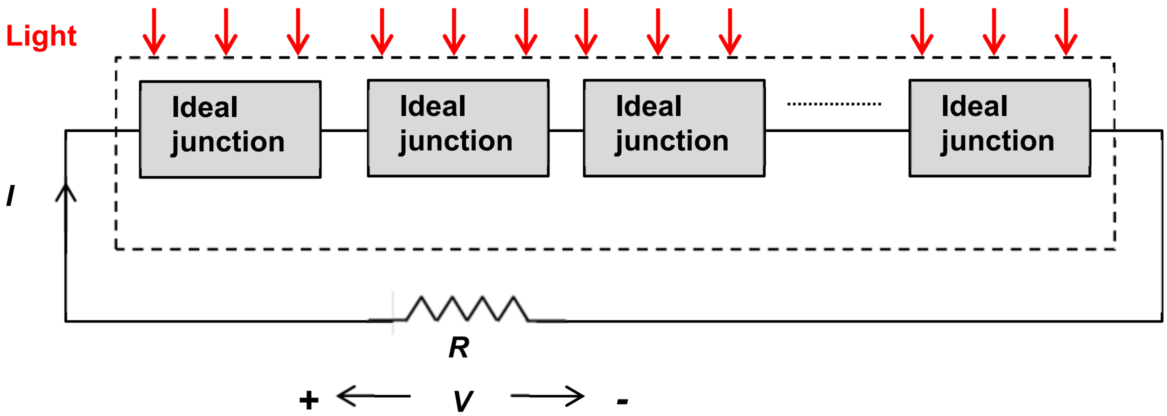
Problem 5.2: (Solar cell circuits)

a) Consider a solar cell connected as shown below.



Most solar cells have parasitic series resistance due to metal contacts, quasineutral regions, etc. We model it as a resistance R_S in series with an ideal junction. The problem now is how to choose the value of the external resistor R in order to get the maximum power delivered to the external resistor under given light illumination conditions. Find the optimal value of R and express it in terms of the differential resistance of the ideal junction at the optimal operating point. Award yourself no points if the answer is right but the method is incorrect.

b) Consider a solar cell module in which "m" ideal junctions are connected electrically in series, as shown in the Figure below. The problem is again how to choose the value of the external resistor R in order to get the maximum power delivered to the external resistor under given light illumination conditions. Find the optimal value of R and express it in terms of the differential resistance of an ideal junction at the optimal operating point. Award yourself no points if the answer is right but the method is incorrect.



Problem 5.3: (Fundamental solar cell efficiencies for deep space applications)

The goal of this problem is to find the maximum possible efficiency of a semiconductor solar cell as a function of the bandgap for use in space at a temperature of 2.7K (i.e. $T_a = 2.7\text{K}$), the temperature in space of the cosmic microwave background radiation.

a) Find your answer using the relation in the notes,

$$\eta = \frac{IV}{A\sin^2\theta_s\, F_E(T_s,0,0)}$$
$$= \frac{qA\left[\sin^2\theta_s\, F_N(T_s,0,E_g/\hbar) + \left(1-\sin^2\theta_s\right)F_N(T_a,0,E_g/\hbar) - F_N(T_a,V,E_g/\hbar)\right]V}{A\sin^2\theta_s\, F_E(T_s,0,0)}$$

For each value of the bandgap, you will need to maximize the above expression with respect to the operating voltage 'V' and then this maximum value will be the answer for the particular bandgap value chosen. Don't pick voltage values larger than the bandgap (in eV) when finding the optimal voltage otherwise you will get unphysical results (you will be getting optical gain for some frequencies and the arguments used to derive the above expression will not all work). Assume no solar light concentration. Plot your results as a graph of the maximum possible efficiency vs the bandgap.

b) Compare your results from (a) from those given in the notes for $T_a = 300\text{K}$ and explain physically the reasons/causes for the differences. Award yourself no points for wrong or poor explanations.

c) In doing calculations for part (a) you found the voltages that maximize the efficiencies. Plot these optimal voltages as a function of the bandgap (in eV) and see if you can detect any correlation between the optimal voltage and the bandgap. Explain physically the reasons/causes for the observations you make. Award yourself no points for wrong or poor explanations.

d) Repeat part (a) but now assume full concentration and plot your results. Compare your results with those obtained in part (a) and explain physically the reasons/causes for the differences and/or similarities. Award yourself no points for wrong or poor explanations.

Problem 5.4: (Thermodynamics and the temperature of the planets)

a) In this problem we try to estimate the temperature of planets in the solar system using simple thermodynamics. The values of θ_s (the half-angle of the Sun) for an observer sitting on the surface of

Earth, Jupiter, and Neptune are 0.266, 0.05, and .009 degrees respectively. Assuming the sun and the planets to be ideal black body emitters, find the temperatures of Earth, Jupiter, and Neptune. The exact values of the average temperature of these planets, for your reference, are 288K, 124K, and 59K, respectively.