ECE 5330 Homework #4 Solutions

4.1 .

a) See the attached plot. To get the right answer your integration limits (for the frequency integral) includes all frequencies for which the integrand is non-zero.

b) I chose $A = 10^8$ /sec and $B = 1.2 \times 10^{-10}$ cm^3/sec. See the actual curve and the fit in the attached plot.

c) In the integral for $R_{sp}(\omega)$ we have the factor:

$$f_c(\vec{k})\left[1 - f_v(\vec{k})\right] \qquad \left\{ \text{for} \quad E_c(\vec{k}) - E_v(\vec{k}) = \hbar\omega \right\}$$

and the integral for $\alpha(\omega)$ had: $f_v(\vec{k}) - f_c(\vec{k})$

Their ratio is:

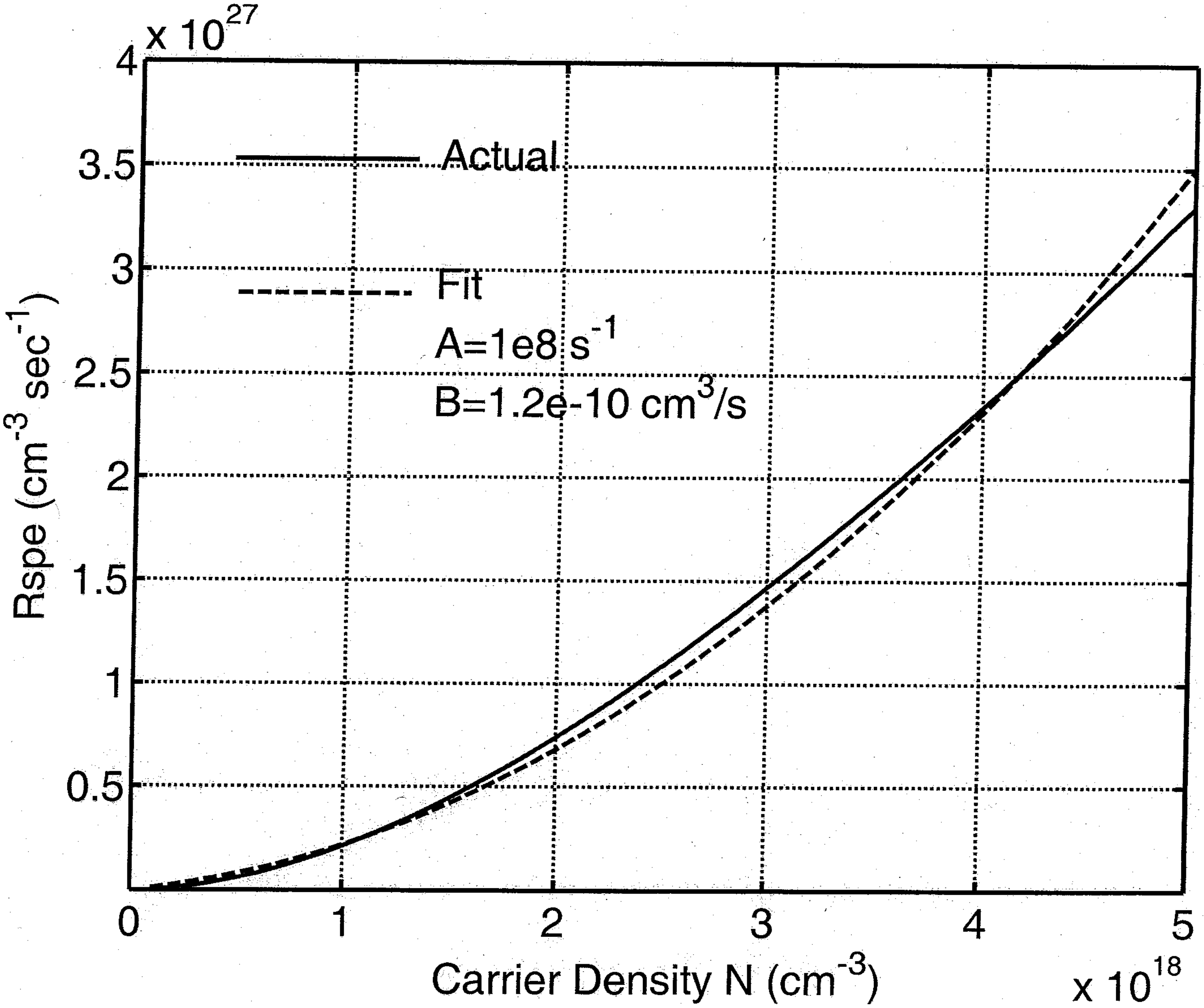
$$\frac{f_c(\vec{k})\left[1 - f_v(\vec{k})\right]}{f_v(\vec{k}) - f_c(\vec{k})} = \frac{1 + e^{\frac{E_v(\vec{k}) - E_{fh}}{KT}} - 1}{e^{\frac{E_c(\vec{k}) - E_{fe}}{KT}} - e^{\frac{E_v(\vec{k}) - E_{fh}}{KT}}} \qquad \left\{ \begin{array}{l} \text{for:} \\ E_c(\vec{k}) - E_v(\vec{k}) = \hbar\omega \end{array} \right.$$

$$= \frac{1}{e^{\frac{E_c(\vec{k}) - E_v(\vec{k}) - (E_{fe} - E_{fh})}{KT}} - 1}$$

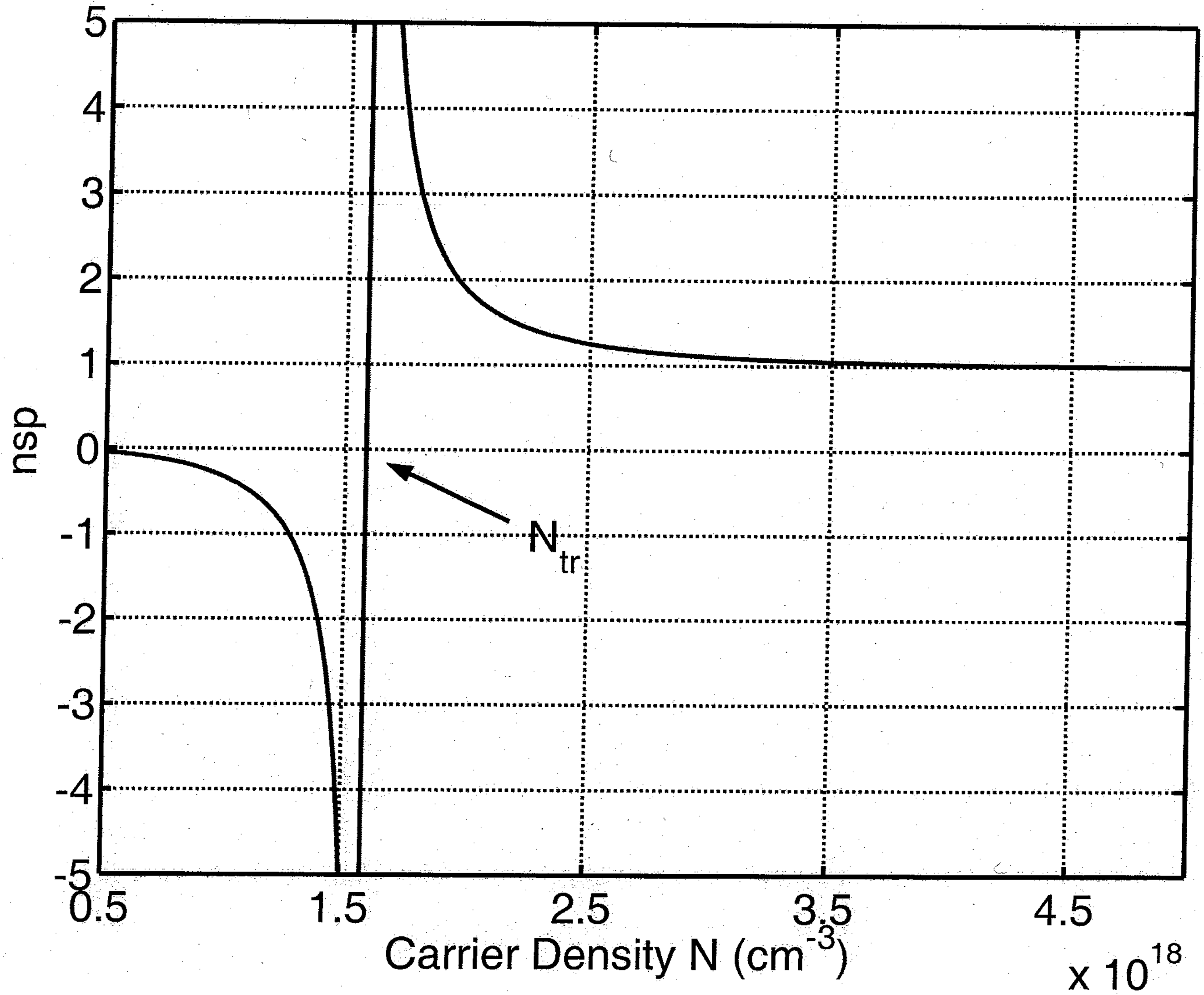
$$= \frac{1}{e^{\frac{\hbar\omega - eV}{KT}} - 1}$$

Therefore, this extra factor appears with $\alpha(\omega)$ in the integral for R_{Tsp}.

d) See the attached plot.



x 10^27
Actual
Fit
A=1e8 s^-1
B=1.2e-10 cm^3/s
Rspe (cm^-3 sec^-1)
Carrier Density N (cm^-3)
x 10^18
0
1
2
3
4
5
0.5
1
1.5
2
2.5
3
3.5
4



5
4
3
2
1
0
-1
-2
-3
-4
-5
nsp
N
tr
0.5
1.5
2.5
3.5
4.5
Carrier Density N (cm-3)
x 10
18

4.2

a) $\hat{p} = m_0 \dfrac{d\hat{x}}{dt}$

Heisenberg Equation:

$$i\hbar \frac{d\hat{x}}{dt} = [\hat{x}, \hat{H}_0] = \hat{x}\hat{H}_0 - \hat{H}_0\hat{x}$$

$$\Rightarrow \quad \hat{p} = -\frac{i}{\hbar} m_0 [\hat{x}\hat{H}_0 - \hat{H}_0\hat{x}]$$

$$\Rightarrow \quad \langle\psi_m|\hat{p}|\psi_n\rangle = -\frac{i}{\hbar} m_0 \{\langle\psi_m|\hat{x}|\psi_n\rangle E_n - \langle\psi_m|\hat{x}|\psi_n\rangle E_m\}$$

$$\Rightarrow \quad \langle\psi_m|\hat{x}|\psi_n\rangle(E_n - E_m) = \frac{i\hbar}{m_0}\langle\psi_m|\hat{p}|\psi_n\rangle$$

b) $[\hat{x}, [\hat{x}, \hat{H}_0]] = \hat{x}[\hat{x}, \hat{H}_0] - [\hat{x}, \hat{H}_0]\hat{x}$

$$\Rightarrow \quad \langle\psi_m|[\hat{x}, [\hat{x}, \hat{H}_0]]|\psi_m\rangle = \sum_p \langle\psi_m|\hat{x}|\psi_p\rangle\langle\psi_p|[\hat{x}, \hat{H}_0]|\psi_m\rangle - \langle\psi_m|[\hat{x}, \hat{H}_0]|\psi_p\rangle\langle\psi_p|\hat{x}|\psi_m\rangle$$

$$= \sum_p \langle\psi_m|\hat{x}|\psi_p\rangle\langle\psi_p|\hat{x}|\psi_m\rangle(E_m - E_p) - \langle\psi_m|\hat{x}|\psi_p\rangle\langle\psi_p|\hat{x}|\psi_m\rangle(E_p - E_m)$$

$$= 2\sum_p |\langle\psi_m|\hat{x}|\psi_p\rangle|^2 (E_m - E_p).$$

But $[\hat{x}, \hat{H}_0] = \dfrac{i\hbar\hat{p}}{m_0}$

and $[\hat{x}, [\hat{x}, \hat{H}_0]] = -\dfrac{\hbar^2}{m_0}$

$$\Rightarrow \quad \langle\psi_m|[\hat{x}, [\hat{x}, \hat{H}_0]]|\psi_m\rangle = -\frac{\hbar^2}{m_0}$$

$$\Rightarrow \quad 2\sum_p |\langle\psi_p|e\hat{x}|\psi_m\rangle|^2 (E_p - E_m) = \frac{e^2\hbar^2}{m_0}$$

$\Rightarrow$ The momentum matrix element sum-rule follows from results of part (a).

Problem 4.3

a) See attached plot.

b) (i) The peak gain decreases at higher temperatures because for the same carrier density at higher tempartures the carrier distribution in energy (given by the Fermi-Dirac distribution $f(E_c(k)-E_{fe})$) is more spread out resulting in lower value for the difference $\left[f(E_c(k)-E_{fe}) - f(E_v(k)-E_{fh})\right]$ which appears in the gain formula.

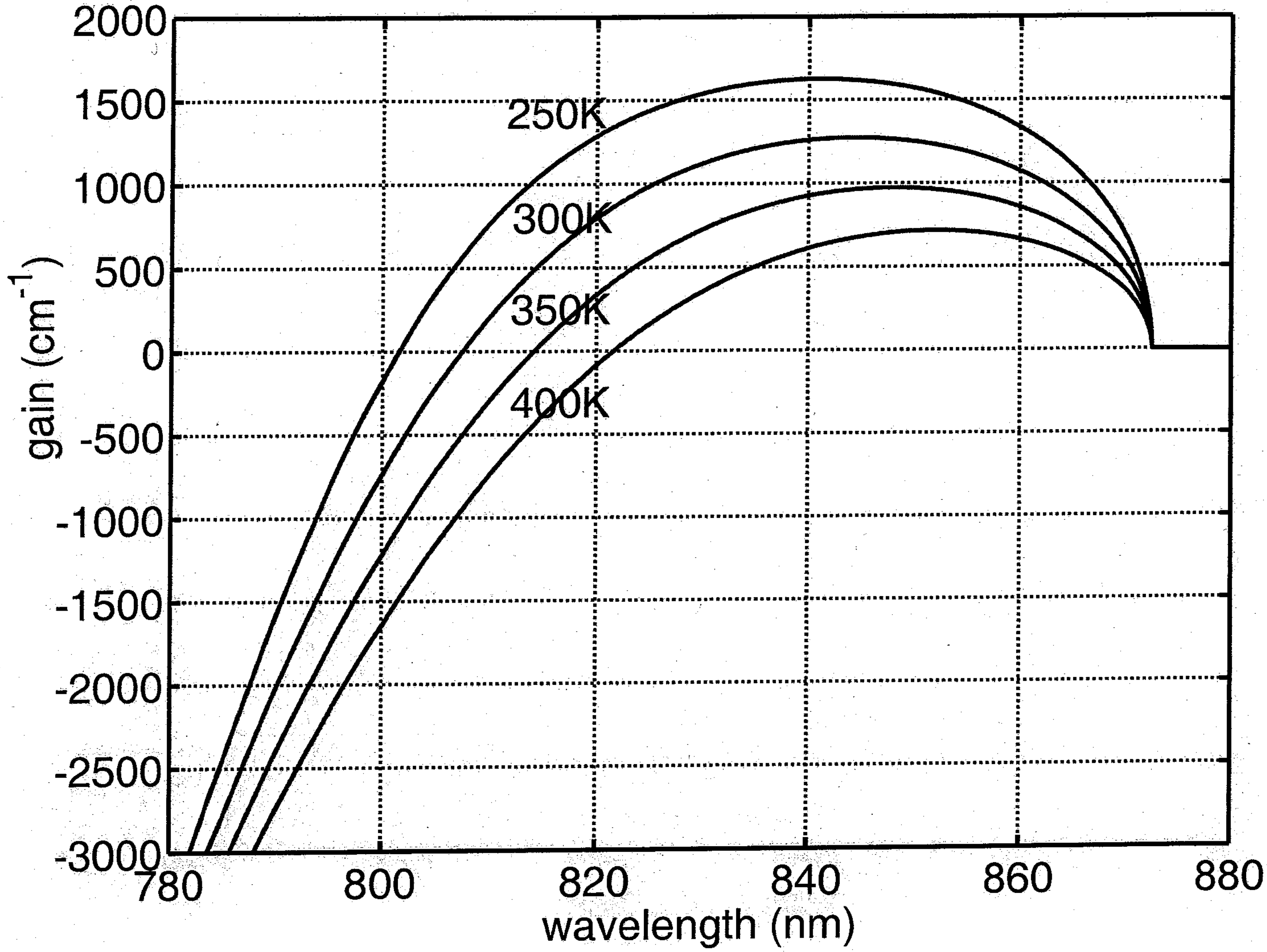
(ii) The gain bandwidth for any temperature can be determined as follows. The lowest frequency ω_L for which $g(\omega)>0$ is related to the bandgap i.e. $\omega_L = \frac{E_g}{\hbar}$.

For all $\omega > \omega_L$, $g(\omega)>0$ if $\left[f(E_c(k)-E_{fe}) - f(E_v(k)-E_{fh})\right] > 0$.

which, as shown in an earlier handout, implies $E_{fe}-E_{fh} > E_c(k)-E_v(k)$

But optical transitions can only happen if $E_c(k) - E_v(k) = \hbar\omega$

$\Rightarrow$ $g(\omega)>0$ if $E_{fe}-E_{fh} > \hbar\omega$. So given a carrier density N, the highest frequency ω_H for which $g(\omega)>0$ equals the Fermi level splitting, i.e $\omega_H = E_{fe}-E_{fh}$. As temperature is increased then for a fixed carrier density the Fermi level splitting $(E_{fe}-E_{fh})$ decreases (you can confirm this in your


2000
1500
1000
500
0
-500
-1000
-1500
-2000
-2500
-3000
gain (cm^{-1})
250K
300K
350K
400K
780
800
820
840
860
880
wavelength (nm)

calculations). So ω_H decreases (ω_L stays the same since the band gap E_g is not assumed to be a function of temperature), and consequently the gain bandwidth decreases.