Department of Electrical and Computer Engineering, Cornell University

ECE 5330: Semiconductor Optoelectronics

Fall 2014

Homework 4 **Due on Oct. 02, 2014**

Suggested Readings:

i) Lecture Notes
iii) Review electrodynamics and dielectric optical slab waveguides from your favorite book for the coming weeks.

Table of Parameter Values of III-V Semiconductors:

Parameters at 300K	GaAs	AlAs	InAs	InP	GaP
Eg (Γ-point) (eV)	1.424	3.03	0.354	1.344	2.78
m_e^* / m_o	0.067	0.15	0.023	0.077	0.25
m_{hh}^* / m_o	0.50	0.79	0.40	0.6	0.67
m_{lh}^* / m_o	0.087	0.15	0.026	0.12	0.17
Relative dielectric constant	13.0	10.0	15.0	12.5	11.0
Ep (eV)	25.7	21.1	22.2	20.7	22.2

Problem 4.1: (Spontaneous emission rate calculation)

In this problem you will calculate the recombination rate R_{Tsp} (units: number of transitions per cm^3 per sec) due to spontaneous emission of photons of all frequencies for bulk GaAs as a function of the carrier density. You will assume, as in the previous homework set, that a certain number of electrons have been taken from the valence band and transferred to the conduction band so that the density N of electrons and holes in the material is the same (i.e. $N = n = p$).

a) Calculate and plot the recombination rate R_{Tsp} in bulk GaAs as a function of the carrier density N for values of N between $0.5x10^{18}$ / cm^3 and $5x10^{18}$ / cm^3. Include heavy hole to conduction band transitions as well as light hole to conduction band transitions when calculating R_{Tsp}. Since the carrier densities will be high you will need to use exact Fermi-Dirac statistics. The calculation algorithm will be:

i) Choose a value for N.
ii) Calculate the position of electron Fermi level and the hole Fermi level.
iii) Calculate the rate R_{Tsp}. You will need to evaluate a one dimensional integral over frequency numerically.
iv) Go to (i).

b) Try to fit the plot obtained in part (a) with the functional form shown below:

$$R_{Tsp} = A\,N + B\,N^2$$

and find out the values of the constants A and B. The constant A has no particular name but the constant B is called the bimolecular recombination coefficient.

c) As in the lecture notes, the total spontaneous emission rate can be written in two different ways,

$$R_{Tsp} = \int_0^\infty R_{sp}(\omega)\,V_p\,g_p(\omega)\,d\omega$$

and, assuming $E_{fe} - E_{fh} = qV$,

$$R_{Tsp} = \int_0^\infty v_g\ \alpha(\omega)\frac{g_p(\omega)}{e^{(\hbar\omega - qV)/KT} - 1}\,d\omega$$

Derive the latter from the former.

d) There is yet a third way of writing the total spontaneous emission rate which will be very useful later in the course,

$$R_{Tsp} = \int_0^\infty \left[v_g\ g(\omega)\,n_{sp}(\omega)\right] g_p(\omega)\,d\omega$$

$n_{sp}(\omega)$ is called the spontaneous emission factor and equals,

$$n_{sp}(\omega) = \frac{1}{1 - e^{(\hbar\omega - qV)/KT}}$$

Calculate and plot the spontaneous emission factor n_{sp} at a wavelength of 860 nm for bulk GaAs for values of the carrier density N between $0.5x10^{18}\ /\mathrm{cm}^3$ and $5x10^{18}\ /\mathrm{cm}^3$. Recall from the previous homework that the transparency carrier density N_{tr} in GaAs is around $1.6x10^{18}\ /\mathrm{cm}^3$, and at this carrier density you should expect to see the value of n_{sp} blow up to infinity (why?).

Problem 4.2: (Matrix elements and sum-rules for optical transitions)

In this problem you will establish matrix elements and certain sum-rules for optical transitions.

a) Consider a 1-D quantum system with Hamiltonian $\hat{H}_o$ with eigenstates $|\psi_n\rangle$ with energies E_n. The Hamiltonian, without loosing generality, can be written as:

$$\hat{H}_o = \frac{\hat{p}^2}{2m_o} + V(\hat{x})$$

Show that the momentum matrix elements between any two states are related to the dipole matrix elements by the relation:

$$\langle\psi_m|\,e\,\hat{x}\,|\psi_n\rangle\,(E_n - E_m) = i\,e\,\hbar\frac{\langle\psi_m|\,\hat{p}\,|\psi_n\rangle}{m_o}$$

b) Starting from the commutation relation:

$$\left[\hat{x},\left[\hat{x},\hat{H}_o\right]\right] = ?$$

Prove the famous f-sum rule (also called the Thomas-Reiche-Kuhn sum-rule) for the dipole matrix elements:

$$\sum_m \left|\langle \psi_m \,|\, e\,\hat{x}\,|\psi_n\rangle\right|^2 \left(E_m - E_n\right) = \frac{e^2\hbar^2}{2m_o}$$

And also for the momentum matrix elements:

$$\sum_m \frac{\left|\langle \psi_m \,|\, \hat{p}\,|\psi_n\rangle\right|^2}{\left(E_m - E_n\right)} = \frac{m_o}{2}$$

Hint: remember the completeness relation for the eigenstates of a Hermitian operator:

$$\sum_p \left|\psi_p\right\rangle\left\langle\psi_p\right| = \hat{1} = \text{identity operator}$$

These sum rules are important because they put a limit on the maximum values of the momentum matrix elements. For example, if somebody tells you that they have discovered a "super" material that has gigantic momentum matrix elements, you can easily verify the claim and see if the alleged results exceed the limits imposed by the f-sum rule.

Problem 4.3: (Gain vs temperature)

The optical gain in semiconductors is a strong function of temperature and the performance of semiconductor optical devices, such as lasers and optical amplifiers, degrade rapidly with increasing temperature. In this problem you will calculate the gain/loss of bulk GaAs for different temperatures. You will (as before) assume that a certain number of electrons have been taken from the valence band and transferred to the conduction band so that the density N of electrons and holes in the material is the same (i.e. $N = n = p$).

a) Calculate and plot the material gain g (in units cm^{-1}) of bulk GaAs as a function of the wavelength λ of photons (in free space) for 4 different values of the temperature: T=250K, 300K, 350K, 400K. Assume the carrier density N equals $4x10^{18} / \text{cm}^3$ in each case. For wavelengths, choose the range from 750 nm to 900 nm. So your final graph should have a total of 4 curves. Include heavy hole to conduction band transitions as well as light hole to conduction band transitions when calculating gain. Assume no transition broadening (so you can use your old code from a previous problem).

b) You should see two main features in your result for part (a):

i) The peak gain decreases with increase in temperature
ii) The gain bandwidth (i.e. the wavelength range for which gain is greater than zero) shrinks with increase in temperature.

Explain physically why (i) and (ii) happen.