ECE533 Homework #3 Solutions
(Farhan Rana)

3.1

a) $N_p = \frac{I_o}{V_g \hbar\omega}$ b) $G_e = G_h = \frac{\alpha(\omega) I_o}{\hbar\omega} = V_g \alpha(\omega) N_p$

c) $$\frac{\partial n}{\partial t} - \frac{\vec{\nabla}\cdot \vec{J}_e}{q} = G_e - \frac{(n-n_o)}{\tau} \quad \& \quad \frac{\partial p}{\partial t} + \frac{\vec{\nabla}\cdot\vec{J}_h}{q} = G_h - \frac{(p-p_o)}{\tau}$$

where $\vec{J}_e = q\mu_e n \vec{E} + q D_e \vec{\nabla} n$ & $J_h = q\mu_h p \vec{E} - q D_h \vec{\nabla} p$

d) For minority carriers $\vec{J}_e \approx q D_e \vec{\nabla} n$ (ignore drift part)

$$\Rightarrow \frac{\partial n}{\partial t} - D_e \nabla^2 n = G_e - \frac{(n-n_o)}{\tau}$$

Steady state $\Rightarrow$ $-D_e \nabla^2 n = G_e - \frac{(n-n_o)}{\tau} \Rightarrow -D_e \nabla^2(\Delta n) = G_e - \frac{\Delta n}{\tau}$

A position independent $\Delta n = G_e \tau$ satisfies all the boundary conditions. $\Rightarrow$ $\Delta p = \Delta n = G_e \tau$.

e) $\Delta n(t) = \Delta p(t) = G_e \tau \left(1 - e^{-t/\tau}\right)$

f) $\Delta n(t) = \Delta p(t) = G_e \tau\, e^{-\frac{(t-T)}{\tau}}$

g) The recombination lifetime τ characterizes the response of the system to light.

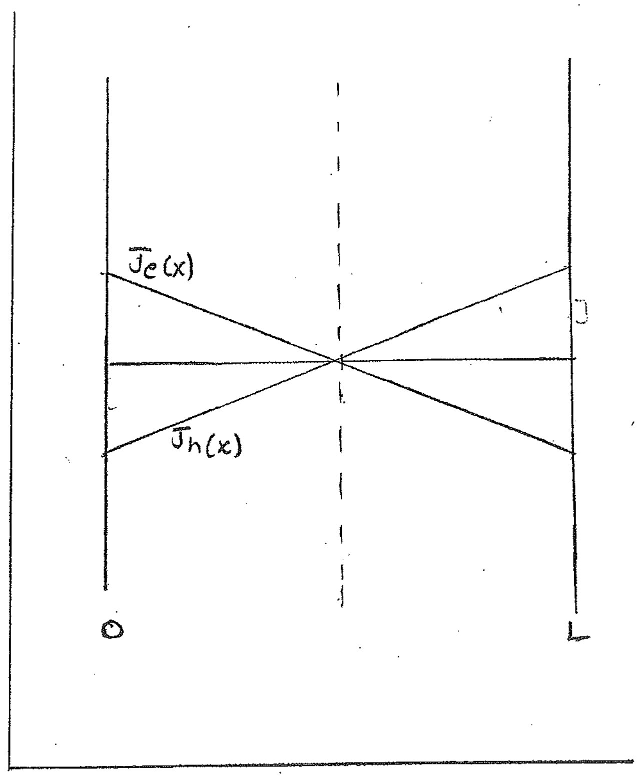
3.2

a) Boundary Condition: $\Delta p(x=0) = \Delta n(x=0) = 0 = \Delta n(x=L) = \Delta p(x=L)$
Assume sol. only depends on x (not y, z, or time t)

$$\frac{\partial^2}{\partial x^2}\Delta n - \frac{\Delta n}{L_e^2} = -\frac{G_e}{D_e} \xrightarrow{L_e \gg L} \frac{\partial^2 \Delta n}{\partial x^2} = -\frac{G_e}{D_e}$$

Solution: $\Delta n(x) = \frac{G_e}{2D_e} x(L-x) = \Delta p(x)$.

b) $J_e(x) = q D_e \frac{\partial \Delta n}{\partial x} = q \frac{G_e}{2}(L-2x)$



c) $J_T = 0 = J_e(x) + J_h(x)$

d) $J_h(x) = J_T - J_e(x) = -J_e(x)$

$= -q \frac{G_e}{2}(L-2x)$

e) $J_{h\,diff} = -q D_h \frac{\partial \Delta p}{\partial x}$

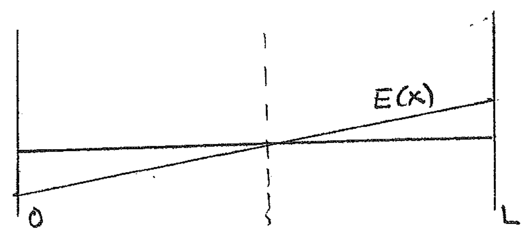
$= -q \left(\frac{D_h}{D_e}\right) \frac{G_e}{2}(L-2x)$

f) $J_{h\,drift} = J_h(x) - J_{h\,diff}$

$= \left\{ -\frac{q G_e}{2} + q \frac{D_h}{D_e}\frac{G_e}{2} \right\}(L-2x) = -q\left(1 - \frac{D_h}{D_e}\right)\frac{G_e}{2}(L-2x)$

g) $J_{h\,drift} = q \mu_h p(x) E(x)$

$p(x) = N_a + \Delta p(x) \approx N_a$



$\Rightarrow E(x) = \frac{J_{h\,drift}}{q \mu_h N_a} = -\left(1 - \frac{D_h}{D_e}\right)\frac{G_e}{2\mu_h N_a}(L-2x)$

The electric field tries to make the holes go where the electrons are to maintain quasineutrality. The electric field is needed since the hole diffusion constant D_h is almost always smaller than the electron diffusion constant D_e, and so the electrons diffuse faster than the holes.

f). The total current in the external circuit is zero. For every electron (flowing via diffusion) that reaches an ohmic contact, a hole (flowing via drif + diffusion) also reaches that ohmic contact. The net charge that flows into an ohmic contact is therefore zero. Another way to see this is as follows (if you are not mathematically challenged):

$$J_T = q\left[u_e n(x) + u_n p(x)\right] E(x) + q D_e \frac{\partial n}{\partial x} - q D_n \frac{\partial p}{\partial x}$$

$$= q\left[u_e n(x) + u_n p(x)\right] E(x) + q(D_e - D_h) \frac{\partial \Delta n(x)}{\partial x}.$$

$$\Rightarrow J_T \frac{1}{(q u_e n(x) + q u_n p(x))} = E(x) + \frac{(D_e - D_h) \frac{\partial \Delta n(x)}{\partial x}}{\left[q u_e n_0 + q u_n p_0 + q(u_e + u_h)\Delta n(x)\right]}$$

Integrate from 0 to L:

$$J_T \int_0^L \frac{dx}{(q u_e n(x) + q u_n p(x))} = \int_0^L E(x)\,dx + \int_0^L \frac{(D_e - D_n)\left(\frac{d\,\Delta n(x)}{dx}\right) dx}{\left[q u_e n_0 + q u_n p_0 + q(u_e + u_h)\Delta n(x)\right]}$$

$$= 0 + 0.$$

$$\Rightarrow J_T = 0.$$

3.3

$$g(\omega) = g_{hh}(\omega) + g_{eh}(\omega)$$

$$g_{hh}(\omega) = \frac{\pi e^2}{\varepsilon_0 m_0^2 n \omega c} \cdot \frac{m_0 E_p}{3} \cdot \int \frac{d^3k}{(2\pi)^3} \delta\left(E_c(k) - E_{hh}(k) - \hbar\omega\right) \left[f\left(E_c(k) - E_{fe}\right) - f\left(E_{hh}(k) - E_{fh}\right) \right]$$

$$E_c(k) - E_{hh}(k) = E_g + \frac{\hbar^2 k^2}{2m_{rhh}} \quad \left\{ \frac{1}{m_{rhh}} = \frac{1}{m_e} + \frac{1}{m_{hh}} \right\}$$

Let $E = \frac{\hbar^2 k^2}{2m_{rhh}}$ & $dE = \frac{\hbar^2 k\,dk}{m_{rhh}}$

$$\Rightarrow \begin{cases} E_c(k) - E_{hh}(k) = E_g + E \\ E_c(k) = E_c + \frac{\hbar^2 k^2}{2m_e} = E_c + \frac{m_{rhh}}{m_e} E \\ E_{hh}(k) = E_v - \frac{\hbar^2 k^2}{2m_{hh}} = E_v - \frac{m_{rhh}}{m_{hh}} E \end{cases}$$

Substituting the above values

$$g_{hh}(\omega) = \frac{\pi e^2}{\varepsilon_0 m_0^2 n \omega c} \cdot \frac{m_0 E_p}{3} \cdot \frac{1}{\sqrt{2}\pi^2} \left(\frac{m_{rhh}}{\hbar^2} \right)^{3/2} \sqrt{\hbar\omega - E_g} \left[f\left(E_c + \frac{m_{rhh}}{m_e}(\hbar\omega - E_g) - E_{fe}\right) - f\left(E_v - \frac{m_{rhh}}{m_{hh}}(\hbar\omega - E_g) - E_{fh}\right) \right]$$

Similarly

$$g_{eh}(\omega) = \frac{\pi e^2}{\varepsilon_0 m_0^2 n \omega c} \cdot \frac{m_0 E_p}{3} \cdot \frac{1}{\sqrt{2}\pi^2} \left(\frac{m_{rlh}}{\hbar^2} \right)^{3/2} \sqrt{\hbar\omega - E_g} \left[f\left(E_c + \frac{m_{rlh}}{m_e}(\hbar\omega - E_g) - E_{fe}\right) - f\left(E_v - \frac{m_{rlh}}{m_{lh}}(\hbar\omega - E_g) - E_{fh}\right) \right]$$

where $\frac{1}{m_{rlh}} = \frac{1}{m_e} + \frac{1}{m_{lh}}$

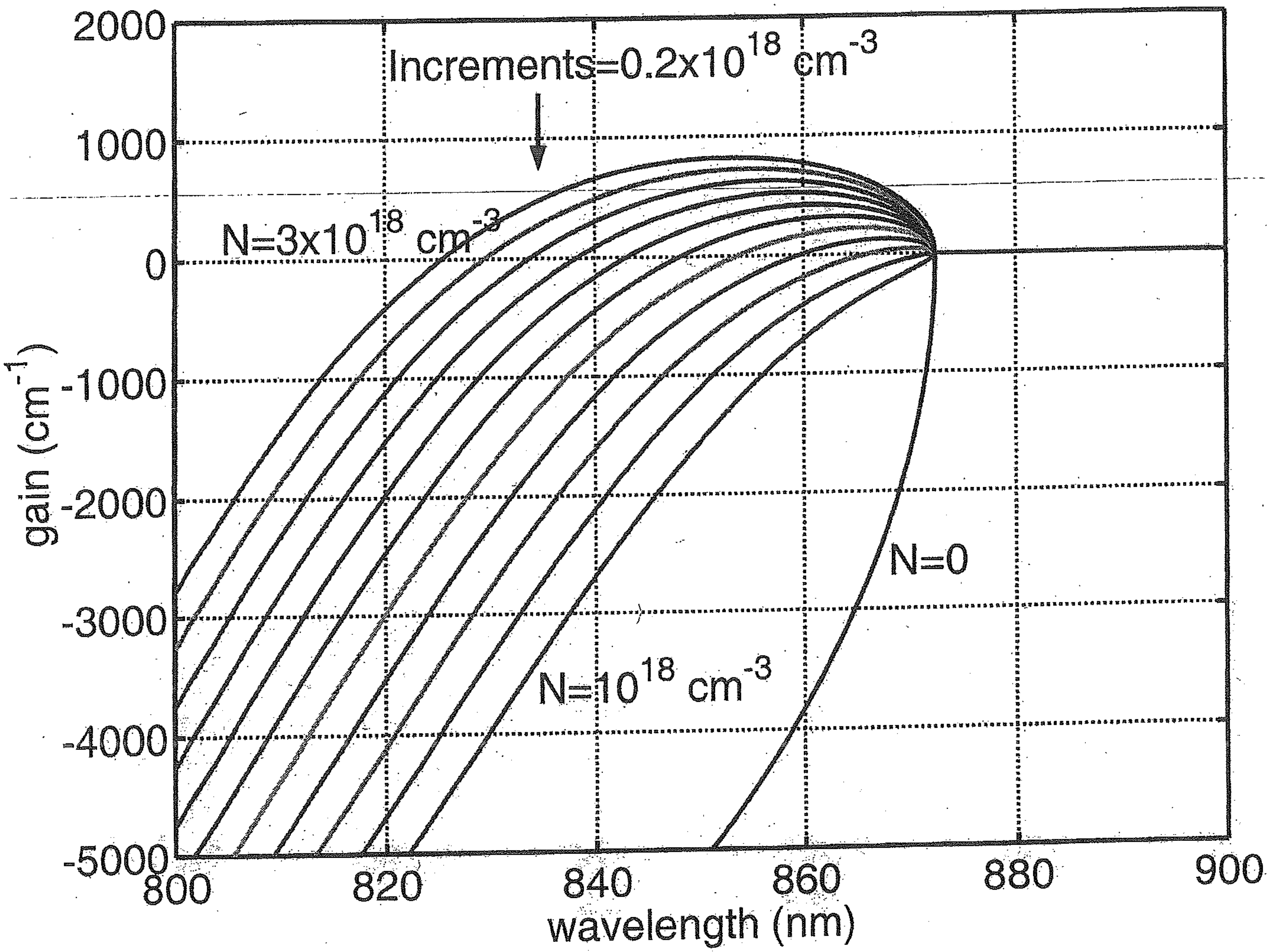
The electron fermi level E_{fe} is calculated from:

$$n = N = N_c \frac{2}{\sqrt{\pi}} F_{1/2}\left(\frac{E_{fe}-E_c}{kT}\right) \qquad N_c = 2\left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2}$$

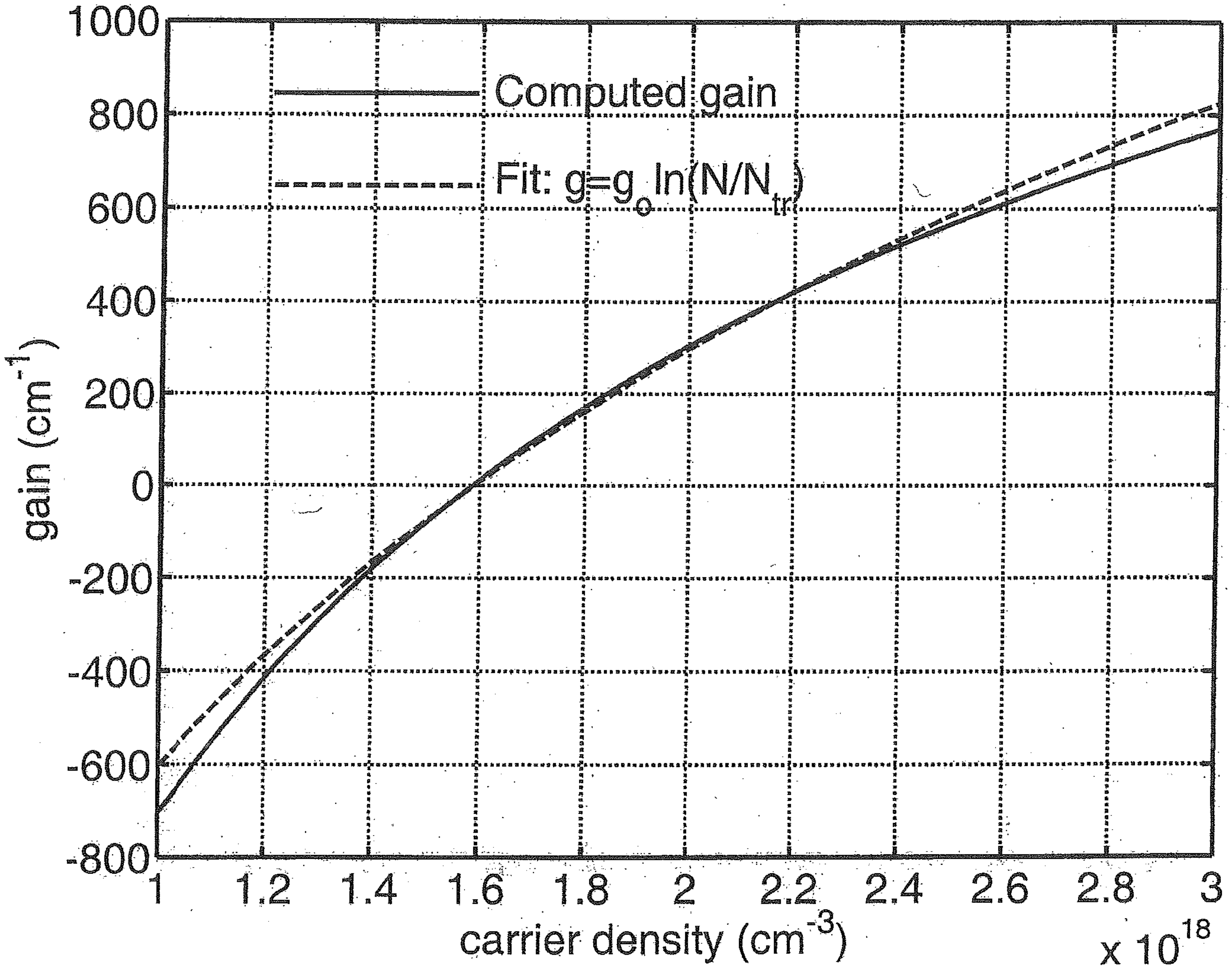
The hole fermi level E_{fh} is calculated from:

$$p = N = N_V \frac{2}{\sqrt{\pi}} F_{1/2}\left(\frac{E_V - E_{fh}}{kT}\right) \quad \text{where} \begin{cases} N_V = 2\left(\dfrac{m_{dh} kT}{2\pi\hbar^2}\right)^{3/2} \\ m_{dh} = \left(m_{hh}^{3/2} + m_{eh}^{3/2}\right)^{2/3} \end{cases}$$

See next page for the results for both parts a and b.



Increments=0.2x10^18 cm^-3
N=3x10^18 cm^-3
N=0
N=10^18 cm^-3
2000
1000
0
-1000
-2000
-3000
-4000
-5000
800
820
840
860
880
900
gain (cm^-1)
wavelength (nm)



For the fit: $g = g_0 \ln\left(\frac{N}{N_{tr}}\right)$ I chose $g_0 = 1300\ cm^{-1}$

$N_{tr} = 1.59 \times 10^{18}\ cm^{-3}$

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a) $I = AJ = qA\,(n_o u_e + p_o u_n)\,E(x)$

Integrate from 0 to L

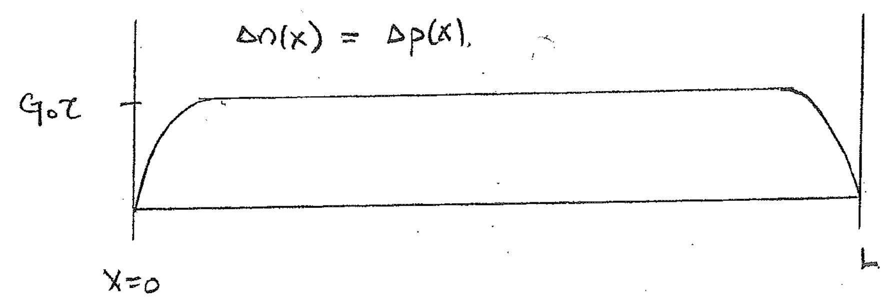
$$IL = qA\,(n_o u_e + p_o u_n)\int_0^L E(x)\,dx = qA\,(n_o u_e + p_o u_n)\,V$$

$I = GV = \frac{V}{R}$ where $G = \frac{qA}{L}(n_o u_e + p_o u_n) = \frac{1}{R}$

b) Solve: $$\frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{L_e^2} = -\frac{G_o}{D_e}$$

Sol. : $$\Delta n(x) = G_o\tau\left[1 - \frac{\cosh\left(\frac{x-\frac{L}{2}}{L_e}\right)}{\cosh\left(\frac{L}{2L_e}\right)}\right] = \Delta p(x)$$

For locations few diffusion lengths (L_e) away from the two ohmic contacts $\Delta n(x) \approx G_o\tau = \Delta p(x) =$ Constant.



The actual electron + hole concentrations are
$$\begin{cases} n(x) = n_o + \Delta n(x) \\ p(x) = p_o + \Delta p(x) \\ p_o \approx N_a, \quad n_o \approx \frac{n_i^2}{N_a} \end{cases}$$

c) Cannot ignore diffusion now since we have non-uniform carrier distribution.

$$I = AJ = qA\left[(n_o + \Delta n(x))\,u_e + (p_o + \Delta p(x))\,u_n\right]E(x) + qA\,D_e\frac{\partial \Delta n}{\partial x} - qA\,D_h\frac{\partial \Delta p}{\partial x}.$$

$$I = qA\left[(n_0+\Delta n(x))\mu_e + (P_0+\Delta n(x))\mu_n\right]E(x) + qA(D_e - D_n)\frac{\partial \Delta n}{\partial x}$$

$$\int_0^L \frac{I\,dx}{qA\left[(n_0+\Delta n(x))\mu_e + (P_0+\Delta n(x))\mu_n\right]} = \int_0^L E(x)\,dx + \int_0^L dx \frac{qA(D_e - D_n)\cdot\frac{\partial \Delta n}{\partial x}}{qA\left[(n_0+\Delta n(x))\mu_e + (P_0+\Delta n(x))\mu_n\right]}$$

$$I\int_0^L \frac{dx}{qA\left[(n_0+\Delta n(x))\mu_e + (P_0+\Delta n(x))\mu_n\right]} = V + 0$$

{ Diffusive terms give zero }

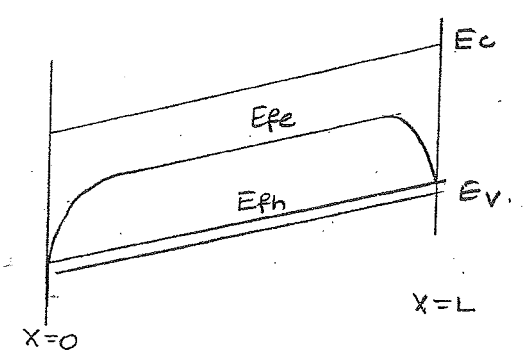
Since $L_e \lll L$, we can assume $\Delta n(x) \approx G_0\tau \approx \Delta p(x)$ in the integral on the LHS above

$$I = q\frac{A}{L}\left[(n_0+G_0\tau)\mu_e + (P_0+G_0\tau)\mu_n\right]V$$

$$I = GV + \Delta I \qquad \left\{ \Delta I = q\frac{A}{L}G_0\tau(\mu_e+\mu_n)V \right.$$

d). $$g_p = \frac{\Delta I/q}{G_0 A L} = \frac{\tau(\mu_e+\mu_n)\frac{V}{L}}{L} = \frac{\tau}{\tau_t}$$

e)



{ Fermi levels split due to electron-hole generation in the device.

f) We are not violating anything. The device is not a photovoltaic detector. Its operation is based on the simple fact that the resistance of the device changes when light shines on it. Light generates extra carriers that decrease the resistance resulting in an increase in the current through the device.