Department of Electrical and Computer Engineering, Cornell University

ECE 5330: Semiconductor Optoelectronics

Fall 2014

Homework 3 **Due on Sep. 25, 2014**

Suggested Readings:

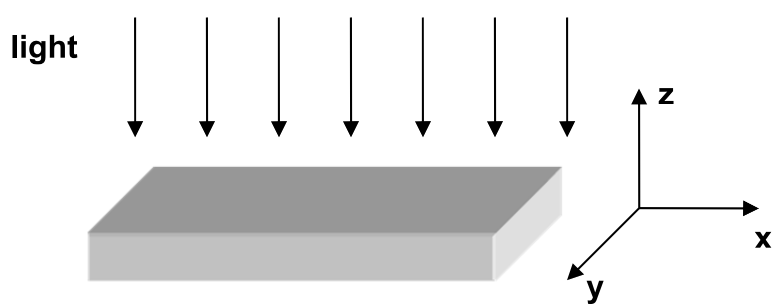
i) Lecture Notes
ii) Review minority carrier transport in semiconductor devices.

Table of Parameter Values of III-V Semiconductors:

Parameters at 300K	GaAs	AlAs	InAs	InP	GaP
Lattice constant (A)	5.6533	5.6600	6.0584	5.8688	5.4505
Eg(Γ-point) (eV)	1.424	3.03	0.354	1.344	2.78
m_e^*/m_o	0.067	0.15	0.023	0.077	0.25
m_{hh}^*/m_o	0.50	0.79	0.40	0.6	0.67
m_{lh}^*/m_o	0.087	0.15	0.026	0.12	0.17
Relative dielectric constant	13.0	10.0	15.0	12.5	11.0
Δ (eV)	0.34	0.28	0.38	0.11	0.08
Ep (eV)	25.7	21.1	22.2	20.7	22.2
C_{11} (dynes/cm^2)	11.88	12.5	8.33	10.11	14.05
C_{12} (dynes/cm^2)	5.37	5.34	4.526	5.61	6.203

Problem 3.1 (Shockley equations with light – part 1 – time transients)

Consider a p-doped bar of a direct-gap semiconductor as shown below. The p-doping of the semiconductor results in an equilibrium hole concentration of p_o, and an electron concentration of n_o.



Light with photon energy $\hbar\omega$ and uniform intensity I_o (units: energy per cm^2 per sec) is incident on the semiconductor bar as shown above. Ignore any reflections of light on the surface of the semiconductor or any decay of the light inside the semiconductor. The absorption coefficient of the semiconductor material is $\alpha(\omega)$ (units: per cm), and is known, and the group velocity of light in the semiconductor material is v_g.

a) What is the photon density N_p (units: per cm^3) inside the semiconductor material in terms of I_o?

b) What are the generation rates G_e and G_h (units: # per cm^3 per sec) for electrons and holes, respectively, inside the semiconductor material?

c) Assuming the excess minority carrier (i.e. electrons) recombination lifetime to be τ, and diffusion lengths to be D_e and D_h, respectively, write down the continuity equations for electrons and holes.

d) Find the excess hole and electron concentrations, Δp and Δn, in the sample in the steady state (i.e. the light has been switched on for a long long time). Your solution MUST NOT violate quasi-neutrality. There is no position dependence in this problem.

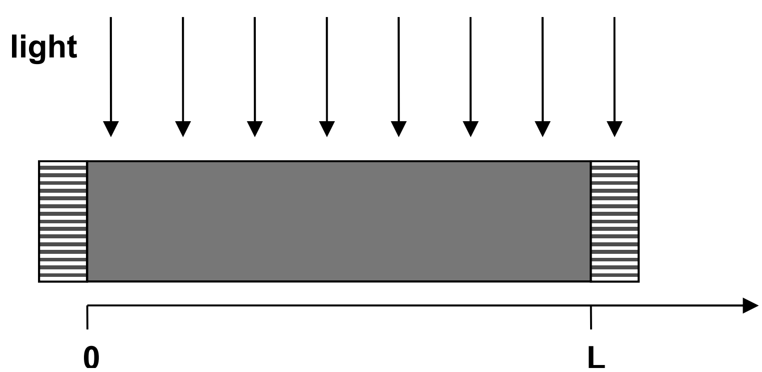
e) Suppose the light is switched on at time $t = 0$. Light was off for all times $t < 0$. Find the excess electron and hole concentrations, $\Delta p(t)$ and $\Delta n(t)$, for all time $t \geq 0$.

f) Suppose after being on for a long long time, the light is switched off at time $t = T$. Find the excess electron and hole concentrations, $\Delta p(t)$ and $\Delta n(t)$, for all time $t \geq T$.

g) Based upon the behavior you saw in parts (e) and (f) what time scale characterizes the temporal response of the material to light?

Problem 3.2 (Shockley equations with light – part 2 – ohmic contacts)

This problem will test your understanding of basic semiconductor device physics, quasi-neutrality, and minority carrier transport. Again consider a p-doped bar of a direct-gap semiconductor now with metal ohmic contacts on each end in the x-direction, as shown below.



Light with photon energy $\hbar\omega$ and uniform intensity I_o (units: energy per cm^2 per sec) is incident on the semiconductor as shown above. Ignore any reflections on the surface of the semiconductor or any decay of the light inside the semiconductor. The absorption coefficient of the material is $\alpha(\omega)$ (units: per cm) and the group velocity of light in the semiconductor material is v_g. The p-doping of the semiconductor results in an equilibrium hole concentration of p_o, and electron concentration of n_o. Assume steady state in all parts of this problem (i.e. the light has been switched on for a long long time).

a) Assuming that the electron and hole diffusion lengths are much much larger than the length L of the sample, find the excess hole and electron concentrations, $\Delta p(x)$ and $\Delta n(x)$, in the sample as a function of position in the steady state. You will need to use appropriate boundary conditions at the two ohmic contacts. Your solution MUST NOT violate quasi-neutrality.

b) Using your answer in part (a) find the electron current $J_e(x)$? And sketch it.

c) What is the total current, electron plus hole, $J_h(x)+J_e(x)$? Don't need to do any calculations to answer this.

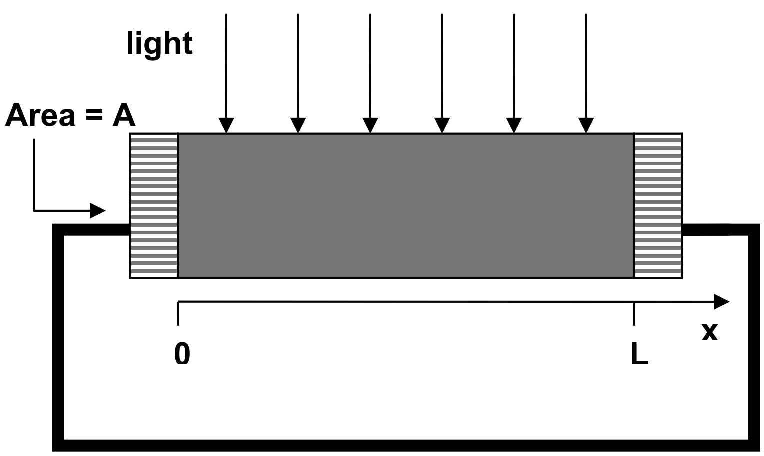
d) Find the hole current $J_h(x)$? Sketch it.

e) Find the diffusion component of the hole current?

f) Find the drift component of the hole current?

g) Using your answer in (f) find the electric field $E(x)$ in the sample. Sketch it.

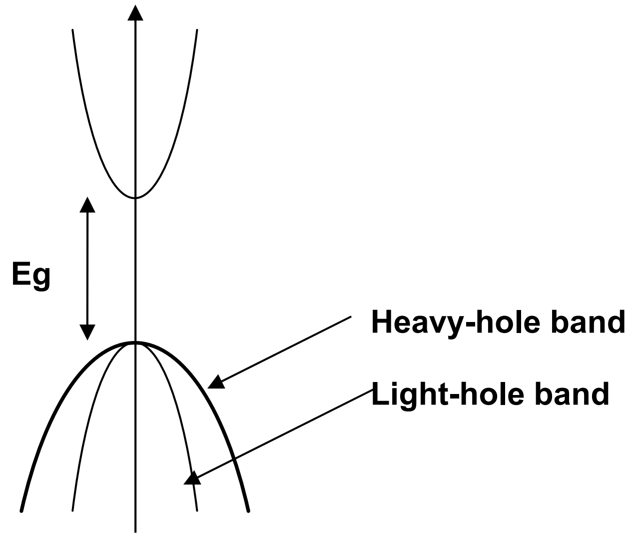
Now the two ohmic contacts are connected with an electric wire, as shown below.



f) What is the total current flowing in the external circuit and in what direction (sketch the direction)? Can this device be used as a solar cell?

Problem 3.3: (Optical gain/loss calculation)

In this problem you will calculate the gain/loss of bulk GaAs. You will assume that a certain number of electrons have been taken from the valence band and transferred to the conduction band so that the density N of electrons and holes in the material is the same (i.e. $N = n = p$). There are two valence bands: A heavy-hole (HH) band with effective mass m_{hh} and a light-hole (LH) band with effective mass m_{lh}, as shown below.



For bulk III-V material (and only for bulk), the momentum matrix element $|\hat{n} \cdot \vec{p}|^2$ for the conduction band and HH-valence band transitions, and also for the conduction band and LH-valence band transitions, are independent of the direction of the polarization of field and are given in terms of an energy parameter E_p as:

$$|\hat{n} \cdot \vec{p}|^2 = \frac{m_o E_p}{6}$$

The values of E_p are given in the table above.

a) Calculate and plot the material gain g **(in units cm^{-1})** of bulk GaAs as a function of the wavelength λ of photons (in free space) for different values of the carrier density N. For wavelength, choose the range from 800 nm to 900 nm. For carrier density, choose $N = 0$, and also ten equally spaced values from 10^{18} / cm^3 to $3x10^{18}$ / cm^3. So your final graph should have a total of 11 curves. Of course, in $N = 0$ case you will actually be calculating the material loss α of bulk GaAs. Include heavy hole to conduction band transitions as well as light hole to conduction band transitions when calculating gain. Since the carrier densities will be high you will need to use exact Fermi-Dirac statistics. The calculation algorithm will be:

i) Choose a value for N.
ii) Calculate the position of electron Fermi level and the hole Fermi level.
iii) Calculate the gain $g(\lambda)$ for all wavelengths. For each value of wavelength (or frequency) you will need to evaluate a one dimensional integral numerically. See the lecture notes.
iv) Go to (i).

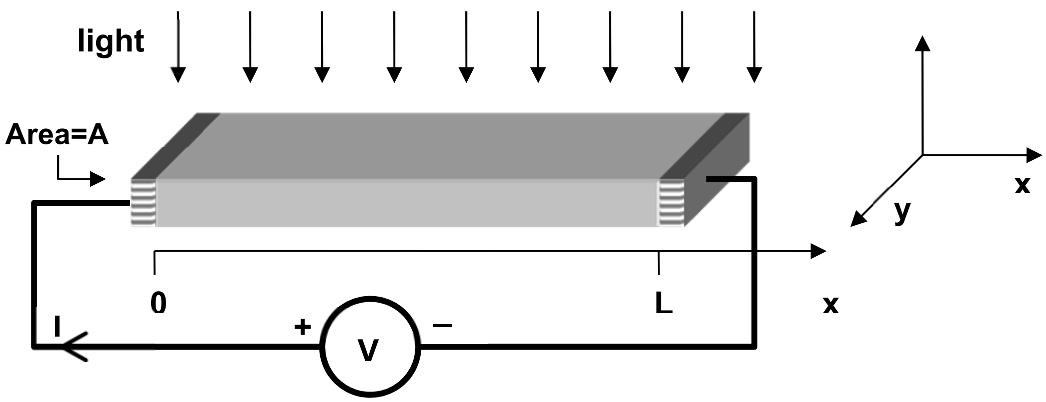
b) Now fix the wavelength at 860 nm, and plot the gain as a function of carrier density N. Choose values of carrier density between 10^{18} / cm^3 and $3x10^{18}$ / cm^3. Try to best fit the resulting plot with the functional form given below:

$$g = g_o \ln\left(\frac{N}{N_{tr}}\right)$$

and give the values of the quantities g_o and N_{tr} (with proper units).

Problem 3.5: (A photoconductor with recombination)

A photoconductor is a piece of semiconducting material with ohmic contacts on two ends (as shown in the figure below). It is the simplest device used to detect photons.



Assume the semiconductor is p-doped with doping N_a. The electron and hole recombination lifetime in the p-doped region is τ. The electron and hole diffusion constants are D_e and D_h, respectively. The intrinsic carrier concentration is n_i. Assume $L_e/L <<<< 1$ and $L_h/L <<<< 1$ (i.e. the sample is much much longer than the carrier diffusion lengths). Note that this does not mean that you ignore the recombination term in the calculations below. It only means that spatial variations in the carrier density will take place only within a few diffusion lengths of the two ohmic contacts.

Assume for part (a) that no light is present.

a) If a voltage V is applied across the device (as shown in the figure) find an expression for the current I flowing in the external circuit.

Now assume for the remaining parts that the light is switched on. The presence of light results in a uniform electron-hole pair generation rate of G_o (units: per cm^3 per sec) in the entire device. Assume steady state.

b) Calculate and sketch the electron and hole concentrations in the entire device as a function of position x (paying due regard to the conditions $L_e/L <<<< 1$ and $L_h/L <<<< 1$). Assume that the presence of voltage across the device does not effect the minority carrier distribution, and this is a pretty good assumption even for large voltages applied across the device provided $L_e/L <<<< 1$ and $L_h/L <<<< 1$.

c) For a voltage V applied across the device (as shown in the figure) find an expression for the current I flowing in the external circuit in the presence of light. Your final answer should contain no unevaluated integrals. An (almost) exact answer can be obtained in the limit $L_e/L <<<< 1$ and $L_h/L <<<< 1$ (if you paid close attention to your answer in part (b)).

Comparing answers to part (a) and part (c), you should see an increase ΔI in the current in the external circuit in the presence of light. The photoconductive gain g_p of a photoconductor is defined as the ratio of the **extra** light-induced charge flow rate $\Delta I/q$ in the external circuit to the total number of electron-hole pairs generated per second by photon absorption inside the device.

d) Show that the photoconductive gain can be written in the form:

$$g_p = \frac{\tau}{\tau_t}$$

where τ is the carrier recombination lifetime, and τ_t is the time taken by the carriers **flowing via drift** to cross the device (also called the drift transit time). In this device there are two different types of carriers (electrons and holes), and the expression for τ_t should come out to be:

$$\frac{1}{\tau_t} = \frac{1}{\tau_{te}} + \frac{1}{\tau_{th}} = \frac{\mu_e}{L}\left(\frac{V}{L}\right) + \frac{\mu_h}{L}\left(\frac{V}{L}\right)$$

where τ_{te} is the drift transit time for electrons and τ_{th} is the drift transit time for holes.

e) Sketch (and label) the complete band diagram of the device (with electron and hole Fermi levels) for $0 \le x \le L$ under light illumination and applied voltage V.

f) The carrier drift transit time τ_t can be made smaller than the carrier recombination time τ by applying a large voltage across the device (which will produce a large electric field). Under such circumstances the photoconductive gain will be greater than unity. Explain how is this possible i.e. how can the extra light-induced charge flow rate in the external circuit be greater than the total number of electron-hole pairs produced per second by light inside the device? Or are we violating something here and such a scenario is impossible?