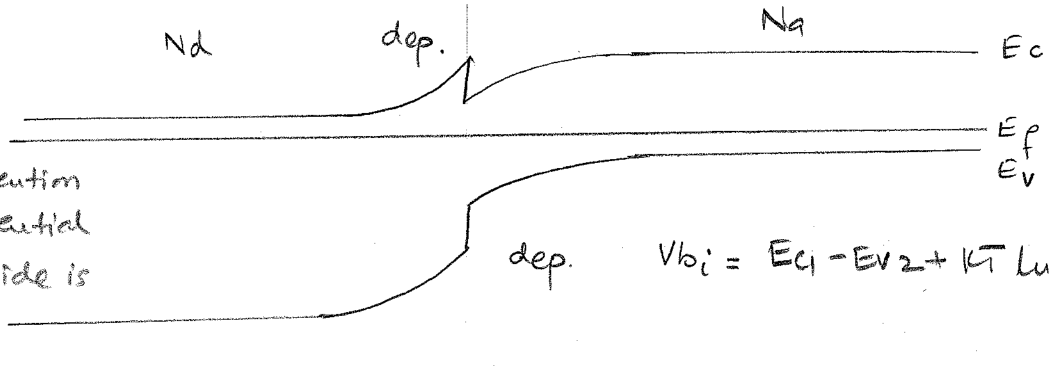
ECE 5330 Homework #2 Solutions (Farhan Rana)

2.1

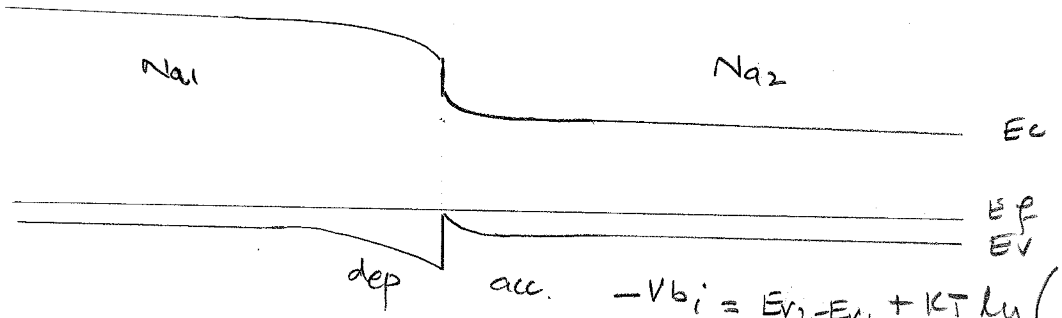
a)



By my convention $V_{bi} > 0$ if potential on the left side is higher.

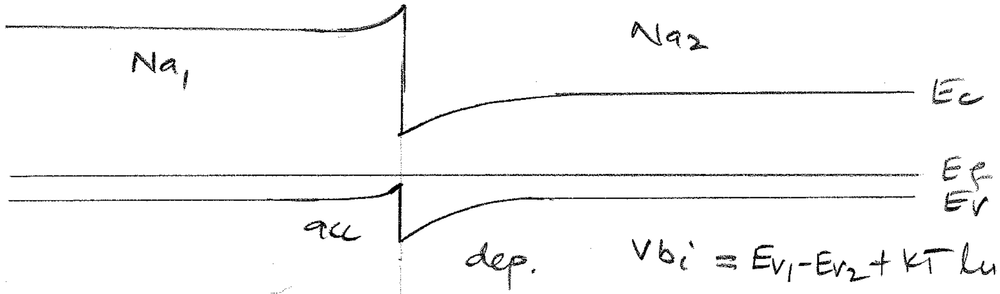
$$V_{bi} = E_{c1} - E_{v2} + KT \ln\left(\frac{N_d N_a}{N_{c1} N_{v2}}\right).$$

b)



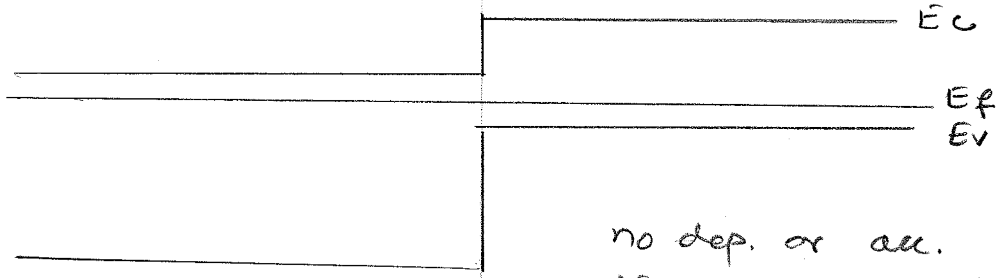
$$-V_{bi} = E_{v2} - E_{v1} + KT \ln\left(\frac{N_{a1}}{N_{a2}} \frac{N_{v2}}{N_{v1}}\right)$$

c)



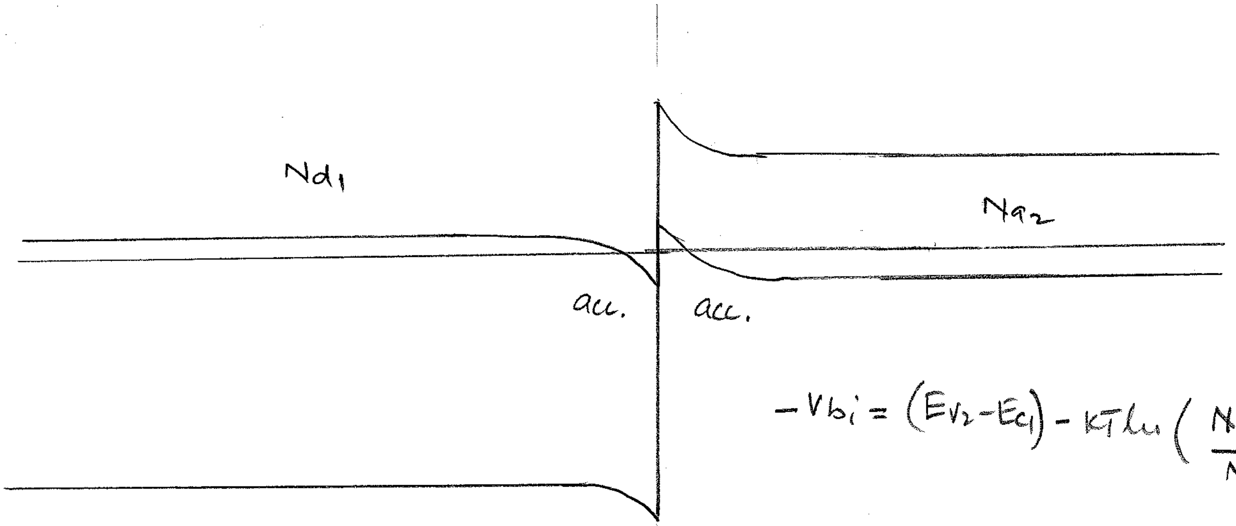
$$V_{bi} = E_{v1} - E_{v2} + KT \ln\left(\frac{N_{a2}}{N_{a1}} \frac{N_{v1}}{N_{v2}}\right)$$

d) Fermi levels already aligned.



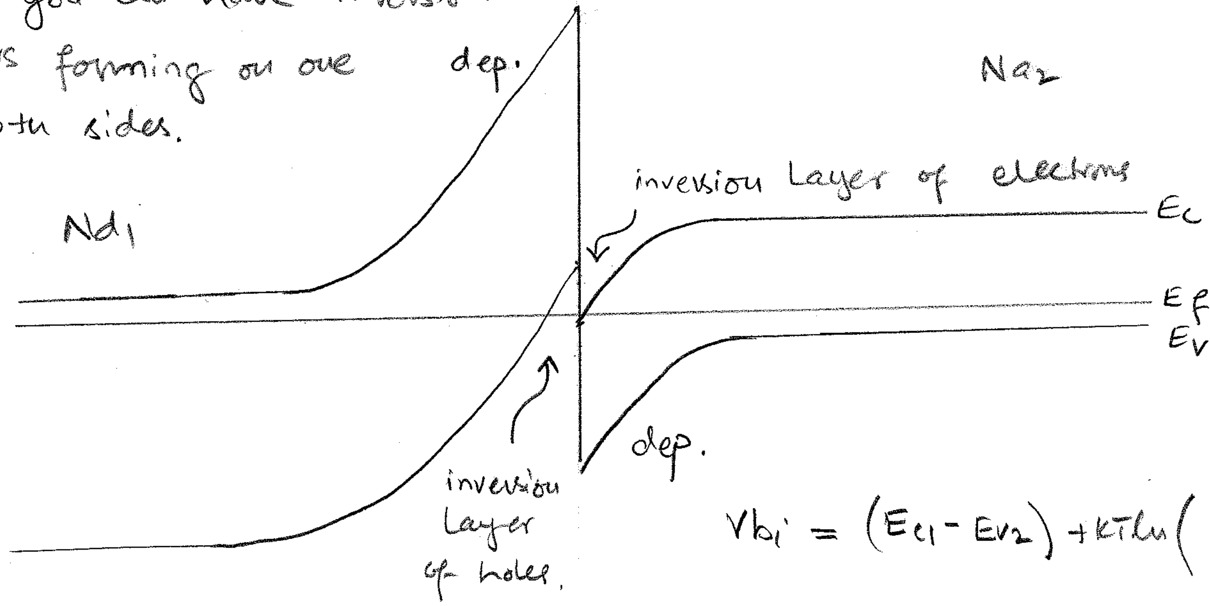
no dep. or acc. regions. $V_{bi} = 0$.

e)



$$-V_{bi} = (E_{v2} - E_{c1}) - kT\ln\left(\frac{N_{d1}N_{a2}}{N_{c1}N_{v2}}\right).$$

f) Here you can have inversion Layers forming on one or both sides.



$$V_{bi} = (E_{c1} - E_{v2}) + kT\ln\left(\frac{N_{d1}N_{a2}}{N_{c1}N_{v2}}\right).$$

2.2

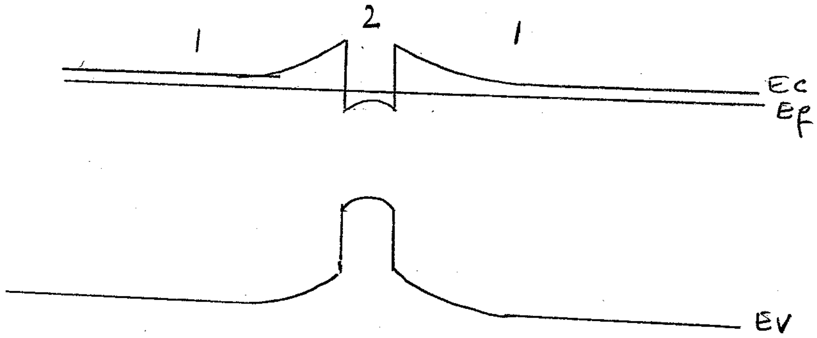
a) $\Delta E_C = 115$ meV $\quad \varepsilon[\text{AlGaAs}] = \varepsilon_1 = 12.58 \quad \varepsilon[\text{GaAs}] = \varepsilon_2 = 13.0$

$m_e^*[\text{AlGaAs}] = 0.0726 \quad m_e^*[\text{GaAs}] = 0.067$ ⇒ I can assume $m_e^*[\text{AlGaAs}] = .067$

b) Since $\frac{2m_e^* e\phi_B}{\hbar^2} < \left(\frac{\pi}{L}\right)^2$ $\{\phi_B = 115 \text{ meV}\}$ ⇒ only one confined level.

Solving: $k_z = 0.8695 \times \frac{2}{L}$ ⇒ $E_1 = \frac{\hbar^2 k_z^2}{2m_e^*} = 47.9$ meV

c)

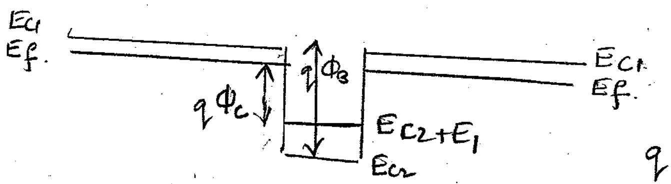


d) From symmetry $V_{bi} = \phi(\infty) - \phi(-\infty) = 0$.

e) Let depletion region width $= x_n$. Then charge neutrality implies

$$2N_d x_n = n = \frac{m_e^*}{\pi\hbar^2} kT \ln\left[1 + e^{\frac{E_f - (E_{c2}+E_1)}{kT}}\right]$$

Define quantities as shown below in the "raw" band diagram.



$q\phi_c = E_f - (E_{c2}+E_1)$

⇒ $q\phi_c = q\phi_B - (E_{c1} - E_f) - E_1$.

$n = N_c e^{\frac{E_f-E_c}{kT}}$ } → MB statistics ⇒ $q\phi_c = q\phi_B - kT\ln\left(\frac{N_c}{N_d}\right) - E_1$

$q\phi_c = (115 + 22 - 47.9)$ meV.

$q\phi_c = 89$ meV.

$n = N_c \frac{2}{\sqrt{\pi}} F_{1/2}\left(\frac{E_f - E_c}{kT}\right)$ } → FD statistics ⇒ $q\phi_c = (115 + 42.5 - 47.9)$ meV

$= 109.6$ meV.

Potential drop in AlGaAs depletion regions $= \dfrac{q N_d X_n^2}{2\varepsilon_1} = \phi_D$

$$\Rightarrow \quad 2 N_d X_n = \sqrt{\frac{8 \varepsilon_1 N_d \phi_D}{q}}$$

Charge neutrality condition becomes

$$\Rightarrow \quad \sqrt{\frac{8 \varepsilon_1 N_d \phi_D}{q}} = \frac{m_e^*}{\pi \hbar^2} kT \ln\left[1 + e^{\frac{q(\phi_c - \phi_D)}{kT}}\right]$$

Solution: $\phi_D = 39.5$ meV (MB)

$= 51.5$ meV (FD)

Position of fermi level relative to $E_{c2} = E_1 + q\phi_c - q\phi_D$

$= 47.9 + 89 - 39.5$

$= 97.4$ meV (MB)

$= 47.9 + 109.6 - 51.5$

$= 106$ meV (FD)

2.3

a) a for $Ga_x In_{1-x} As_y P_{1-y}$ is:

$$a = xy(5.6533) + x(1-y)\,5.4505 + y(1-x)\,6.0584 + (1-x)(1-y)\,5.8688$$

Also

$$E_g = 1.35 + 0.668x - 1.068y + 0.758x^2 + 0.078y^2 - 0.069xy - 0.322x^2y + 0.03xy^2$$

Need $a = 5.8688$ $E_g = 0.80155$ eV ($\lambda = 1550$ nm)

$$\Rightarrow \begin{cases} x = 0.42 \\ y = 0.90 \end{cases}$$

b) Need $a = 5.8688$ $E_g = 0.95569$ eV. ($\lambda = 1300$ nm)

$$\Rightarrow \begin{cases} x = 0.28 \\ y = 0.60 \end{cases}$$

c). Smallest bandgap when $y = 1.0$ & $x = 0.468$ $\{In_{0.532} Ga_{0.468} As\}$

The band gap is 0.75 eV.

Largest bandgap when $y = 0.0$ & $x = 0.0$ $\{InP\}$

The band gap is 1.35 eV.

2.4

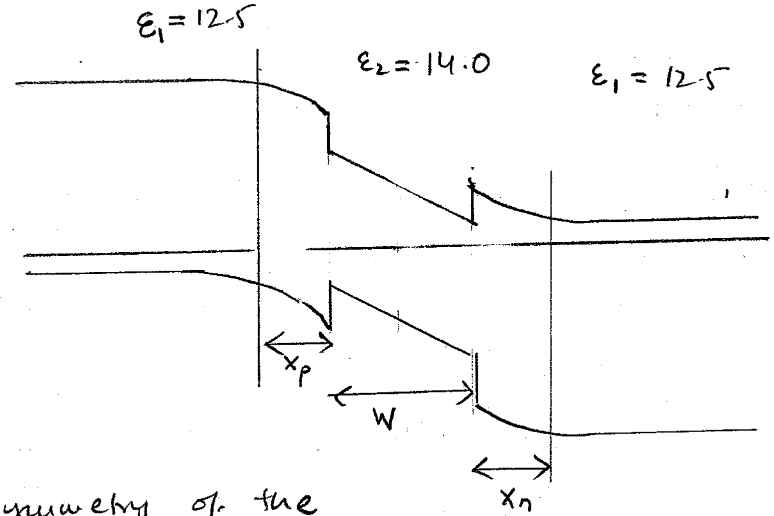
a) Already done that in 2.3(c). $In_{0.532}Ga_{0.468}As$ is Lattice matched to InP. $\{ x = 0.468 \}$.

$E_g = 0.75$ eV.

b). For InP $N_C = 5.7 \times 10^{17}/cm^3$
$N_V = 1.27 \times 10^{19}/cm^3$.

Assuming MB statistics :
$$qV_{bi} = E_g + kT \ln\left[\frac{N_a N_d}{N_C N_V}\right]$$
$$= (1.35 - 0.05)\text{ eV}$$
$$V_{bi} = 1.30 \text{ Volts.}$$

c)



d) From symmetry of the problem $x_n = x_p$

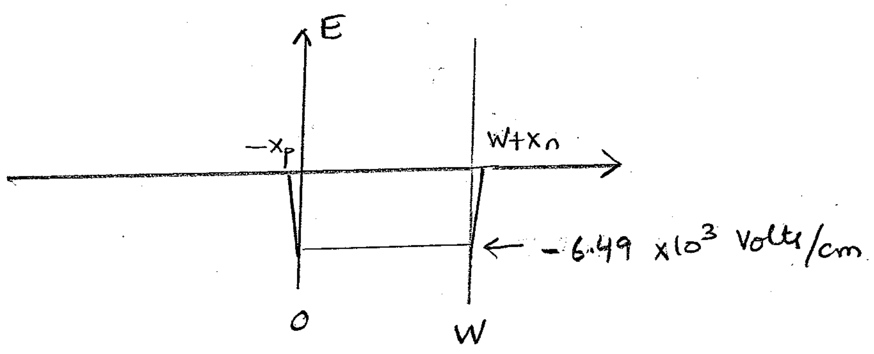
Also
$$V_{bi} = \frac{qN_d x_n^2}{2\varepsilon_1} + \frac{qN_a x_p^2}{2\varepsilon_1} + EW$$ $\left\{ E = \text{electric field in the intrinsic region} \right\}$

$$V_{bi} = \frac{qN_d x_n^2}{\varepsilon_1} + \frac{qN_d x_n W}{\varepsilon_2}$$

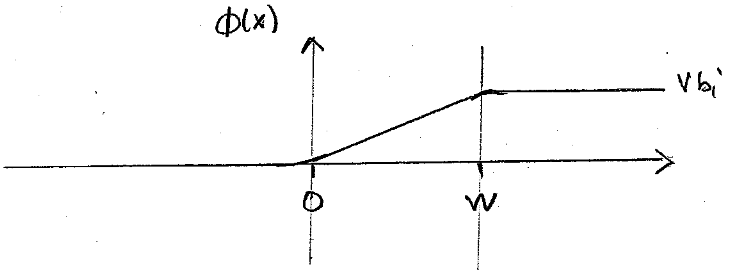
$$\Rightarrow x_n^2 + \left(\frac{\varepsilon_1}{\varepsilon_2} W\right) x_n = \frac{\varepsilon_1 V_{bi}}{qN_d} \Rightarrow x_n = x_p = -\frac{\varepsilon_1}{2\varepsilon_2}W + \sqrt{\left(\frac{\varepsilon_1 W}{\varepsilon_2 2}\right)^2 + \frac{\varepsilon_1 V_{bi}}{qN_d}}$$
$= 0.5$ nm very small.

all the potential falls across the intrinsic InGaAs region.

e)



f)



g) Since the n and p regions are heavily doped all the applied potential will fall across the intrinsic region.

Exact expression:

$$\left\{ \begin{array}{l} x_n = -\frac{\varepsilon_1 W}{2\varepsilon_2} + \sqrt{\left(\frac{\varepsilon_1 W}{2\varepsilon_2}\right)^2 + \frac{\varepsilon_1 (V_{bi}-V)}{qN_d}} \\ \text{and } |E| = \frac{qN_d x_n}{\varepsilon_2} \end{array} \right.$$

$\{ -50 \le V \le 0$

Approximate expression:

$$|E| = \frac{V_{bi}-V}{W}$$

$$= \frac{V_{bi}+|V|}{W}$$

(for $V \le 0$)