ECE 5330: Semiconductor Optoelectronics

Fall 2014

Homework 2 **Due on Sep. 16, 2011**

Suggested Readings:

i) Lecture Notes
ii) Review tight-binding method from your favorite solid state text.

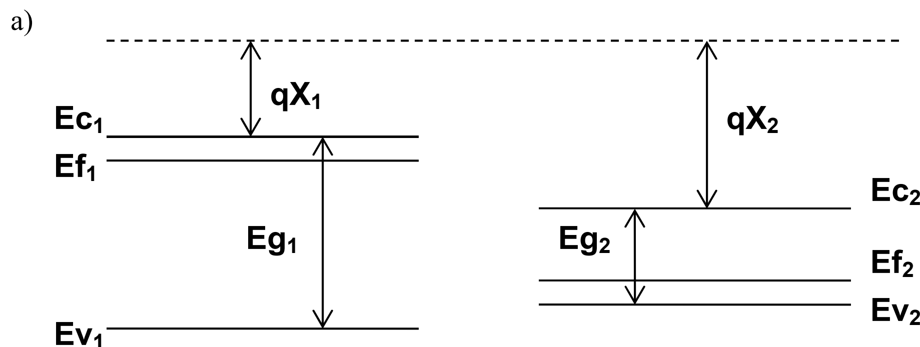
Table of Parameter Values of III-V Semiconductors:

Parameters at 300K	GaAs	AlAs	InAs	InP	GaP
Lattice constant (A)	5.6533	5.6600	6.0584	5.8688	5.4505
$Eg(\Gamma\text{-point})$ (eV)	1.424	3.03	0.354	1.344	2.78
m_e^*/m_o	0.067	0.15	0.023	0.077	0.25
m_{hh}^*/m_o	0.50	0.79	0.40	0.6	0.67
m_{lh}^*/m_o	0.087	0.15	0.026	0.12	0.17
Relative dielectric constant	13.0	10.0	15.0	12.5	11.0

Problem 2.1: (Semiconductor heterojunctions)

In each of the parts given below band diagrams of two different semiconductors are drawn (one on the left side and the other on the right side). In each case you are supposed to **sketch** the **equilibrium** band diagram when a heterojunction is formed between the two semiconductors and give an expression for the built-in potential V_{bi} . In each case indicate the **depletion** and/or **accumulation** regions that may exist in equilibrium on either side of the heterointerface. For your convenience, I have already taken the liberty of aligning the band diagrams such that the alignment shown corresponds to the electron affinity rule (as shown explicitly in part (a)). All the labels are also shown in more detail in part (a), and which you can use for the other parts as well.

a)



b)

Ec_1

Ec_2

Ef_2

Ev_2

Ef_1

Ev_1

c)

Ec_1

Ec_2

Ef_1

Ev_1

Ef_2

Ev_2

d)

Ec_2

Ec_1

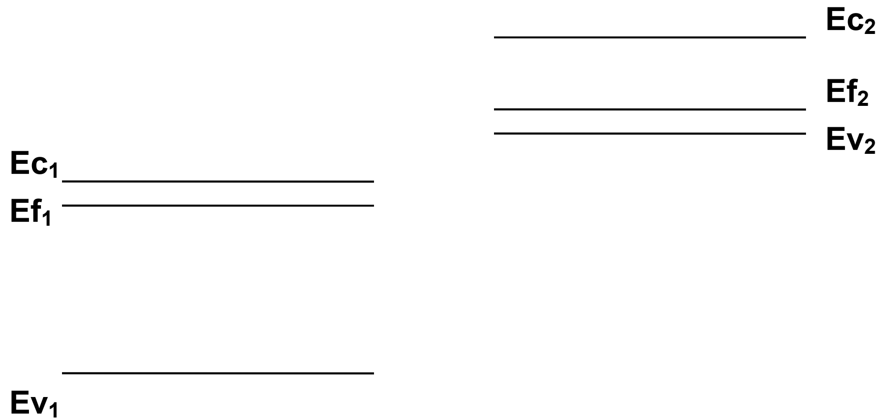
Ef_1

Ef_2

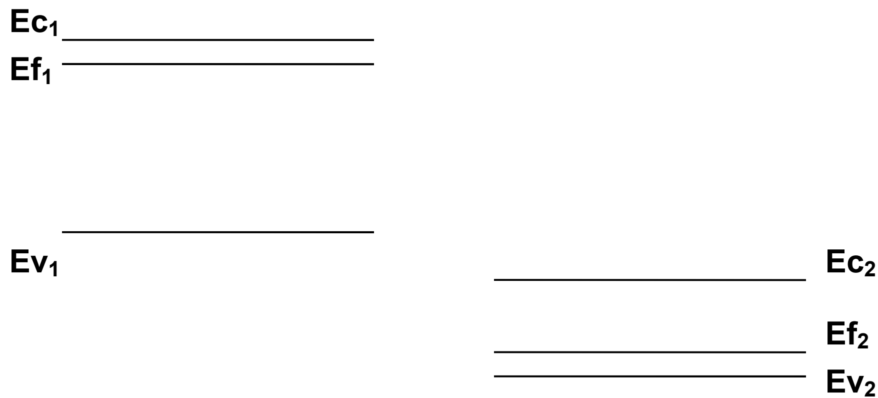
Ev_2

Ev_1

e)



f)



Problem 2.2: (A quantum well problem)

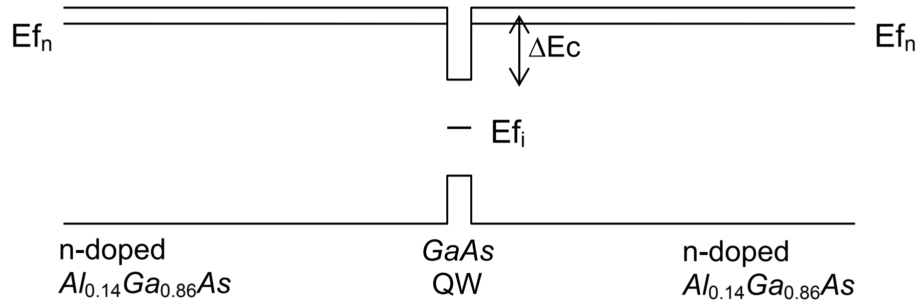
Additional notes:

i) The bandgap of $Al_xGa_{1-x}As$ at the Γ-point is given by the relation,
$E_g(\Gamma) = 1.424 + 1.206x + 0.4x^2$

ii) For most III-V semiconductor heterostructures the band alignment as predicted by the electron affinity rule does not does not agree well with experiments. So some heuristic rules based on experimental evidence are used instead. For $Al_xGa_{1-x}As$ the rule-of-thumb during the 80's and the 90's was that 85% of the bandgap difference ΔE_g in $Al_xGa_{1-x}As$ heterostructures ends up as the conduction band offset ΔE_c and 15% of ΔE_g ends up as ΔE_v. Recently, this 0.85/0.15 ratio for $\Delta E_c / \Delta E_v$ has been revised and it is now believed to be closer to 0.65/0.35. In this problem you will use this new value of the $\Delta E_c / \Delta E_v$ ratio for $Al_xGa_{1-x}As$ heterostructures.

iii) The **bulk** conduction bands of $GaAs$ and $AlAs$ at the Γ-point may be assumed to be parabolic and isotropic.

Consider a quantum well made up of a 6 nm thick $GaAs$ layer sandwiched between very thick $Al_{0.14}Ga_{0.86}As$ layers. The $GaAs$ layer is undoped. The $Al_{0.14}Ga_{0.86}As$ layers are n-doped with N_d = 10^{17} cm^{-3}. The "raw" (i.e. without any band bending) conduction and valence bands of the structure are shown below. Assume that Maxwell-Boltzmann statistics will work for the $Al_{0.14}Ga_{0.86}As$ layers.



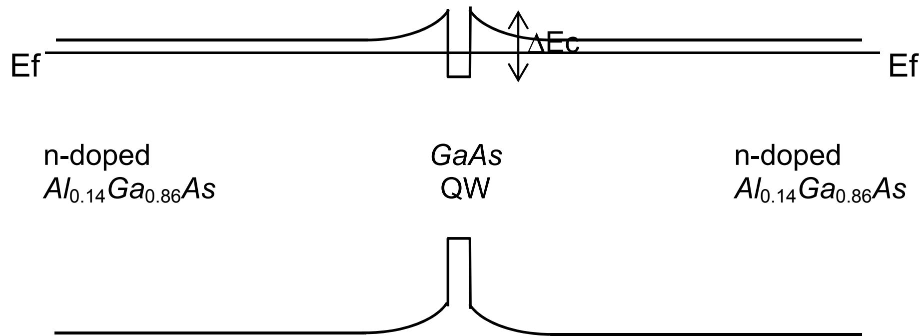
A *GaAs-AlGaAs* quantum well

a) Find the values of the electron effective mass and the dielectric constant for the alloy $Al_{0.14}Ga_{0.86}As$. Also, find the value of the conduction band offset ΔE_c (see the figure above).

b) For this part assume that the electron effective mass in the alloy $Al_{0.14}Ga_{0.86}As$ is the same as in $GaAs$. This is not a bad approximation (check with part (a) results). Find the energies of all the quantized confined energy levels in the conduction band for the quantum well.
Hint: You should find only one confined energy level.

In parts (c) and (d) you will take into account band bending.

c) The band diagram shown above is certainly not the equilibrium band diagram. Equilibrium will be established when some electrons will move from the doped layers into the quantum well leaving behind depleted regions. Sketch the band diagram in equilibrium (you don't need to do any math for this part).
Hint: the answer is on the next page.



A *GaAs-AlGaAs* quantum well

d) What is the value of the equilibrium built-in potential for this structure? i.e. the difference in the electrostatic potential between the far-left and far-right points.

e) For the equilibrium situation with band bending, we need to calculate i) the electrostatic potential drop in the $Al_{0.14}Ga_{0.86}As$ layers and ii) the position of the Fermi level relative to the conduction band bottom in the quantum well. Both these quantities are related and will need to be determined simultaneously. In doing this problem, treat the electrons in the quantum well as a sheet charge density since the quantum well so thin.
Hint: First assume some thickness of the depletion region on both sides in the $Al_{0.14}Ga_{0.86}As$ layers. Figure out how much charge (per unit area) has moved from these layers into the quantum well. Then write an expression for the charge inside the quantum well in terms of the Fermi level and set it equal to the charge lost from the depletion region. In the equation thus obtained try to express both the Fermi level and the depletion region thickness in terms of the potential drop in the $Al_{0.14}Ga_{0.86}As$ layers. This should give you a nonlinear equation for the potential drop in the $Al_{0.14}Ga_{0.86}As$ layers that can be solved graphically.

The technique described in this problem to get electrons (or holes) into a quantum well by n-doping (or p-doping) adjacent wider bandgap layers is called "modulation doping" and is used in many electronic and optical devices.

Problem 2.3: (Quaternaries for 1550 nm and 1300 nm lasers)

In this problem you will figure out the compositions for 1550 nm and 1300 nm $Ga_xIn_{1-x}As_yP_{1-y}$ lasers that are grown on InP substrates. The beauty of the $Ga_xIn_{1-x}As_yP_{1-y}$ quaternaries is that you can lattice match to InP and also get the desired bandgap. Assume:

$$E_g(\Gamma) = 1.35 + 0.668x - 1.068y + 0.758x^2 + 0.078y^2 - .069xy - 0.322x^2y + .03xy^2 \quad (\text{eV})$$

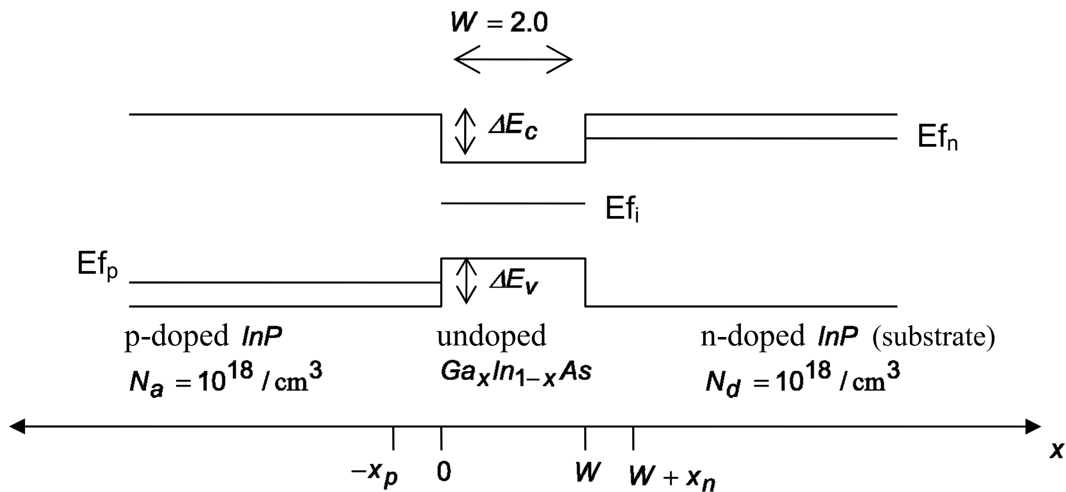
a) Find the composition of $Ga_xIn_{1-x}As_yP_{1-y}$ (i.e. find the values of x, y) that is lattice matched to InP **and** has a bandgap at 1550 nm. When I say "has a bandgap at 1550 nm" I mean that a photon in **free space** with energy equal to the bandgap would have a wavelength equal to 1550 nm.

b) Find the composition of $Ga_xIn_{1-x}As_yP_{1-y}$ (i.e. find the values of x, y) that is lattice matched to InP **and** has a bandgap at 1300 nm.

c) What are the smallest and the widest bandgaps (in nm) that can be achieved with $Ga_xIn_{1-x}As_yP_{1-y}$ alloys that are lattice matched to InP ? What are the compositions needed to achieve these extreme values of the bandgaps?

Problem 2.4: (Heterojunction pin diodes)

In a p-i-n diode, an intrinsic (undoped) layer is sandwiched between a p-doped layer and an n-doped layer. Most photodetectors that are used for operation at 1300 nm and at 1550 nm in fiber optical communication systems are $Ga_xIn_{1-x}As$ p-i-n diodes grown on InP substrates. We will look at the operation of photodetectors later in the course. In this problem you will work out the electrostatics of a heterojunction p-i-n diode. The "raw" band diagram of a heterojunction p-i-n diode is shown in the figure below.



A p-i-n Heterojunction Diode

A layer of undoped $Ga_xIn_{1-x}As$ is sandwiched between a p-doped and a n-doped InP layer. The $Ga_xIn_{1-x}As$ layer is not a quantum well – it is too thick (2 μm) to be a quantum well, and since it is a thick layer it has to be lattice matched to the InP substrate to avoid defects. Assume that the $\Delta E_c / \Delta E_v$ ratio for $Ga_xIn_{1-x}As - InP$ heterostructure interface is 0.4/0.6.

a) Find the composition and the bandgap (in nm) of $Ga_xIn_{1-x}As$ that is lattice matched to InP .

b) Calculate (i.e. give a numerical value) the built-in potential for the p-i-n heterojunction diode shown in the figure above.

c) Sketch the equilibrium band diagram of the p-i-n diode (Hint: assume no carriers in the undoped $Ga_xIn_{1-x}As$ layer in equilibrium).

c) Calculate (i.e. give a numerical value) the thicknesses x_p and x_n of the depletion regions in the p- and n-sides. For this part you will have to solve the electrostatic problem (Hint: assume no carriers in the undoped $Ga_xIn_{1-x}As$ layer in equilibrium).

d) Plot (with units and labels) the electric field in the region $-x_p \le x \le W + x_n$.

e) Plot (with units and labels) the electrostatic potential in the region $-x_p \le x \le W + x_n$.

f) Suppose the p-i-n diode is reversed biased with an external voltage source. Plot the magnitude of the peak electric field in the region $0 \le x \le W$ as a function of the magnitude $|V|$ of the reversed bias voltage for $0 \le |V| \le 50$ volts.