ECE 533 Homework 1 Solutions (Farhan Rana)

1.1

a) & b) Density of states from the two valence bands can be added to give the total density of states:

$$g_v(E) = \frac{\sqrt{2}}{\pi^2}\left(\frac{m^*_{hh}}{\hbar^2}\right)^{3/2}\sqrt{E_v - E} + \frac{\sqrt{2}}{\pi^2}\left(\frac{m^*_{lh}}{\hbar^2}\right)^{3/2}\sqrt{E_v - E}$$

$$= \frac{\sqrt{2}}{\pi^2}\left(\frac{m^*_{dh}}{\hbar^2}\right)^{3/2}\sqrt{E_v - E} \quad \text{where} \quad m^*_{dh} = \left[(m^*_{hh})^{3/2} + (m^*_{lh})^{3/2}\right]^{2/3}$$

1.2

a) $E(\vec{k}) = E_c + \frac{\hbar^2}{2}\vec{k}\cdot\overline{m}_e^{-1}\cdot\vec{k}$. By rotating the co-ordinate system the matrix m_e^{-1} can be diagonalized. In this rotated co-ordinate system m_e is $= \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} m_o$. From lecture notes, the density of state effective mass $= m^*_{de} = (0.4m_o \times 0.3m_o \times 0.2m_o)^{1/3}$ $= 0.2884\, m_o$ and $g_c(E) = \frac{\sqrt{2}}{\pi^2}\left(\frac{m^*_{de}}{\hbar^2}\right)^{3/2}\sqrt{E - E_c}$

1.3

a) $n_i = \sqrt{N_c N_v}\, e^{-\frac{E_g}{2KT}}$ $\qquad N_c = 2\left[\frac{m^*_e KT}{2\pi\hbar^2}\right]^{3/2} = 4.3\times10^{17}/cm^3$

$N_v = 2\left[\frac{m^*_{dh} KT}{2\pi\hbar^2}\right]^{3/2} = 9.4\times10^{18}/cm^3$

$\left\{(m^*_{dh})^{3/2} = (m^*_{hh})^{3/2} + (m^*_{lh})^{3/2}\right\} \Rightarrow n_i = 2.44\times10^6/cm^3.$

b) At $T\approx 0$, E_f cannot be below E_d (otherwise it would mean that all dopants are ionized). E_f cannot be above E_c (since there should be no electrons in the conduction band because the donors are not ionized). So E_f can only be somewhere between E_c and E_d. Electron density is given by:

$n = N_c e^{\frac{E_f - E_c}{kT}}$ { valid if $E_c - E_f >> kT$; which is true at $T \sim 0K$ }

Also

$N_d^+ = \dfrac{N_d}{1 + 2e^{\frac{E_f - E_d}{kT}}}$. Charge neutrality $\Rightarrow$ $n = N_d^+ + p$

Forget about holes at $T \sim 0K$ (i.e no thermal electron-hole pair generation at $T \sim 0K$).

$$\Rightarrow \quad n \simeq N_d^+ \Rightarrow \quad N_c e^{\frac{E_f - E_c}{kT}} = \frac{N_d}{1 + 2e^{\frac{E_f - E_d}{kT}}}$$

Since $T \sim 0K$ and $E_f > E_d$ $\Rightarrow$ $\dfrac{N_d}{1+2e^{\frac{E_f - E_d}{kT}}} \approx \dfrac{N_d}{2} e^{-\frac{(E_f - E_d)}{kT}}$

$$\Rightarrow \quad N_c e^{\frac{E_f - E_c}{kT}} = \frac{N_d}{2} e^{-\frac{(E_f - E_d)}{kT}} \Rightarrow E_f = \frac{E_c + E_d}{2} + \frac{kT}{2} \ln\left(\frac{N_d}{2N_c}\right)$$

In the limit $T \to 0K$

$E_f \longrightarrow \dfrac{E_c + E_d}{2}$ (i.e exactly between the donor level and the conduction band edge).

c) As $T \to \infty$, thermal electron-hole pair generation puts far more electrons in the conduction band than the fully ionized donor atoms. So $n \approx p$ (forget about doping as $T \to \infty$). $n \approx p \Rightarrow E_f = \dfrac{E_c + E_v}{2} + \dfrac{kT}{2}\left(\dfrac{N_v}{N_c}\right)$. (i.e at the same position as if the material was undoped).

d). $n = N_c \dfrac{2}{\sqrt{\pi}} F_{1/2}\left(\dfrac{E_f - E_c}{kT}\right)$. $\quad N_d^+ = \dfrac{N_d}{1 + 2e^{\frac{E_f - E_d}{kT}}}$.

& $n \approx N_d^+$ (forget holes at this high doping).

when $E_f = E_c$ $\quad n = N_c \dfrac{2}{\sqrt{\pi}} F_{1/2}(0) = \dfrac{N_d}{1 + 2e^{\frac{E_c - E_d}{kT}}}$

$$\Rightarrow \quad N_d = \left(1 + 2\, e^{\frac{E_c - E_d}{kT}}\right) \times N_c \times \frac{2}{\sqrt{\pi}} \times F_{1/2}(0)$$

$$F_{1/2}(0) = 0.678 \quad \Rightarrow \quad N_d = 1.3 \times 10^{18} / cm^3 .$$

1.4

a) $\psi(z) \begin{cases} \sin(k_z z) \\ \cos(k_z z) \end{cases} \quad |z| < \frac{L}{2} \qquad \begin{cases} e^{-\kappa |z|} \end{cases} \quad |z| > \frac{L}{2}$.

$$E = \frac{\hbar^2 k_z^2}{2m_e^*} = e\phi - \frac{\hbar^2 \kappa^2}{2m_e^*}$$

For the Sine solutions;

$$\cot\left(k_z \frac{L}{2}\right) = -\frac{\kappa}{k_z} = -\frac{1}{k_z}\sqrt{\frac{2m_e^* e\phi}{\hbar^2} - k_z^2}$$

No solution if $\quad \frac{2m_e^* e\phi}{\hbar^2} < \left(\frac{\pi}{L}\right)^2$

b) For the Cosine Solutions:

$$\tan\left(\frac{k_z L}{2}\right) = \frac{1}{k_z}\sqrt{\frac{2m_e^* e\phi}{\hbar^2} - k_z^2}$$

No third confined level if $\frac{2m_e^* e\phi}{\hbar^2} < \left(\frac{2\pi}{L}\right)^2$.

c) $m_e^* = 0.1 m_0$ $\quad \phi = 100$ meV $\quad L = 10$ nm

Since $\left(\frac{\pi}{L}\right)^2 < \frac{2m_e^* e\phi}{\hbar^2} < \left(\frac{2\pi}{L}\right)^2 \Rightarrow$ two confined levels.

From graphical analysis: $\frac{k_z L}{2} \approx 1.12$ for the first level.

and $\frac{k_z L}{2} \approx 2.15$ for the second.

$\Rightarrow E_1 = \frac{\hbar^2 k_z^2}{2m_e^*} = 19.2$ meV.

$\Rightarrow E_2 = \frac{\hbar^2 k_z^2}{2m_e^*} = 70.6$ meV.