# Chapter 15

# Widely Tunable Photonics

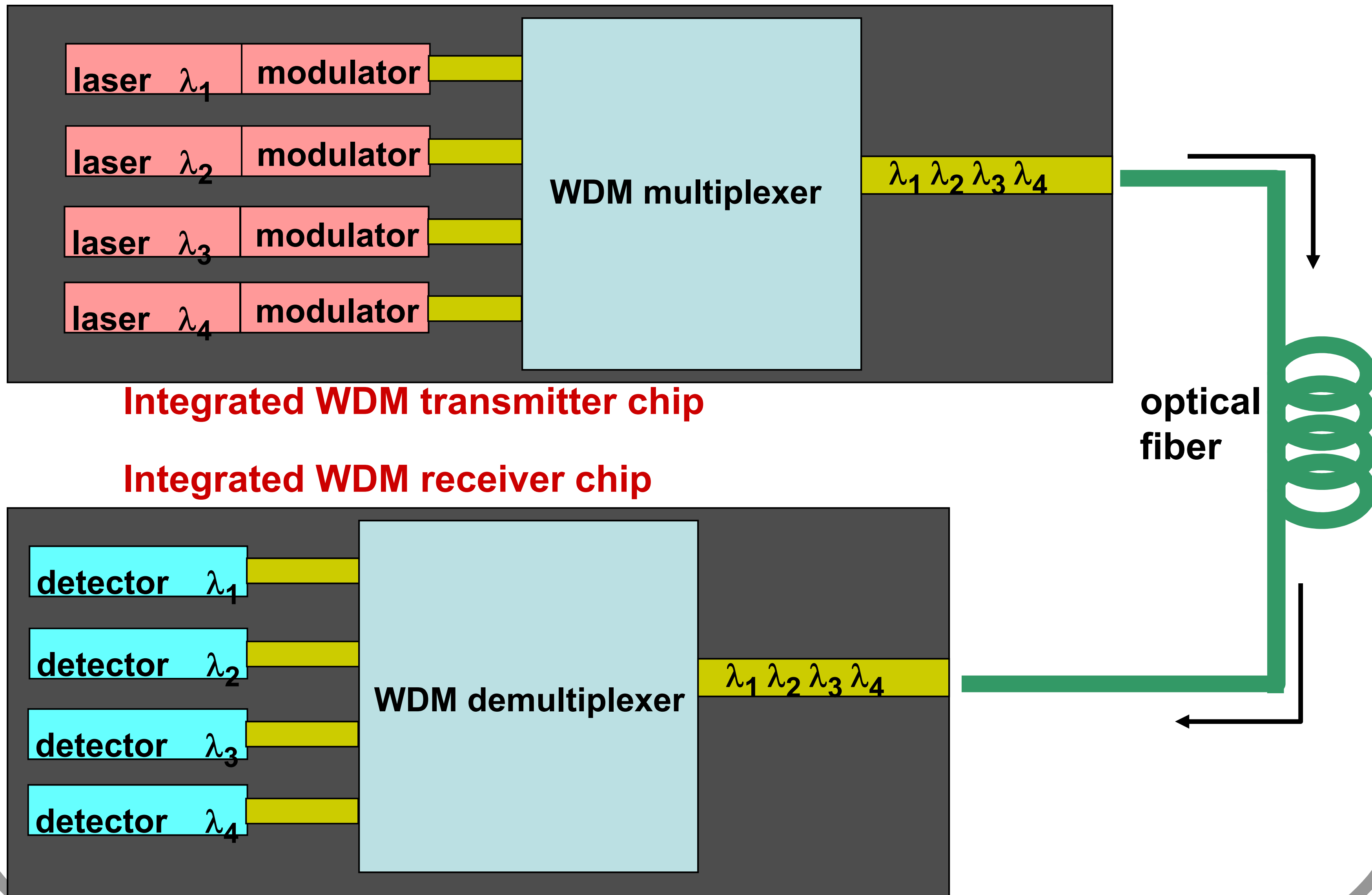
**In this lecture you will learn:**

- DBR lasers
- Tuning semiconductor lasers
- Sampled DBR reflectors
- Sampled DBR Lasers

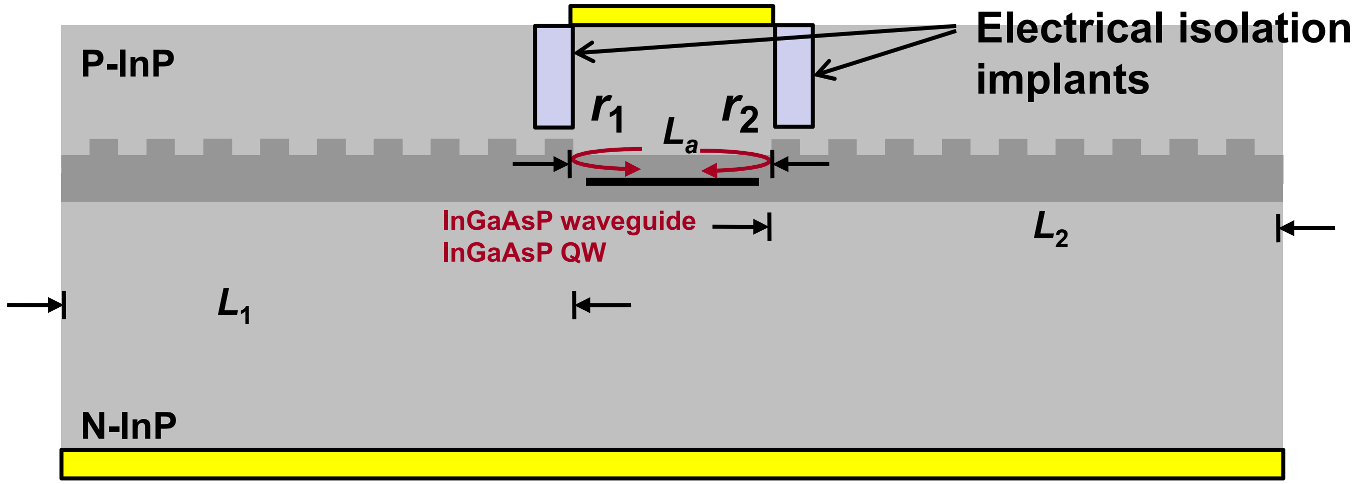
**Refractive Index:** $n$

**Carrier Density:** $N$

# Wavelength Division Multiplexed (WDM) Systems



# Distributed Bragg Reflector (DBR) Lasers




**Lasing Condition:**

$$r_1 r_2\, e^{\left(\Gamma_a \frac{\tilde{g}}{2} - \frac{\tilde{\alpha}}{2}\right)2L_a}\, e^{i\beta 2L_a} = 1$$

Reflection phases

$$\Rightarrow |r_1|\,|r_2|\, e^{\left(\Gamma_a \frac{\tilde{g}}{2} - \frac{\tilde{\alpha}}{2}\right)2L_a}\, e^{i\beta_a 2L_a + \phi_1 + \phi_2} = 1$$

$$\sqrt{R_1}\sqrt{R_2}\, e^{\left(\Gamma_a \frac{\tilde{g}}{2} - \frac{\tilde{\alpha}}{2}\right)2L_a} = 1$$

$$2\beta_a L_a + \phi_1 + \phi_2 = 2\pi\, p \qquad \{\, p = \text{integer}$$

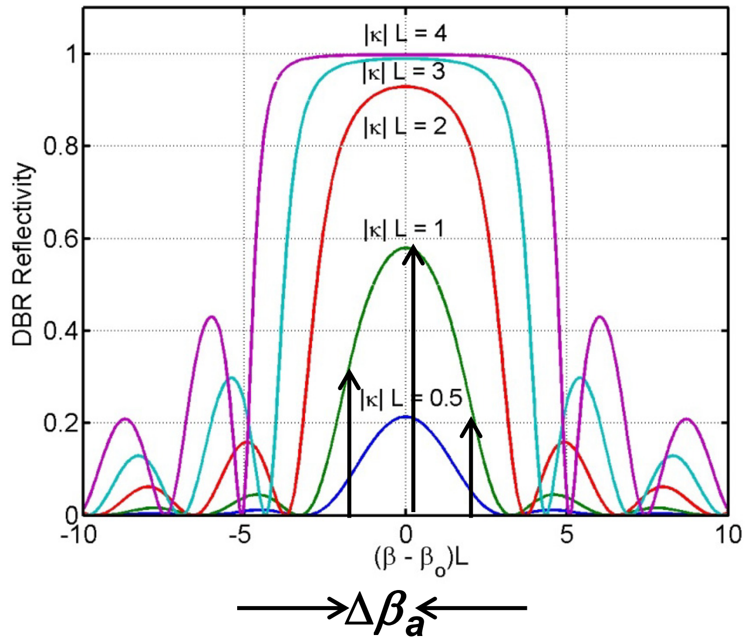
# Distributed Bragg Reflector (DBR) Lasers

$$2\beta_a L_a + \phi_1 + \phi_2 = 2\pi\, p \qquad \{\, p = \text{integer}$$

## Laser Cavity Longitudinal Mode Spacing:

$$2\Delta\beta_a L_a + \frac{\partial \phi_1}{\partial \beta_1}\Delta\beta_1 + \frac{\partial \phi_2}{\partial \beta_1}\Delta\beta_2 = 2\pi$$

$$\Rightarrow \Delta\beta_a = \frac{\pi\, n_{ag}}{n_{ag}L_a + \frac{n_{1g}}{2}\frac{\partial \phi_1}{\partial \beta_1} + \frac{n_{2g}}{2}\frac{\partial \phi_2}{\partial \beta_2}}$$



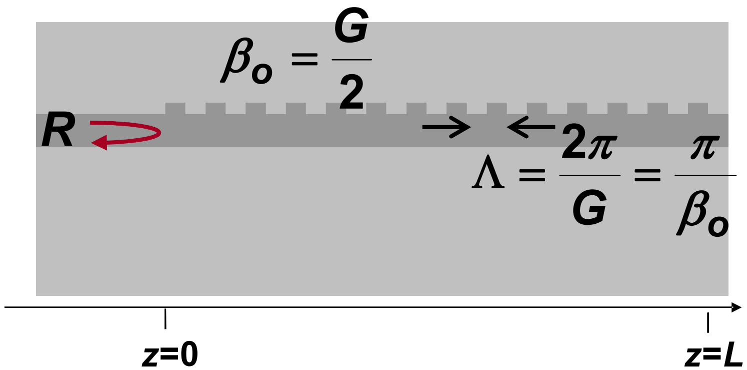

When $\beta \approx \beta_o$ :

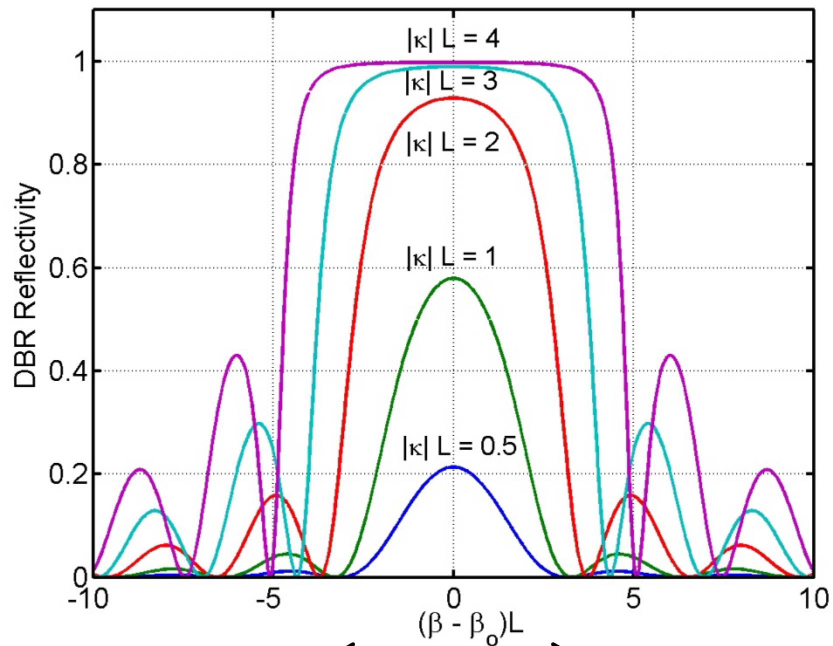
$$\frac{\partial \phi_1}{\partial \beta_1} = \frac{\tanh(|\kappa| L_1)}{2|\kappa|} = 2L_{eff\,1}$$

$$\frac{\partial \phi_2}{\partial \beta_1} = \frac{\tanh(|\kappa| L_2)}{2|\kappa|} = 2L_{eff\,2}$$

$$\Delta\beta_a \approx \frac{\pi\, n_{ag}}{n_{ag}L_a + n_{1g}L_{eff\,1} + n_{2g}L_{eff\,2}}$$

# Distributed Bragg Reflectors (Side Note)




$$\Delta\varepsilon(x,y,z) = f(x,y)\left[d_1 e^{iGz} + d_{-1} e^{-iGz}\right]$$

$$G = 2\beta_o$$

$$\kappa \approx \frac{\omega\, d_1}{2 n n_g^M v_g}\Gamma_G$$

When $\beta = \beta_o$ :

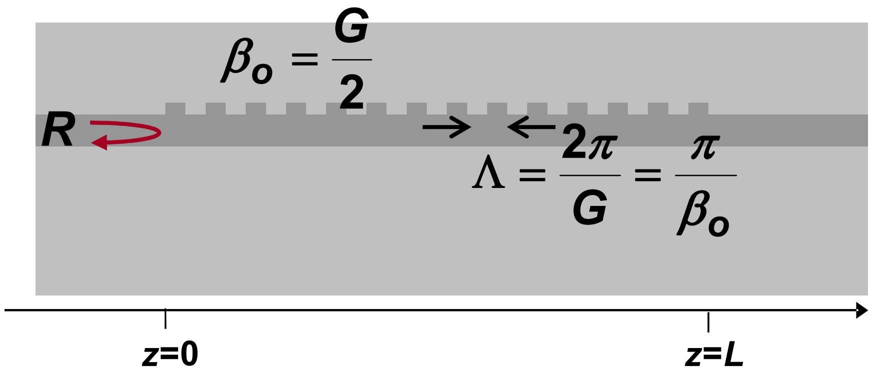
$$R_{max} = \tanh^2\left(|\kappa| L\right)$$

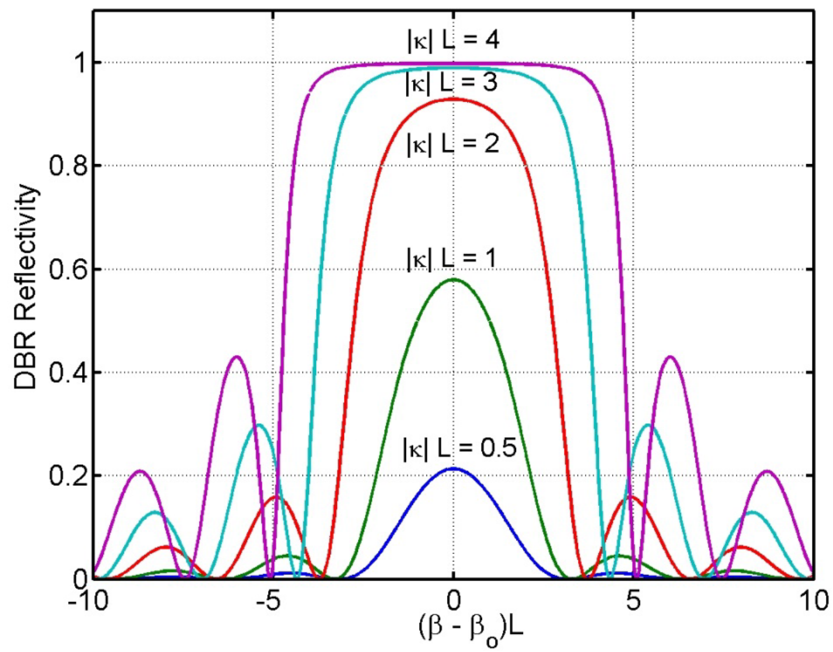
When $|\kappa| L < 1$:

$$\Delta\beta_{DBR} = 2\sqrt{|\kappa|^2 + \left(\frac{\pi}{L}\right)^2} \approx \frac{2\pi}{L}$$

$$\Rightarrow \Delta\omega_{DBR} = 2 v_g \sqrt{|\kappa|^2 + \left(\frac{\pi}{L}\right)^2} \approx v_g \frac{2\pi}{L}$$

# Distributed Bragg Reflectors (Side Note)




$$\Delta\varepsilon(x,y,z) = f(x,y)\int_{-\infty}^{\infty}\frac{dq}{2\pi}h(q)\,e^{iqz}$$

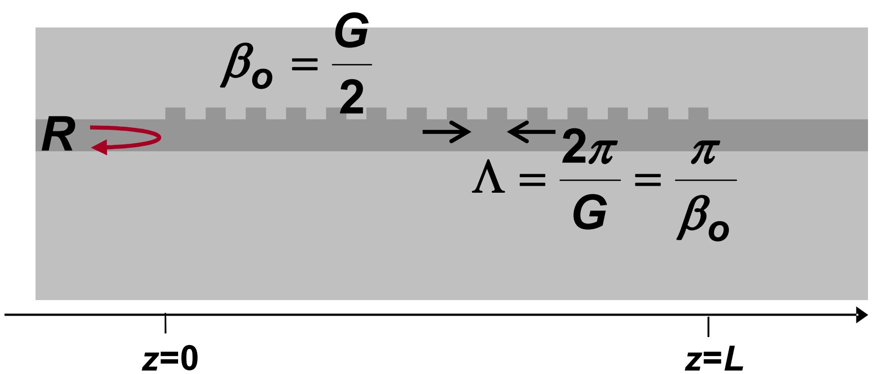
**For infinitely long grating:**

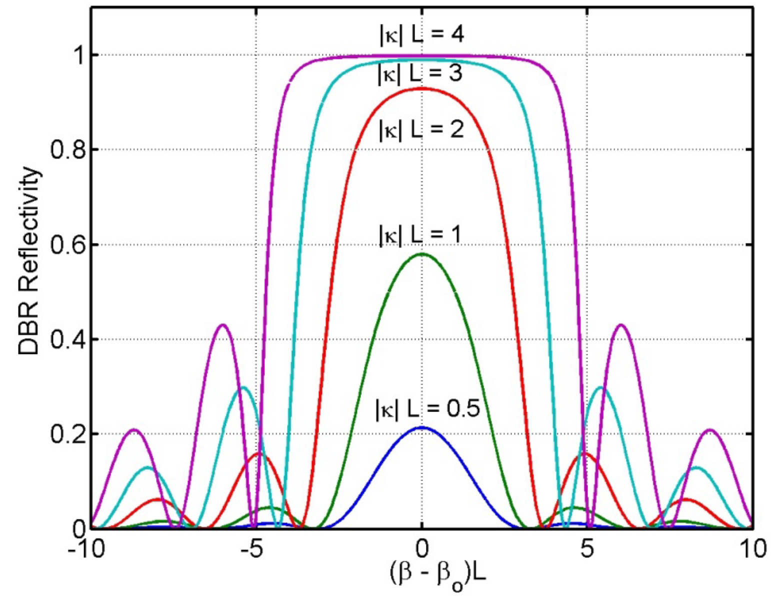
$$h(q) = 2\pi\, d_1\delta(q-G) + 2\pi\, d_{-1}\delta(q+G)$$

**For grating of length *L*:**

$$h(q) = d_1 L\,\frac{\sin\left((q-G)\frac{L}{2}\right)}{(q-G)\frac{L}{2}} + d_{-1}L\,\frac{\sin\left((q+G)\frac{L}{2}\right)}{(q+G)\frac{L}{2}}$$

# Distributed Bragg Reflectors (Side Note)




$$\Delta\varepsilon(x,y,z) = f(x,y)\int_{-\infty}^{\infty}\frac{dq}{2\pi}h(q)\,e^{iqz}$$

**For infinitely long grating:**

$$h(q) = 2\pi\, d_1\delta(q-G) + 2\pi\, d_{-1}\delta(q+G)$$

$$\kappa \approx \frac{\omega\, d_1}{2nn_g^M v_g}\Gamma_G$$

**When** $\beta = \beta_o$ **:**

$$R\left(\beta = \beta_o = \frac{G}{2}\right) = \tanh^2\left(|\kappa| L\right)$$

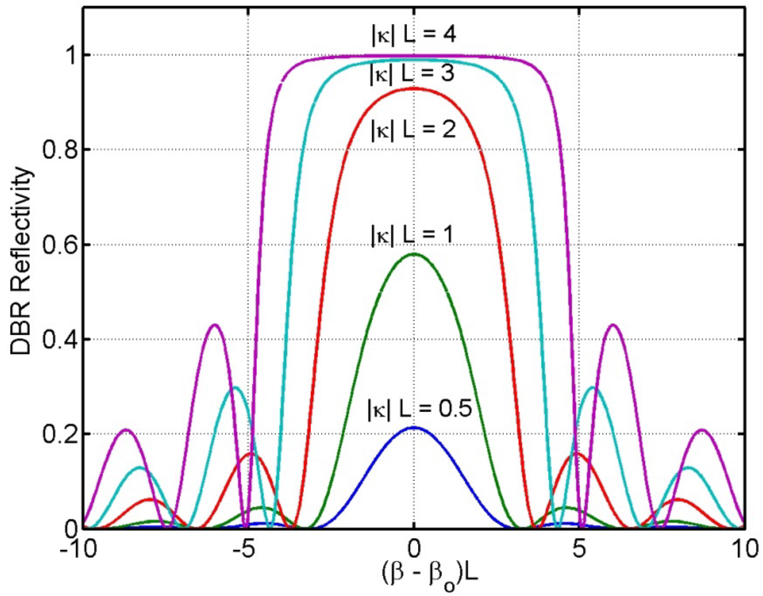
**For grating of length *L*:**

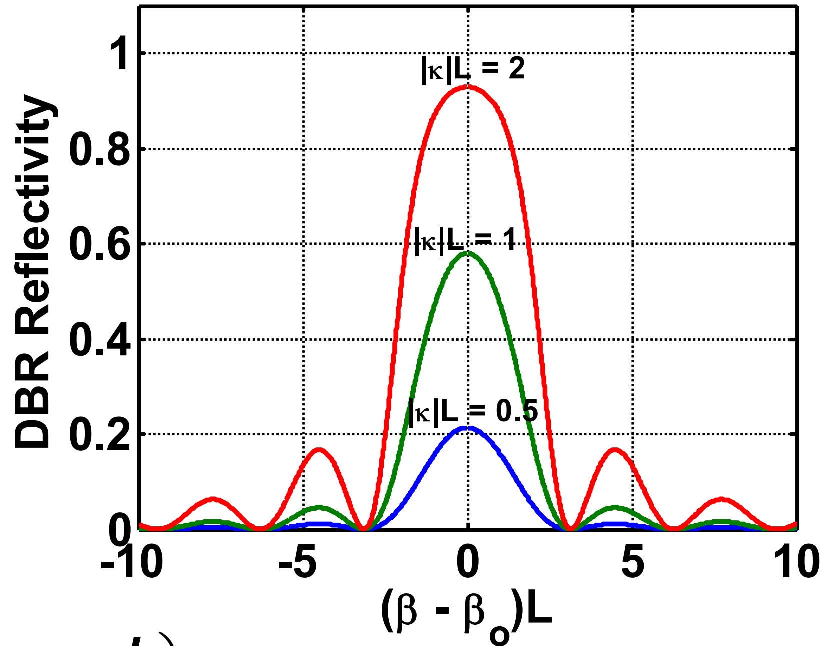
$$h(q) = d_1 L\,\frac{\sin\left((q-G)\frac{L}{2}\right)}{(q-G)\frac{L}{2}} + d_{-1}L\,\frac{\sin\left((q+G)\frac{L}{2}\right)}{(q+G)\frac{L}{2}}$$

$$\kappa(\beta)L \approx \frac{\Gamma_G\,\omega(\beta)}{2nn_g^M v_g}h(q=2\beta) = \kappa(\beta_o)L\,\frac{\sin[(\beta-\beta_o)L]}{(\beta-\beta_o)L}$$

$$\Rightarrow R(\beta) = \tanh^2\left(|\kappa(\beta)|L\right)$$

# Distributed Bragg Reflectors (Side Note)




$$h(q) = d_1 L \frac{\sin\left((q-G)\frac{L}{2}\right)}{(q-G)\frac{L}{2}} + d_{-1} L \frac{\sin\left((q+G)\frac{L}{2}\right)}{(q+G)\frac{L}{2}}$$

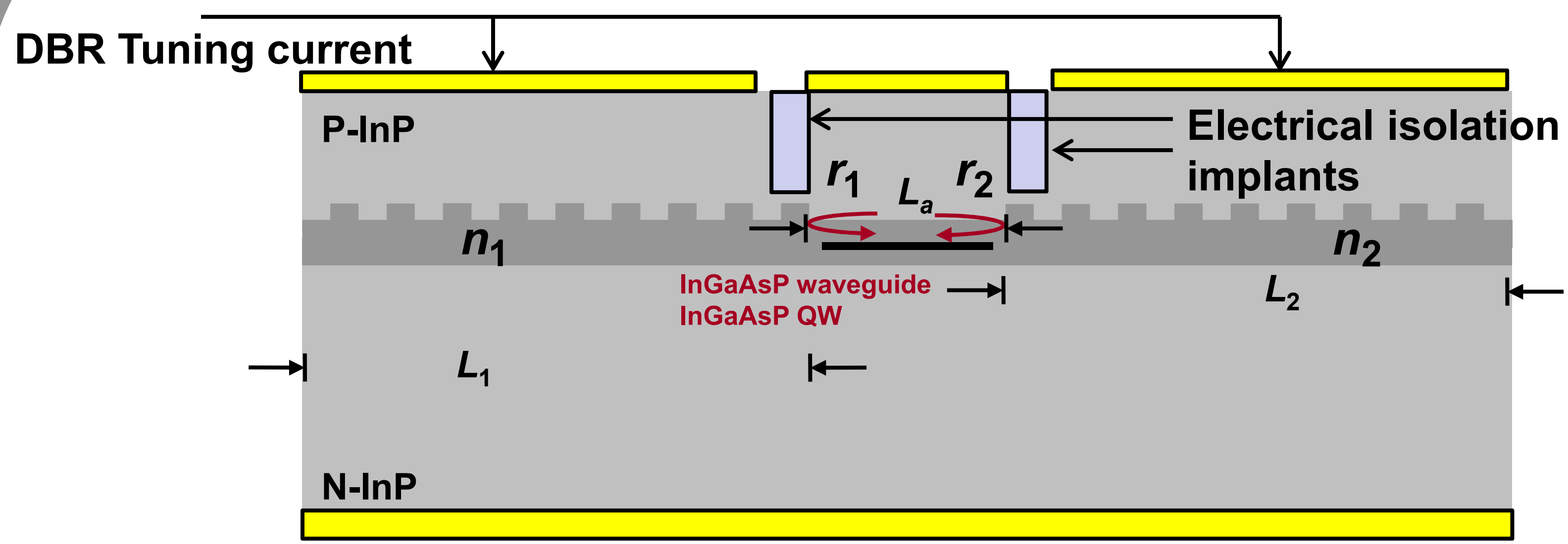
$$\kappa(\beta)L \approx \frac{\Gamma_G \omega(\beta)}{2nn_g^M v_g} h(q = 2\beta) = \kappa(\beta_o)L \frac{\sin[(\beta-\beta_o)L]}{(\beta-\beta_o)L}$$

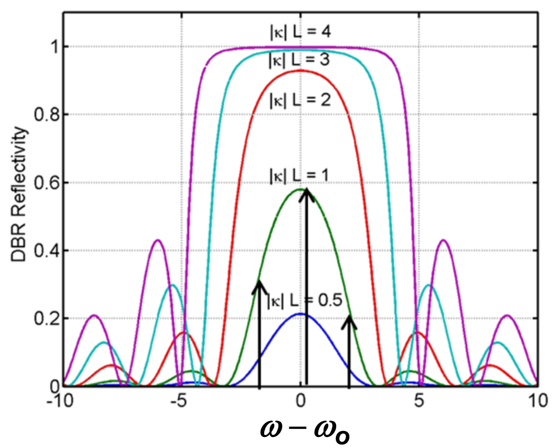
$$\Rightarrow R(\beta) = \tanh^2(|\kappa(\beta)|L)$$

This relation works well only when:

$$|\kappa(\beta_o)|L < 1$$

# Electrical Tuning of DBR Lasers




The wavevector for which maximum reflectivity occurs is:

$$\beta_1(\omega, n_1) = \beta_o \longrightarrow \omega_o$$

Now suppose the index of the grating region core is changed by the injection of electrons and holes:

$$\Delta\beta_1\big|_{index} + \Delta\beta_1\big|_{freq} = 0$$

$$\Rightarrow \frac{\omega_o}{c}\Gamma_1 \frac{n_{1g}}{n_{1g}^M}\Delta n_1 + \frac{\Delta\omega_o}{v_{1g}} = 0$$

$$\Rightarrow \frac{\Delta\omega_o}{\omega_o} = -\frac{\Gamma_1}{n_{1g}^M}\Delta n_1$$ ⟶ change in the frequency for which maximum reflectivity occurs

# Index Change from Carrier Injection - I

Recall the $\alpha$-parameter was defined as:

$$\alpha_H = \frac{dn'/dN}{dn''/dN} = -\frac{4\pi}{\lambda}\frac{dn'/dN}{dg/dN}$$

The subscript "*H*" is there so you don't confuse it with loss

Or

$$\Rightarrow \frac{dn'}{dN} = -\alpha_H \frac{\lambda}{4\pi}\frac{dg}{dN} = \alpha_H \frac{\lambda}{4\pi}\frac{d\alpha}{dN}$$

Gain *g* is the negative of loss $\alpha$

For most InGaAsP semiconductors, when the wavelength is below the bandgap (as in the DBR sections):

$$\frac{d\alpha}{dN} \sim 40\times 10^{-18}\ \text{cm}^2 \qquad \alpha_H \sim -20$$

$$\Rightarrow \frac{dn'}{dN} \sim -0.01\times 10^{-18}\ \text{cm}^3$$

Refractive index decreases by 0.01 when the electron density (and the hole density – assumed to be equal during carrier injection) are increased by $10^{18}$ cm$^{-3}$

Most of the index change comes from the free-carrier plasma effect (Drude model or the intraband contribution to the dielectric constant)

## Index Change from Carrier Injection - II

For most InGaAsP semiconductors, when the wavelength is above the bandgap (as in the gain regions):

$$\frac{dg}{dN} \sim 4 \times 10^{-16}\ \mathrm{cm}^2 \qquad \alpha_H \sim +5$$

$$\Rightarrow \frac{dn'}{dN} \sim -0.025 \times 10^{-18}\ \mathrm{cm}^3$$

Refractive index decreases by 0.025 when the electron density (and the hole density – assumed to be equal in the gain region) are increased by $10^{18}$ $\mathrm{cm}^{-3}$

Most of the index change comes from the interband contribution to the dielectric constant)

# Index Change from Temperature Change

**Change in the temperature results in a change in the material bandgap and carrier distributions, both of which change the material refractive index**

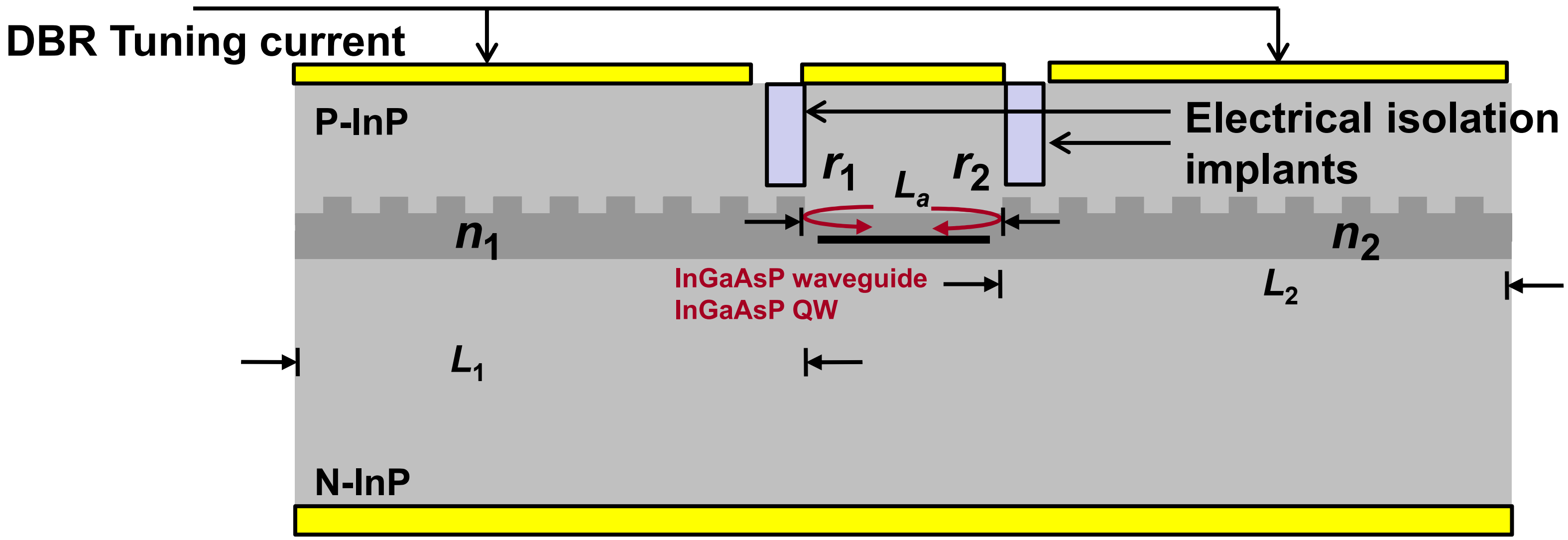
**Change in refractive index is approximately:**

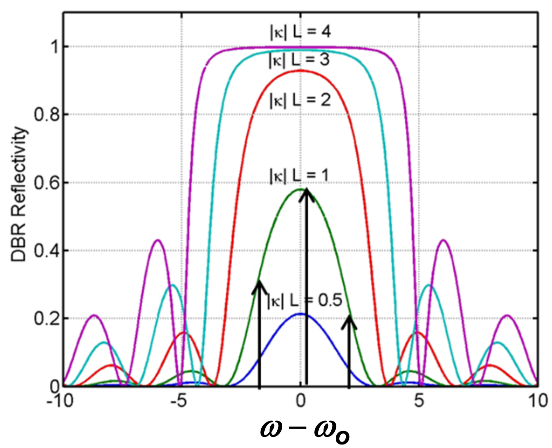
$$\frac{dn'}{dT} \sim +5 \times 10^{-4}\ \mathrm{K}^{-1}$$

**Note that if carrier injection were to result in excessive heating in a poorly designed device, then the index increase from heating would work against index decrease from carrier injection!!**

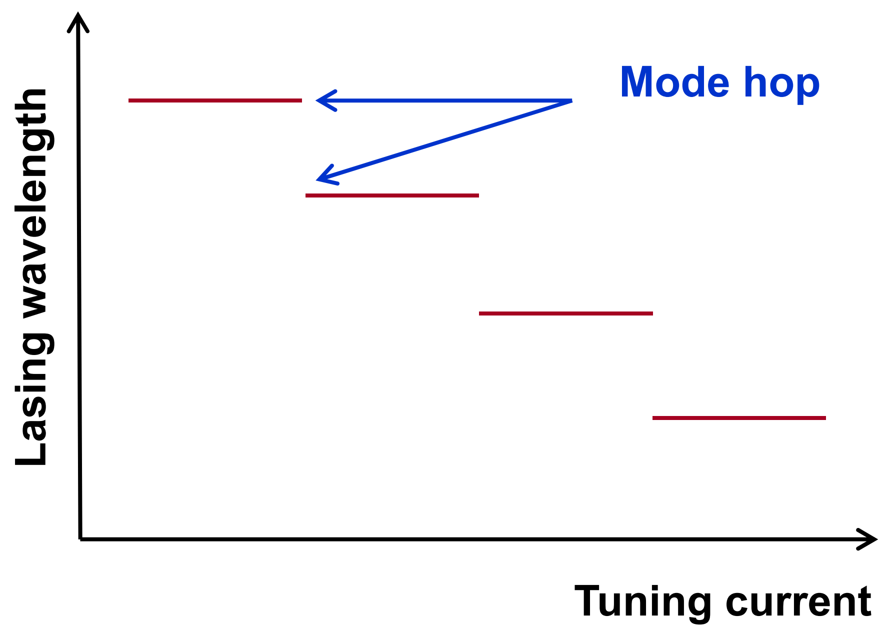
**In well designed device, temperature can be used together with carrier injection to extend the tuning range**

# Electrical Tuning of DBR Lasers

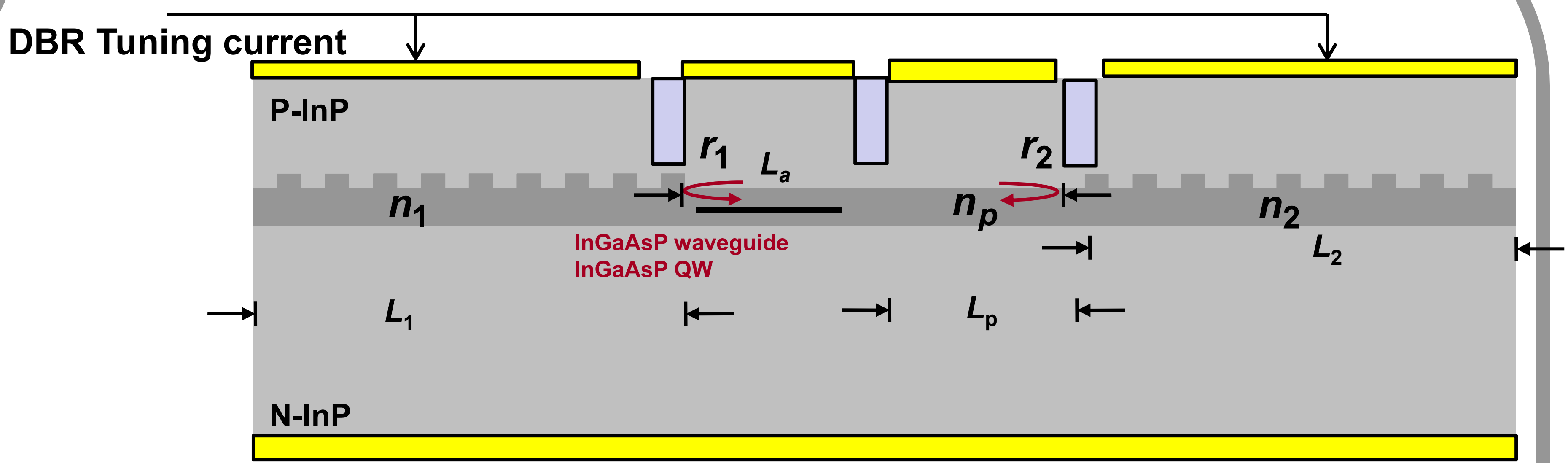



$$\frac{\Delta\omega_o}{\omega_o} = -\frac{\Gamma_1}{n_{1g}^M}\Delta n_1$$



# Continuous Electrical Tuning of DBR Lasers

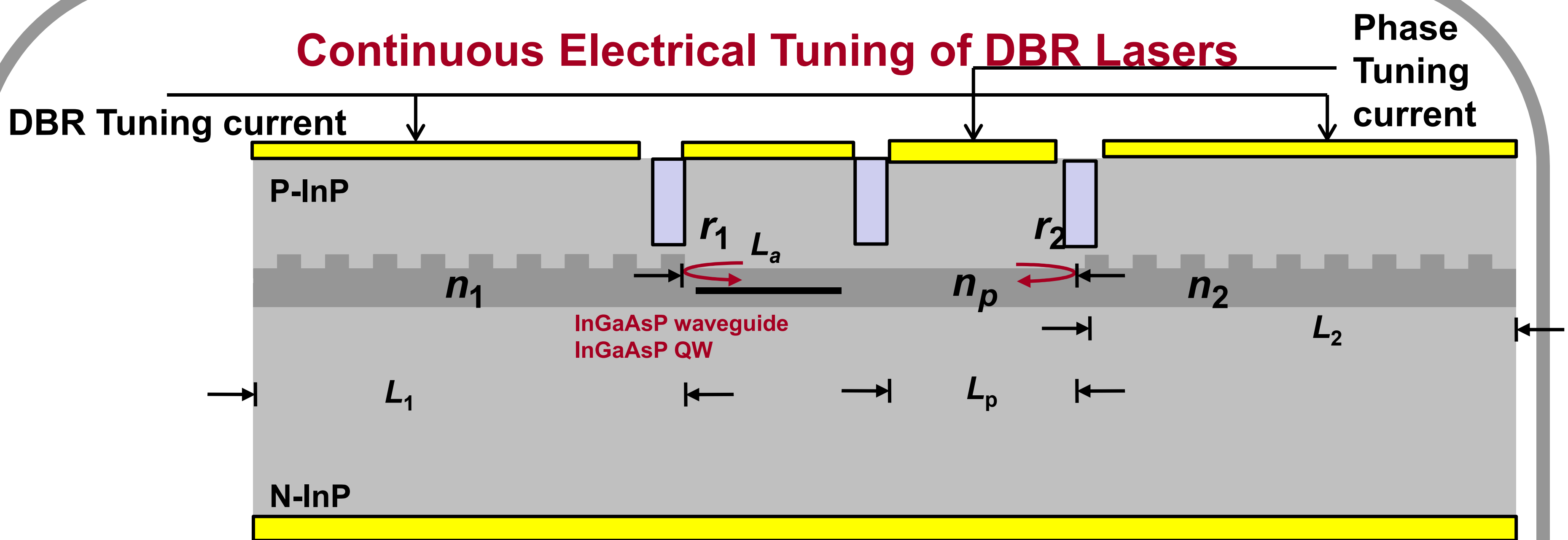



Laser cavity longitudinal modes are determined by the condition:

$$2\beta_a L_a + 2\beta_p L_p + \phi_1 + \phi_2 = 2\pi\ p \qquad \{\ p = \text{integer}$$

One can change the index of the core in the phase tuning section by injecting electrons and holes and thereby change the frequency of the cavity modes!

# Continuous Electrical Tuning of DBR Lasers




When the index in the phase tuning region is changed, the cavity mode frequency change is given by:

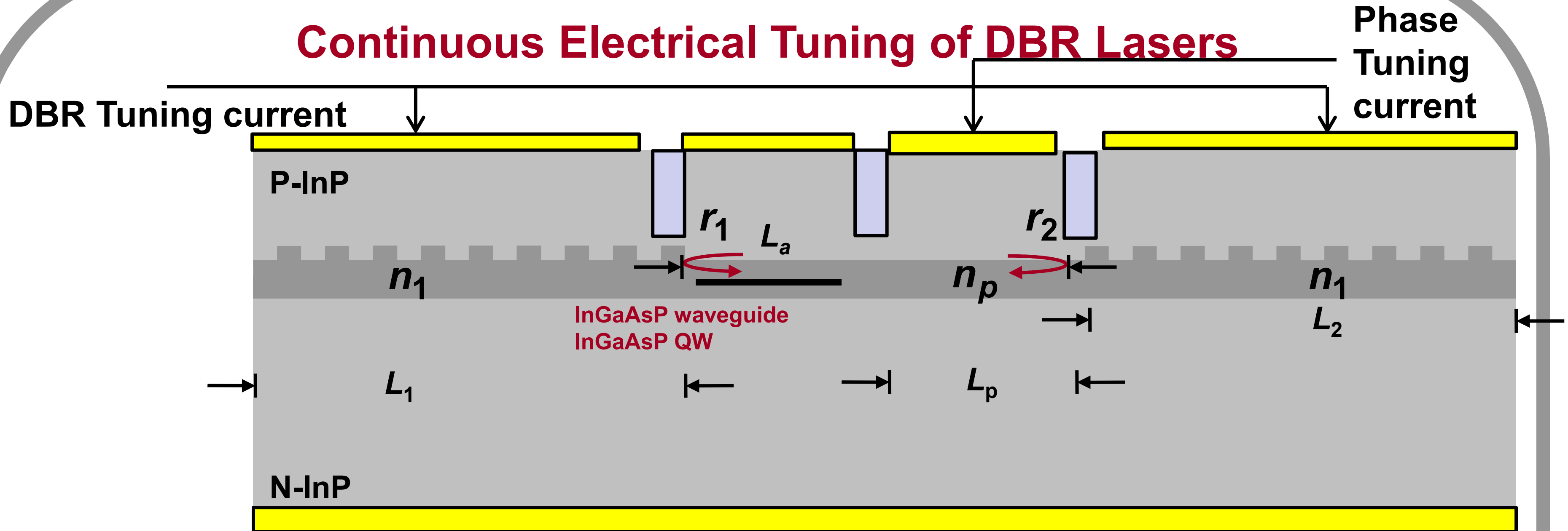
$$2\Delta\beta_a L_a + 2\Delta\beta_p L_p + \frac{\partial\phi_1}{\partial\beta_1}\Delta\beta_1 + \frac{\partial\phi_2}{\partial\beta_1}\Delta\beta_1 = 0$$

$$\Rightarrow 2\frac{\Delta\beta_a}{\Delta\omega}\Delta\omega L_a + 2\left[\frac{\Delta\beta_p}{\Delta\omega}\Delta\omega + \frac{\omega}{c}\Gamma_p\frac{n_{pg}}{n_{pg}^M}\Delta n_p\right]L_p + \frac{\partial\phi_1}{\partial\beta_1}\frac{\Delta\beta_1}{\Delta\omega}\Delta\omega + \frac{\partial\phi_2}{\partial\beta_1}\frac{\Delta\beta_1}{\Delta\omega}\Delta\omega = 0$$

$$\Rightarrow \left(n_{ag}L_a + n_{1g}L_{eff1} + n_{2g}L_{eff2} + n_{pg}L_p\right)\Delta\omega = -\frac{\omega}{c}\Gamma_p\frac{n_{pg}}{n_{pg}^M}\Delta n_p L_p$$

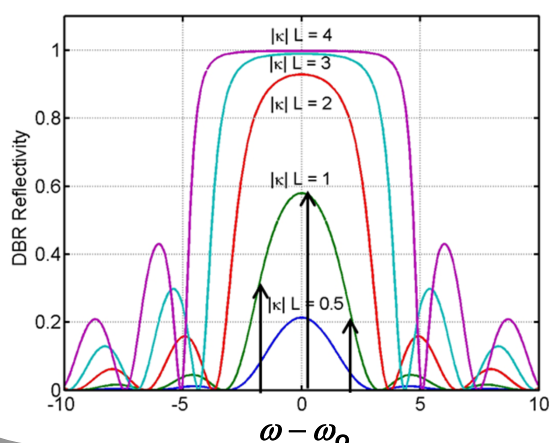
$$\Rightarrow \frac{\Delta\omega}{\omega} = -\frac{\Gamma_p}{n_{pg}^M}\Delta n_p\frac{n_{pg}L_p}{n_{ag}L_a + n_{1g}L_{eff1} + n_{2g}L_{eff2} + n_{pg}L_p} = -\frac{\Gamma_{\ell p}\Gamma_p}{n_{pg}^M}\Delta n_p$$

# Continuous Electrical Tuning of DBR Lasers



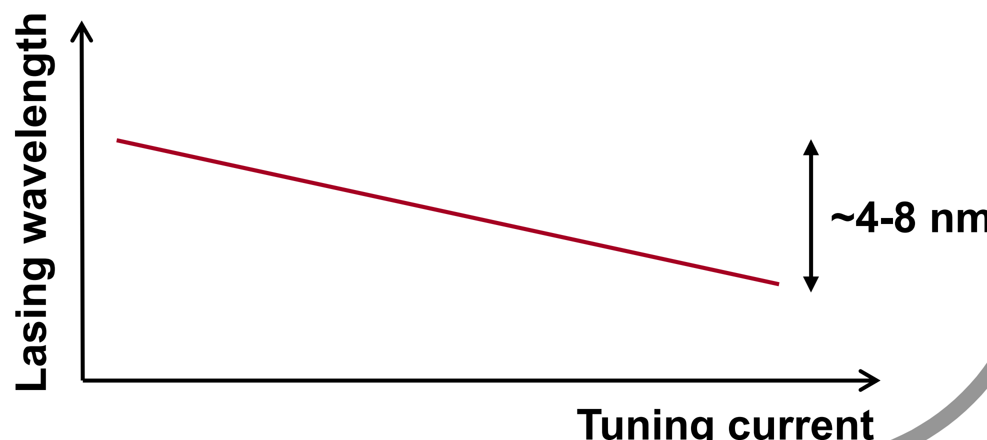

## Cavity Mode Tuning:

$$\frac{\Delta\omega}{\omega} = -\frac{\Gamma_{\ell p}\Gamma_p}{n_{pg}^M}\Delta n_p$$

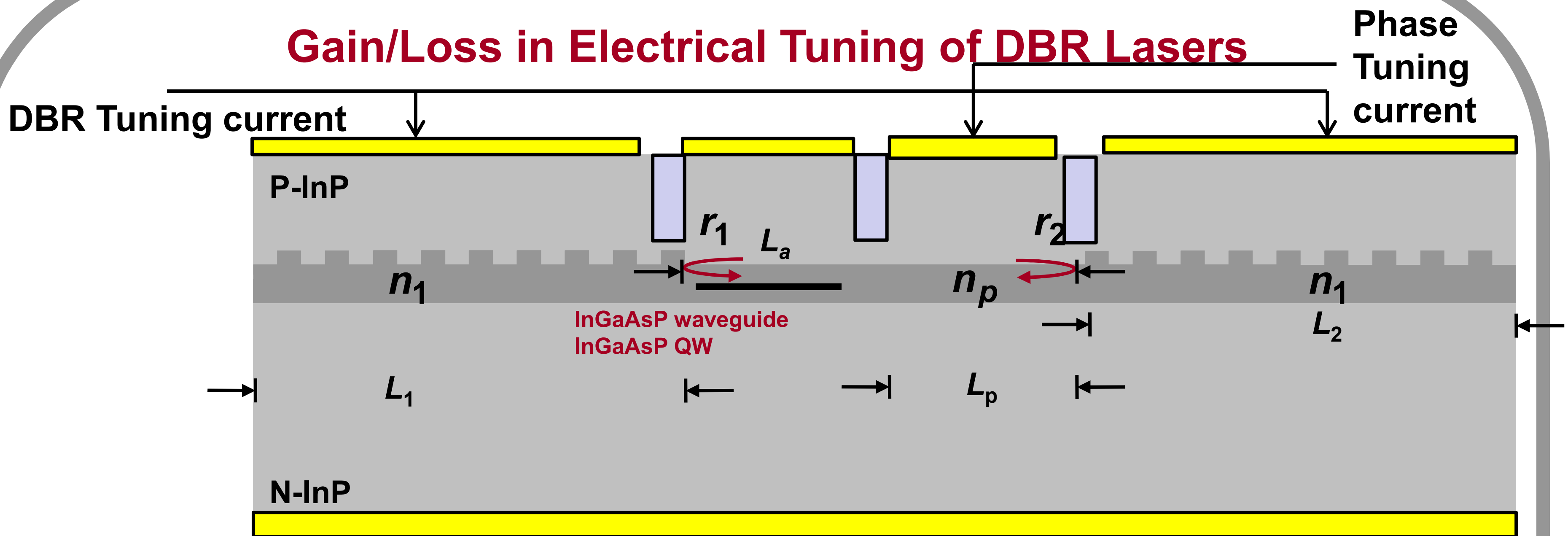



## DBR Tuning:

$$\frac{\Delta\omega_o}{\omega_o} = -\frac{\Gamma_1}{n_{1g}^M}\Delta n_1$$



# Gain/Loss in Electrical Tuning of DBR Lasers




**Cavity Mode Tuning:**

$$\frac{\Delta\omega}{\omega} = -\frac{\Gamma_{\ell p}\Gamma_p}{n_{pg}^M}\Delta n_p$$ ⟶ **Not entirely correct!**

**Tuning introduces extra loss in the phase tuning section that must be compensated by the increase in gain in the gain region:**

$$\Gamma_a \Delta g_a L_a = \Gamma_p \Delta\alpha_p L_p$$

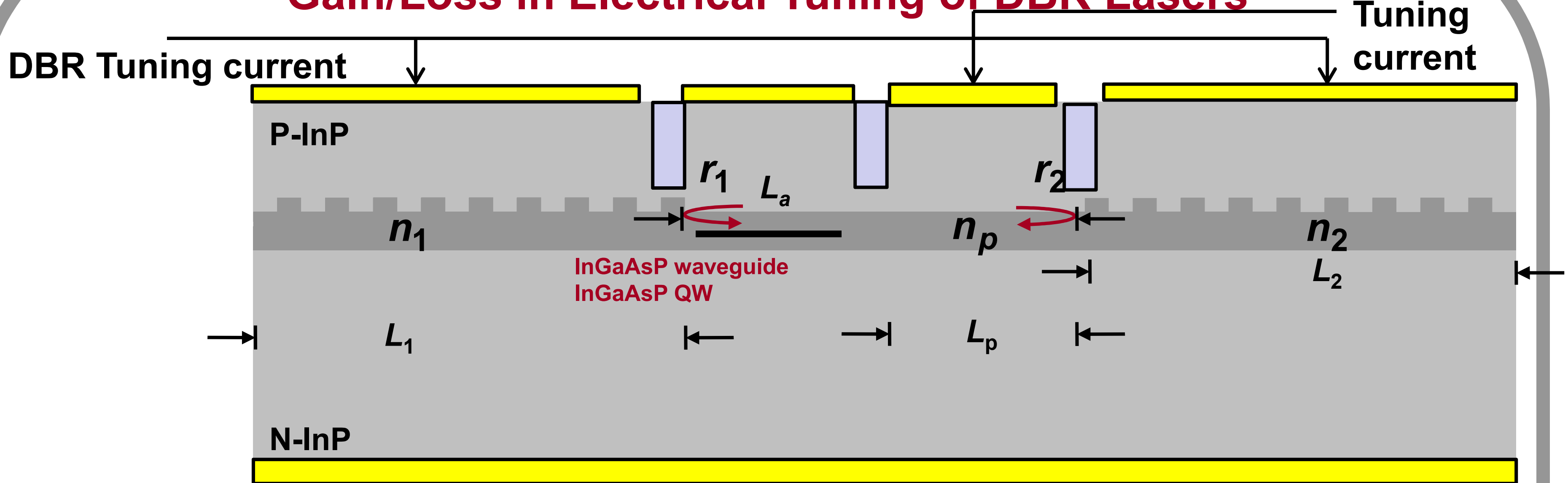
$$\Rightarrow \frac{\Gamma_a \Delta n_a L_a}{\alpha_{aH}} = -\Gamma_c \frac{\Delta n_p}{\alpha_{pH}} L_p$$

$$\Rightarrow \Gamma_a \Delta n_a L_a = -\frac{\alpha_{aH}}{\alpha_{pH}}\Gamma_p \Delta n_p L_p$$

**Recall the $\alpha$-parameter:**

$$\alpha_H = \frac{dn'/dN}{dn''/dN} = -\frac{4\pi}{\lambda}\frac{dn'/dN}{dg/dN} = \frac{4\pi}{\lambda}\frac{dn'/dN}{d\alpha/dN}$$

# Gain/Loss in Electrical Tuning of DBR Lasers




When the index in the phase tuning region is changed, the cavity mode frequency change is given by:

$$2\Delta\beta_a L_a + 2\Delta\beta_p L_p + \frac{\partial\phi_1}{\partial\beta_1}\Delta\beta_1 + \frac{\partial\phi_2}{\partial\beta_2}\Delta\beta_2 = 0$$

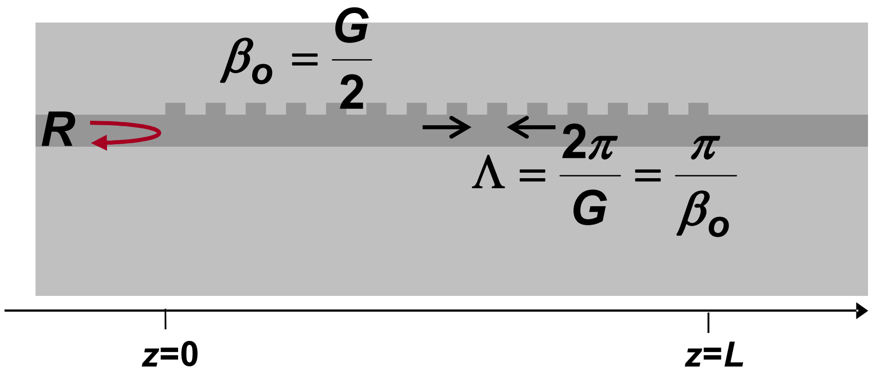
$$\Rightarrow 2\left[\frac{\Delta\beta_a}{\Delta\omega}\Delta\omega + \frac{\omega}{c}\Gamma_a\frac{n_{ag}}{n_{ag}^M}\Delta n_a\right]L_a + 2\left[\frac{\Delta\beta_p}{\Delta\omega}\Delta\omega + \frac{\omega}{c}\Gamma_p\frac{n_{pg}}{n_{pg}^M}\Delta n_p\right]L_p + \frac{\partial\phi_1}{\partial\beta_1}\frac{\Delta\beta_1}{\Delta\omega}\Delta\omega + \frac{\partial\phi_2}{\partial\beta_2}\frac{\Delta\beta_2}{\Delta\omega}\Delta\omega = 0$$

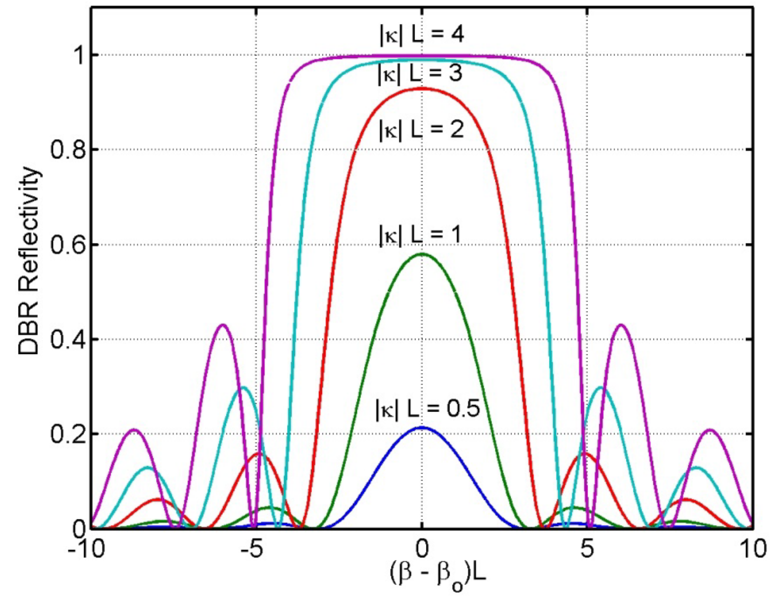
$$\frac{\Delta\omega}{\omega} = -\left(1 - \frac{\alpha_{aH}}{\alpha_{pH}}\right)\frac{\Gamma_{\ell p}\Gamma_p}{n_{pg}^M}\Delta n_p$$

$$\left\{ \frac{n_{ag}}{n_{ag}^M} \approx \frac{n_{pg}}{n_{pg}^M} \right.$$

Also, loss and gain regions have opposite signs for the $\alpha$-parameter

# Distributed Bragg Reflectors (Side Note)




$$\Delta\varepsilon(x,y,z) = f(x,y)\int_{-\infty}^{\infty}\frac{dq}{2\pi}h(q)\,e^{iqz}$$

**For infinitely long grating:**

$$h(q) = 2\pi\, d_1\delta(q-G) + 2\pi\, d_{-1}\delta(q+G)$$

$$\kappa \approx \frac{\omega\, d_1}{2nn_g^M v_g}\Gamma_G$$

When $\beta = \beta_o$ :

$$R\left(\beta = \beta_o = \frac{G}{2}\right) = \tanh^2(|\kappa|L)$$

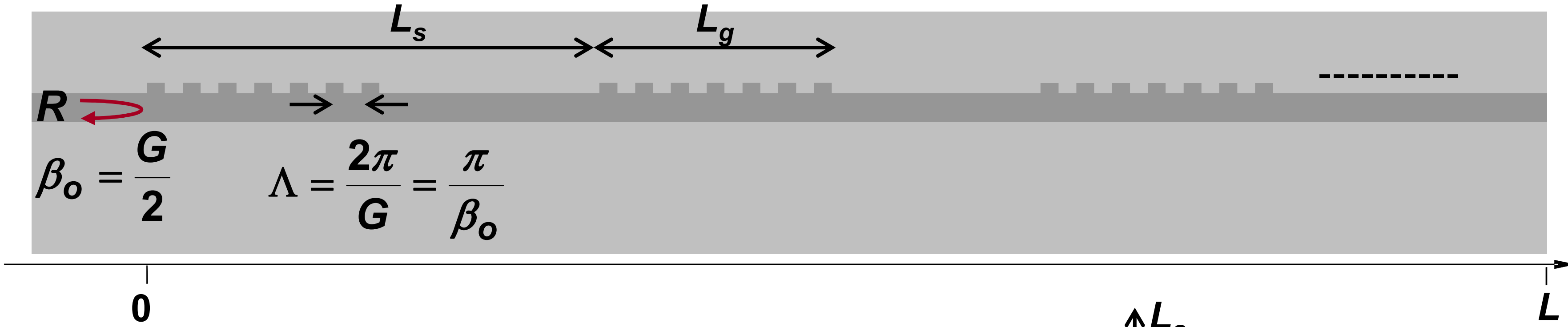
**For grating of length *L*:**

$$h(q) = d_1 L\frac{\sin\left((q-G)\frac{L}{2}\right)}{(q-G)\frac{L}{2}} + d_{-1}L\frac{\sin\left((q+G)\frac{L}{2}\right)}{(q+G)\frac{L}{2}}$$

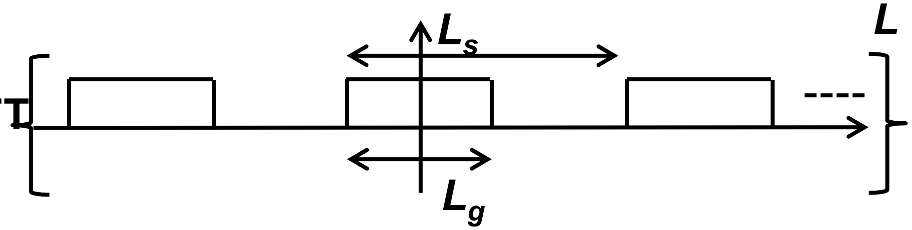
$$\kappa(\beta)L \approx \frac{\Gamma_G\omega(\beta)}{2nn_g^M v_g}h(q=2\beta) = \kappa(\beta_o)L\frac{\sin[(\beta-\beta_o)L]}{(\beta-\beta_o)L}$$

$$\Rightarrow R(\beta) = \tanh^2(|\kappa(\beta)|L)$$

# Sampled DBR Reflectors




$$h(q) = [2\pi\ d_1 \delta(q-G) + 2\pi\ d_{-1}\delta(q+G)] \otimes \mathrm{FT}$$

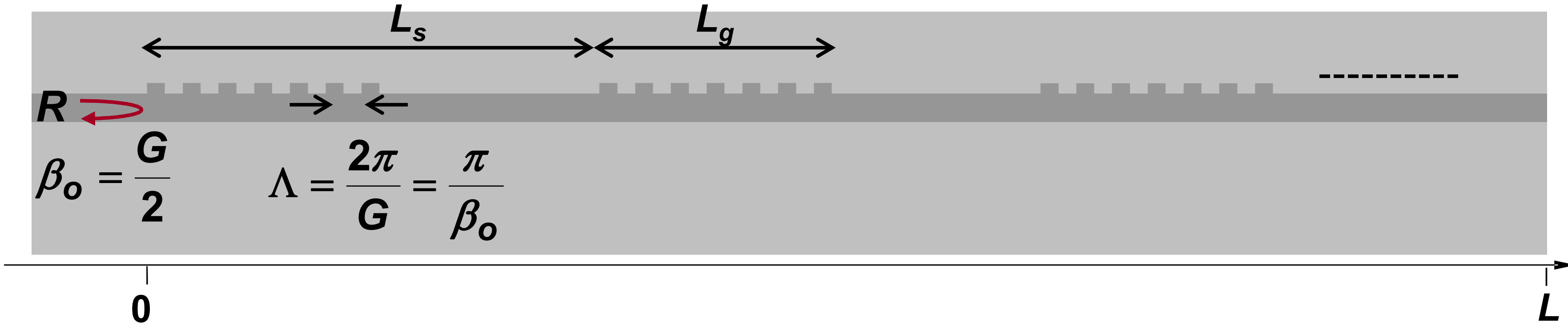



$$\otimes\ L \frac{\sin\left(q\frac{L}{2}\right)}{q\frac{L}{2}}$$

$$h(q) \approx d_1 L\left(\frac{L_g}{L_s}\right)\sum_{p=-\infty}^{\infty} \frac{\sin\left(\pi p\frac{L_g}{L_s}\right)}{\pi p\frac{L_g}{L_s}} \frac{\sin\left(\left(q-G-2\pi\frac{p}{L_s}\right)\frac{L}{2}\right)}{\left(q-G-2\pi\frac{p}{L_s}\right)\frac{L}{2}}$$

$$+ d_{-1} L\left(\frac{L_g}{L_s}\right)\sum_{p=-\infty}^{\infty} \frac{\sin\left(\pi p\frac{L_g}{L_s}\right)}{\pi p\frac{L_g}{L_s}} \frac{\sin\left(\left(q+G-2\pi\frac{p}{L_s}\right)\frac{L}{2}\right)}{\left(q+G-2\pi\frac{p}{L_s}\right)\frac{L}{2}}$$

# Sampled DBR Reflectors

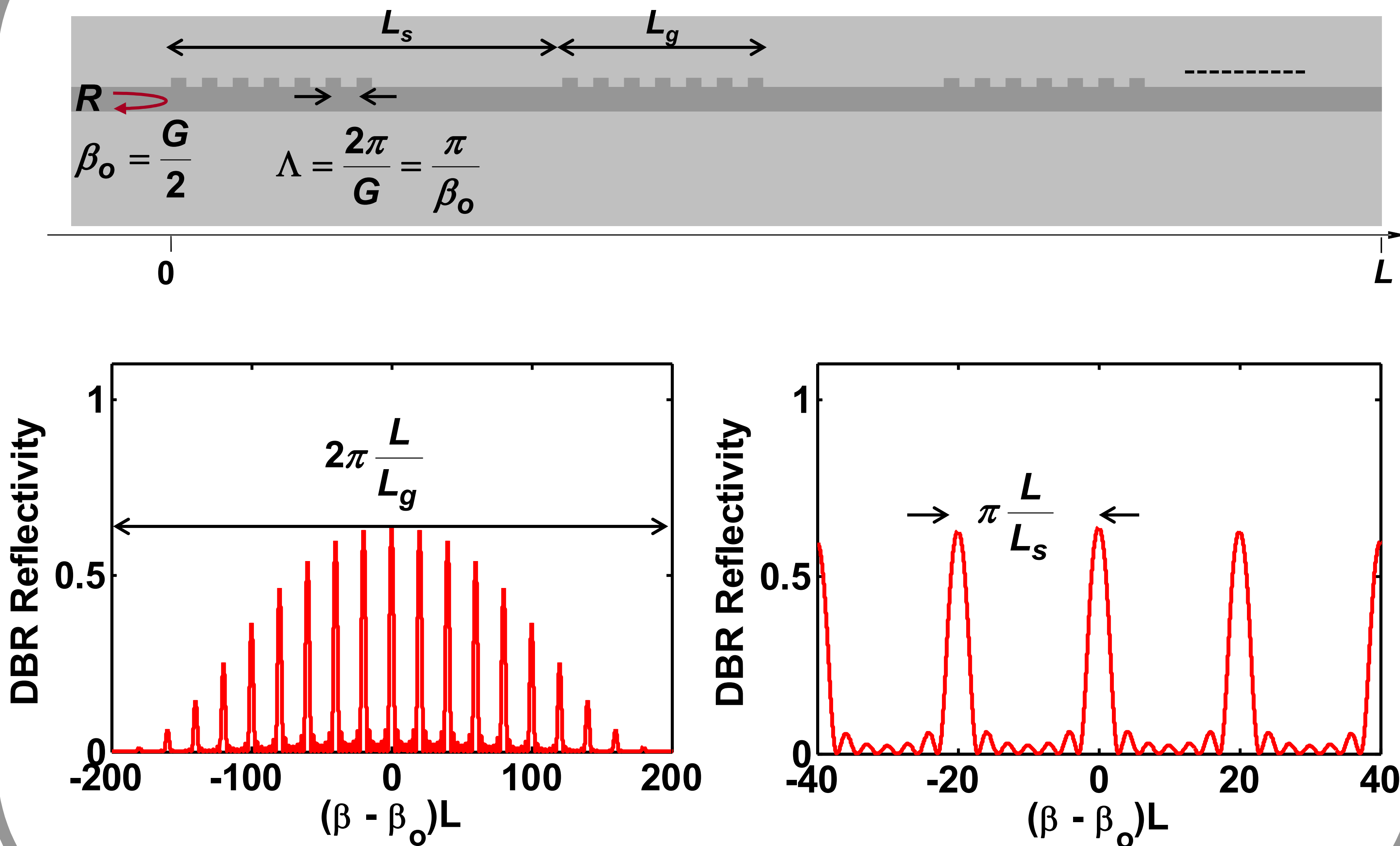



$$\kappa(\beta)L \approx \frac{\Gamma_G \omega(\beta)}{2nn_g^M v_g} h(q = 2\beta)$$

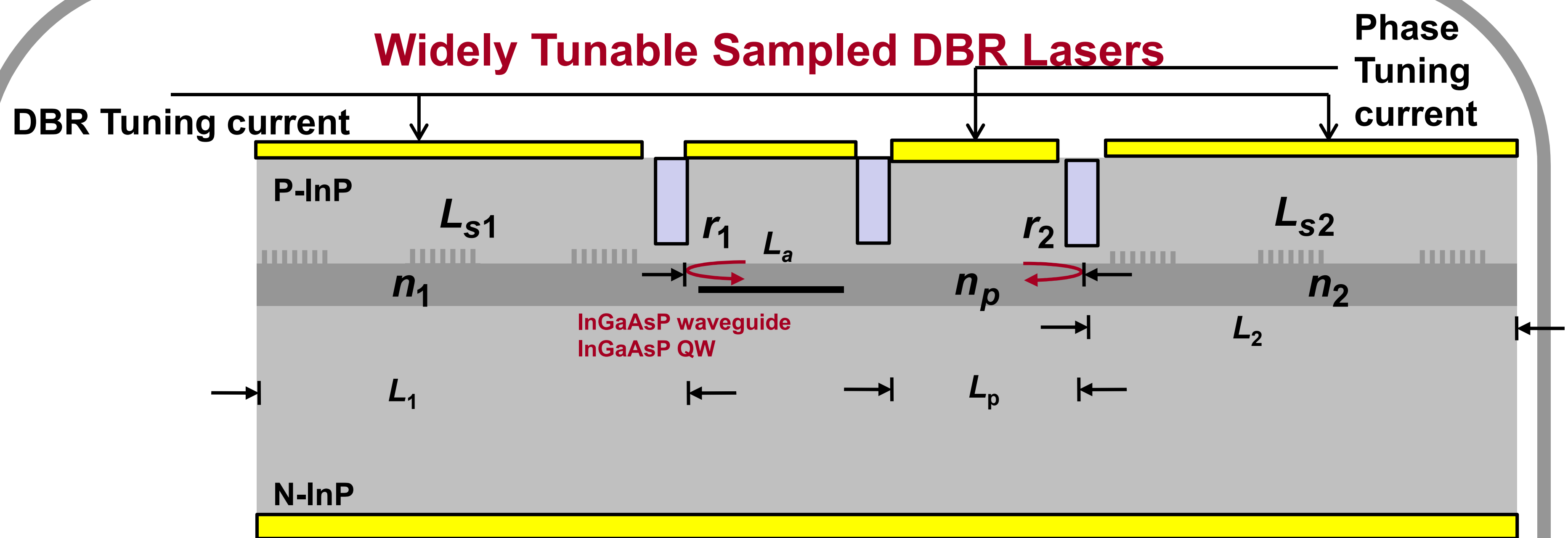
$$\kappa(\beta)L \approx \frac{\Gamma_G \omega(\beta)}{2nn_g^M v_g} d_1 L\left(\frac{L_g}{L_s}\right)\sum_{p=-\infty}^{\infty} \frac{\sin\left(\pi p \frac{L_g}{L_s}\right)}{\pi p \frac{L_g}{L_s}} \frac{\sin\left(\left(\beta - \beta_o - \pi \frac{p}{L_s}\right)L\right)}{\left(\beta - \beta_o - \pi \frac{p}{L_s}\right)L}$$

$$\Rightarrow R(\beta) = \tanh^2\left(|\kappa(\beta)|L\right)$$

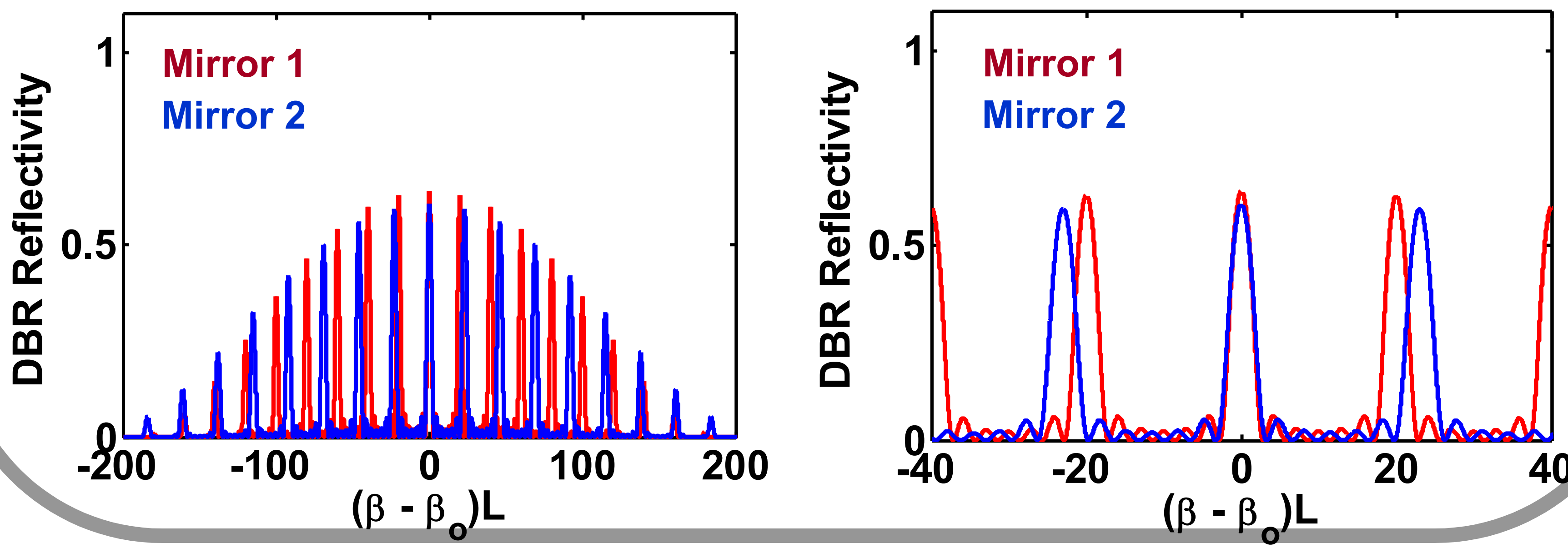
# Sampled DBR Reflectors



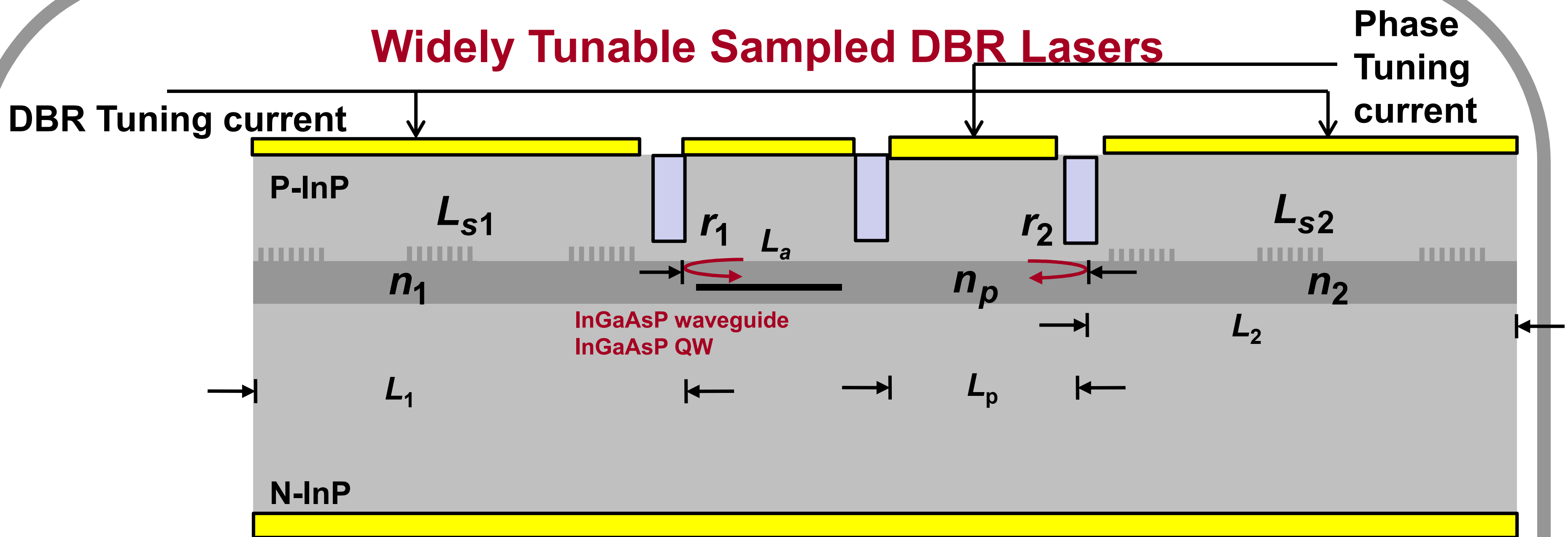
# Widely Tunable Sampled DBR Lasers




Use the Vernier effect with the reflectivity combs of the two mirrors:



# Widely Tunable Sampled DBR Lasers




**Maximum tuning range limited by the Vernier beat period and by the reflectivity envelope:**

$$\Delta\beta_{beat} = \frac{\pi}{|L_{s1} - L_{s2}|}$$

$$\Delta\beta_{env} \sim \frac{\pi}{L_g}$$

$\longrightarrow$

**Choose:**

$$L_g \approx |L_{s1} - L_{s2}|$$

**to get the widest tuning range given by:**

$$\Delta\omega = \frac{c}{n_{1g}} \frac{\pi}{|L_{s1} - L_{s2}|}$$

$$\Delta\lambda = \frac{\lambda^2}{2n_{1g}|L_{s1} - L_{s2}|}$$

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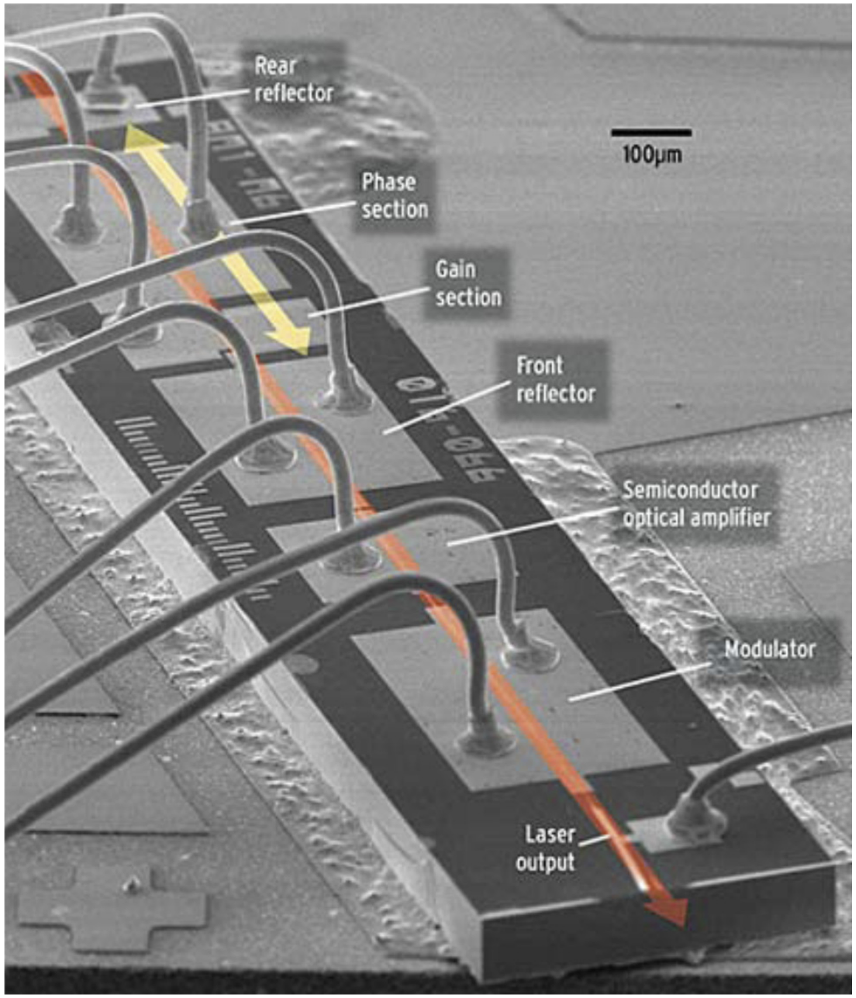
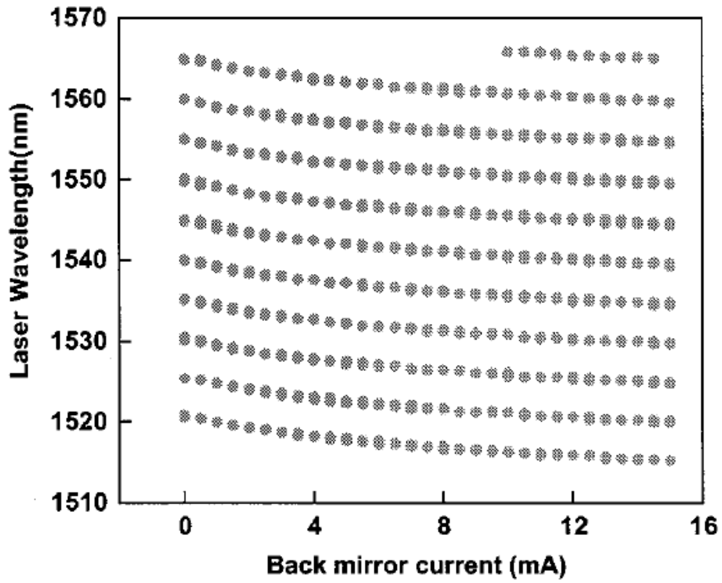
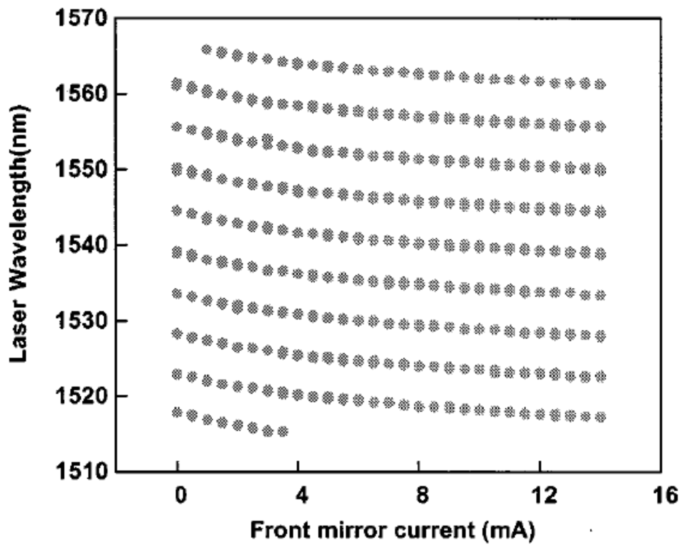
# Widely Tunable Sampled DBR Lasers (> 50 nm Tuning Range)

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# The Last Slide

That's All Folks!