ECE 4960 Spring 2017: Computational and Software Engineering
Homework 2: Differentiation in Local Analysis
Due 2/10 after class

Document your programming environment: Language; development platform; operating system

Prob. 1. (Quadratic function): For $f(x) = x^2$, we know the exact $f'(x=1) = 2$.

1.1 Use Eq. (1) below to estimate $f'(x=1)$ varying the value of h from 0.1 to 10^{-18} to observe the relative error in calculating $f'(x)$. Tabulate your results with sufficient precision in a table.
1.2 Repeat your calculation with $f(x) = x^2 + 10^8$. Add your results to the same table.
1.3 Repeat the above two procedure by using Eq. (2). Add your results to the same table.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + O(h) \qquad (1)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2) \qquad (2)$$

h	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2 + 10^8$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2 + 10^8$
10^{-1}	**0.1**	**0.1**	**Close to 10^{-16}**	**Close to 10^{-8}**
10^{-2}	**0.01**	**Around 0.01**	**Close to 10^{-15}**	**Close to 10^{-7}**
10^{-3}	**0.001**	**Around 0.001**	**Close to 10^{-14}**	**Close to 10^{-6}**
…	**Decreasing and starting to increase around 10^{-8}**	**Decreasing and starting to increase around 10^{-4}**	**Increasing always due to precision pollution**	**Increasing always due to precision pollution**
10^{-17}	**2.0 or NaN**	**2.0 or NaN**	**2.0 or NaN**	**2.0 or NaN**
10^{-18}	**2.0 or NaN**	**2.0 or NaN**	**2.0 or NaN**	**2.0 or NaN**

Prob. 2. (Cubic function): For $f(x) = x^3$, we know the exact $f'(x=1) = 3$.

2.1 Use Eqs. (3) – (5) below to estimate $f'(x=1)$ varying the value of h from 2^{-4} to 2^{-20} to observe the relative error in calculating $f'(x)$. Tabulate your results with sufficient precision in a table.
2.2 Estimate η from Eqs. (6) and (7) for each choice of h. Add your results to the same table.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + E(h); \qquad E(h) = O(h) = \frac{1}{2} h f''(x) + O(h^2) \qquad (3)$$

$$f'(x) = \frac{f(x+2h) - f(x)}{2h} + E(2h); \qquad E(2h) = O(h) = \frac{1}{2} 2h f''(x) + O(h^2) \qquad (4)$$

$$f'(x) = \frac{-1}{2h} f(x+2h) - \frac{3}{2h} f(x) + \frac{2}{h} f(x+h) + O(h^2) \qquad (5)$$

$$R(h) \equiv \frac{E(2h)}{E(h)} \cong \eta \qquad (6)$$

$$R(h) \cong \frac{\hat{A}(4h) - \hat{A}(2h)}{\hat{A}(2h) - \hat{A}(h)} \cong \eta \qquad (7)$$

Note that I change the decrease rate of *h* to show larger range here. *4h, 2h* and *h* are still evaluated for each row below.

h	Error in $f'(x=1)$ by Eq. (3)	Error in $f'(x=1)$ by Eq. (4)	Error in $f'(x=1)$ by Eq. (5)	η by Eq. (6)	η by Eq. (7)
10^{-4}	**6×10^{-4}**	**1.2×10^{-3}**	**8.0×10^{-8}**	**2.0**	**2.0**
10^{-5}	**6×10^{-5}**	**1.2×10^{-4}**	**8.0×10^{-10}**	**2.0**	**2.0**
10^{-6}	**6×10^{-6}**	**1.2×10^{-5}**	**$\sim 2.0\times10^{-3}$**	**2.0**	**2.0**
…	**Decreasing until around 10^{-8} then start increasing due to loss in precision**	**Decreasing until around 10^{-8} then start increasing due to loss in precision**	**Start increasing due to loss in precision**	**Start deviating from 2.0 around 10^{-8}**	**Start deviating from 2.0 around 10^{-8}**
10^{-19}	**3.0 or NaN**	**3.0 or NaN**	**3.0 or NaN**	**1.0**	**1.0 or NaN**
10^{-20}	**3.0 or NaN**	**3.0 or NaN**	**3.0 or NaN**	**1.0**	**1.0 or NaN**

This table is just for illustration of the precision behavior. The answer will be precision, implementation and platform dependent!