

**ECE 4960 Spring 2017: Computational and Software Engineering**  
**Homework 2: Differentiation in Local Analysis**  
 Due 2/10 after class

**Document your programming environment: Language; development platform; operating system**

**Prob. 1. (Quadratic function):** For  $f(x) = x^2$ , we know the exact  $f'(x=1) = 2$ .

- 1.1 Use Eq. (1) below to estimate  $f'(x=1)$  varying the value of  $h$  from 0.1 to  $10^{-18}$  to observe the relative error in calculating  $f'(x)$ . Tabulate your results with sufficient precision in a table.
- 1.2 Repeat your calculation with  $f(x) = x^2 + 10^8$ . Add your results to the same table.
- 1.3 Repeat the above two procedure by using Eq. (2). Add your results to the same table.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + O(h) \quad (1)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2) \quad (2)$$

$h$	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2 + 10^8$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2 + 10^8$
$10^{-1}$				
$10^{-2}$				
$10^{-3}$				
...				
$10^{-18}$				

**Prob. 2. (Cubic function):** For  $f(x) = x^3$ , we know the exact  $f'(x=1) = 3$ .

- 2.1 Use Eqs. (3) – (5) below to estimate  $f'(x=1)$  varying the value of  $h$  from  $2^{-4}$  to  $2^{-20}$  to observe the relative error in calculating  $f'(x)$ . Tabulate your results with sufficient precision in a table.
- 2.2 Estimate  $\eta$  from Eqs. (6) and (7) for each choice of  $h$ . Add your results to the same table.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + E(h); \quad E(h) = O(h) = \frac{1}{2}hf''(x) + O(h^2) \quad (3)$$

$$f'(x) = \frac{f(x+2h) - f(x)}{2h} + E(2h); \quad E(2h) = O(h) = \frac{1}{2}2hf''(x) + O(h^2) \quad (4)$$

$$f'(x) = \frac{-1}{2h}f(x+2h) - \frac{3}{2h}f(x) + \frac{2}{h}f(x+h) + O(h^2) \quad (5)$$

$$R(h) \equiv \frac{E(2h)}{E(h)} \cong \eta \quad (6)$$

$$R(h) \cong \frac{\hat{A}(4h) - \hat{A}(2h)}{\hat{A}(2h) - \hat{A}(h)} \cong \eta \quad (7)$$

$h$	Error in $f'(x=1)$ by Eq. (3)	Error in $f'(x=1)$ by Eq. (4)	Error in $f'(x=1)$ by Eq. (5)	$\eta$ by Eq. (6)	$\eta$ by Eq. (7)
$2^{-4}$					
$2^{-5}$					
$2^{-6}$					
...					
$2^{-20}$					