ECE 4880 RF Systems Fall 2016
Prelim 2 Exam Solution

Load reflection coefficient Γ_L at the load as: $\Gamma_L = \dfrac{V_-}{V_+} = \dfrac{Z_L/Z_o - 1}{Z_L/Z_o + 1}$

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \frac{1}{S_{21}} \begin{bmatrix} 1 & -S_{22} \\ S_{11} & S_{12}S_{21} - S_{11}S_{22} \end{bmatrix} \qquad \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{T_{11}} \begin{bmatrix} T_{21} & T_{11}T_{22} - T_{12}T_{21} \\ 1 & -T_{12} \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{Y_0}{(1+S_{11})(1+S_{22}) - S_{12}S_{21}} \begin{bmatrix} (1-S_{11})(1+S_{22}) + S_{12}S_{21} & -2S_{12} \\ -2S_{21} & (1+S_{11})(1-S_{22}) + S_{12}S_{21} \end{bmatrix}$$

Thermal noise at room temperature: –174dBm + 10log$_{10}$(BW/1Hz)

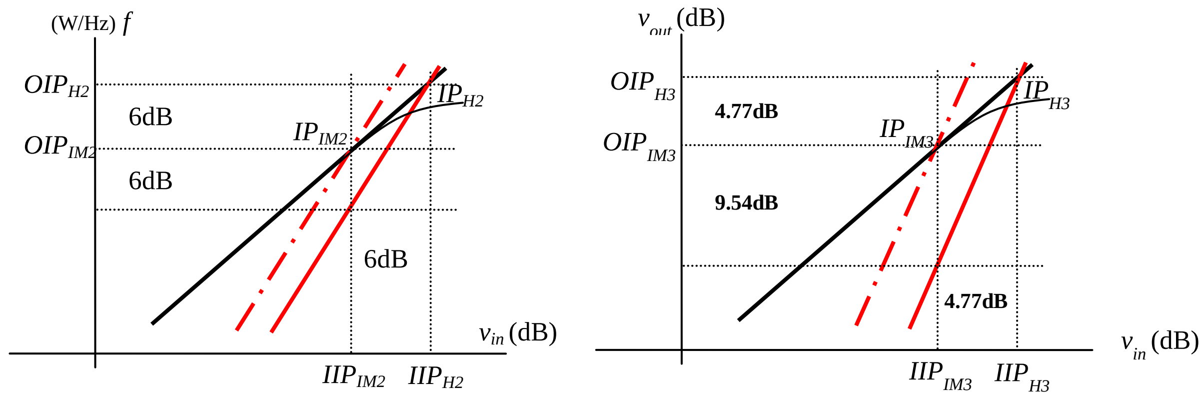


Fig. 1. Use of intercept points to represent nonlinearity from the quadratic and cubic terms.

$I_{1dBcomp} = IIP_{IM3}$ – 9.64dB when only IM3 is dominant.

Nonlinear Taylor coefficients: $A_{IIPIM\,2} = \left|\dfrac{a_1}{a_2}\right|$; $A^2_{IIPIM\,3} = \dfrac{4}{3}\left|\dfrac{a_1}{a_3}\right|$

$$\frac{1}{OIP_{IM\,3,cas}} = \frac{1}{g_2 \cdot OIP_{IM\,3,1}} + \frac{1}{OIP_{IM\,3,2}}$$ (IM3 adding coherently)

The noise factor of the cascade: $f_{cas} = f_1 + \sum_{k=2}^{N} \dfrac{f_k - 1}{\prod_{i=1}^{k-1} g_i}$;

Instantaneous spur-free dynamic range: $ISFDR = P_{in\max} - P_{in\min} = \dfrac{2}{3}(IIP3 - P_{in\min})$

Desensitization signal level in V for two-tone signals: $B = \sqrt{\dfrac{1}{3}\left|\dfrac{a_1}{a_3}\right|} = \dfrac{1}{2} A_{IIPM\,3}$

Name: ______________________ NetID: __________________________

1. (**1dB gain compression, 6 pts**) To obtain an estimate of the input power at the 1dB gain compression point $I_{1dBcomp}$ of an RF amplifer, circle all statements that are true. There can be one or more correct answers. (**6 pts**) No explanation necessary.
   (A) If we know IIP_{H2}, then $I_{1dBcomp}$ can be uniquely determined.
   (B) If we know *ISFDR* (instantaneous spur-free dynamic range), then $I_{1dBcomp}$ can be uniquely determined.
   (C) $I_{1dBcomp}$ can be obtained from a single-tone carrier input testing.
   (D) $I_{1dBcomp}$ is well defined only if the dominant nonlinear term in the VTC Taylor expansion has a negative coefficient (such as a_2 or a_3).
   (E) $I_{1dBcomp}$ is an intrinsic property for the amplifier and will not depend on the testing frequency.
   (F) $I_{1dBcomp}$ can be extracted from the S parameter measurements under constant input power and sweeping frequency.

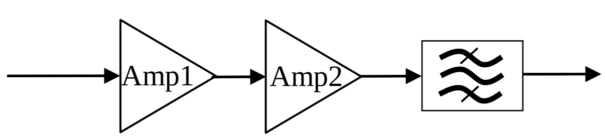
Answer: (C), with or without (D). For (D), if you assume the linear gain is negative, then a_2 or a_3 needs to be positive. The most correct statement should just be the linear coefficient and the dominant VTC Taylor expansion coefficient should be of opposite sign. (A) is incorrect as 2nd harmonic is not in band, and we also need information of the dominance of nonlinearity. (B) is incorrect as we need to know whether the third harmonic is dominant, and the noise behavior of the entire receiver. (E) is incorrect, as $I_{1dBcomp}$, like many other RF parameters, is only constant for a bandwidth. (F) is incorrect as we need to sweep power at a constant frequency.

2. (**Signal cascade, 44 pts**) To achieve a desirable RF gain, we often have to cascade modules in a signal chain of the RF front end. Three components are available for the purpose of the RF frond end of an Wi-Fi receiver (2.4GHz – 2.5GHz), all with good input and output matching at 50Ω:

Module	Gain (dB)	*NF* (dB)	IIP_{H2} (dBm)	IIP_{H3} (dBm)	Bandwidth (GHz)
Amp1	15	2	40	30	2.3 – 2.6
Amp2	Variable: –20 to 10	10	60	15	1.9 – 3.0
Band-pass filter (BPF)	In-band: –1 Out-of-band: –40	5	80	80	2.35 – 2.55

a) Here you are designing to receive to the weakest input p_{in} = –100dBm with SNR_{in} = 10dB to extend the total operating range, and you will need a front-end with the least added noise. How will you arrange the three modules (all needed for the overall functions)? (**4 pts**) What are the noise figure and SNR_{out} of the entire block? (**4 pts**)

To minimize noise, you will put the module with the least NF and reasonable gain at the front:



$$f_{cas} = f_1 + \frac{f_2 - 1}{g_1} + \frac{f_3 - 1}{g_1 g_2} = 10^{2/10} + \frac{10^{10/10} - 1}{10^{15/10}} + \frac{10^{5/10} - 1}{10^{15/10} \cdot 10^{10/10}} = 1.87$$

$NF_{cas} = 10\log_{10}(f_{cas})$ = 2.72dB.

$SNR_{out} = SNR_{in} - NF_{cas} =$ 7.3dB.

b) For the same configuration in a) and Amp2 at its highest gain of 10dB, when p_{in} = –20dBm, estimate the power of the 3rd-order intermodulation p_{outIM3} in each stage. (**6 pts**) Hint: You do not need to deal with modules with very high IIP.

We assume that p_{in} has pure two tones for the intermodulation consideration.

Amp1 has IIP_{IM3} = 30dBm – 4.77dB = 25.23dBm. OIP_{IM3} = 40.23dBm.
Amp2 has IIP_{IM3} = 15dBm – 4.77dB = 10.23dBm. OIP_{IM3} = 20.23dBm.
BPF has IIP_{IM3} so high that we can regard it as a linear element.

After Amp1:
p_{in1} = –20dBm; p_{out1} = –5dBm; $p_{outIM3} = 3p_{out1} - 2(OIP_{IM3})$ = –95.46dBm.

After Amp2:
$p_{in2} = p_{out1}$ = –5dBm; p_{out2} = 5dBm; $p_{outIM3} = 3p_{out1} - 2(OIP_{IM3})$ = –25.46dBm. There is also a component from direct amplification of IM3 out of Amp1, but the power level can be estimated at –95.46dBm + 10dB = –85.46dBm. We can safely ignore it.

After BPF:
No nonlinearity needs to be considered for BPF. p_{outIM3} = –25.46dBm + (–1dB) = –26.46dBm to include in-band insertion loss assuming IM3 is still in-band.

c) To accommodate the full range of p_{in} from –100dBm to –20dBm, Amp2 is set to the lowest gain of –20dB from a feedback when p_{in} = –20dBm is detected. Estimate p_{outIM3} in each stage now. (**4 pts**) This is the only sub-question that Amp2 has a gain of –20dB for gain control.

For variable-gain amplifiers, IIP is most often independent of gain (remember that nonlinearity is coming from gain compression, often close to when v_{out} has a magnitude close to the external power supply).

After Amp1: the same as above.
p_{in1} = –20dBm; p_{out1} = –5dBm; p_{outIM3} = –95.46dBm.

After Amp2:
$p_{in2} = p_{out1}$ = –5dBm; p_{out2} = –25dBm; $p_{outIM3} = 3p_{out1} - 2(OIP_{IM3})$ = –115.5dBm. There is also a component from direct amplification of IM3 out of Amp1, but the power level can be estimated at:
–95.46dBm – 20dB = –115.5dBm. The two power combined to –112.5dBm.

After BPF:
No nonlinearity needs to be considered for BPF. p_{outIM3} = –112.5dBm + (–1dB) = –113.5dBm to include in-band insertion loss assuming IM3 is still in-band.

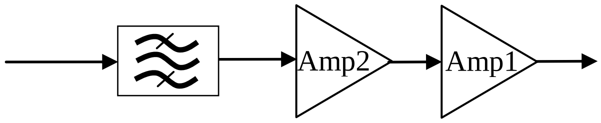
We can see that not only the signal is attenuated, IM3 and all other nonlinear effects are also decreased significantly. This shows the benefit of attenuating the RF end when the signal is too strong. This configuration can also improve the jamming or desensitization situation. Even though both signals and jammers will be attenuated together, the later stage will deal with a jammer with much smaller input power far away from IIP_{IM3}, and hence less nonlinearity.

Notice that the signal power is now degraded, and the attenuator has *NF* close to the attenuation (noise is about the same, but the signal is attenuated). For Amp2, the *NF* is close to 20dB now, and the system *NF* and SNR_{out} degrade significantly. However, since p_{in} and SNR_{in} is now much larger, we can often tolerate the system *NF* and SNR_{out} degradation.

This example explains that when VGA is in the signal chain to avoid system saturation, we do not need to pay special attention to the case when pin is large and the RF gain is reduced in the front end.

d) Here you are expecting a large interference signal at 2.6GHz in your ambient, but you still hope to provide the largest gain in the 2.4 – 2.5GHz band by setting Amp2 at 10dB gain. Due to the interference, IM3 consideration is more important than noise now. How will you arrange the three modules? Briefly justify your answers. (**4 pts**) What are the noise figure and SNR_{out} of the entire block now? (**6 pts**)

As 2.6GHz is in the band of the two amplifiers, the best strategy to avoid its influence is to put the BPF in front to reduce its desensitization effect. Also, when IM3 is the most important design consideration, we will need to put the low IIP_{IM3} module first, so that the second amplifier with a higher IIP_{IM3} will see a smaller input power.



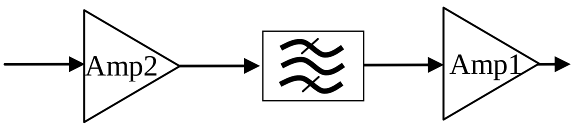
$$f_{cas} = f_1 + \frac{f_2 - 1}{g_1} + \frac{f_3 - 1}{g_1 g_2} = 10^{5/10} + \frac{10^{10/10} - 1}{10^{-1/10}} + \frac{10^{2/10} - 1}{10^{-1/10} \cdot 10^{10/10}} = 14.6$$

NF_{cas} = 11.6dB.
$SNR_{out} = SNR_{in} - NF_{cas}$ = –1.6dB.

We can see that to avoid the penalty of the low IIP (highly nonlinear) of Amp2, the noise performance also suffers.

e) Here you are expecting a large jamming signal at 2.45GHz in your ambient as you are close to a microwave oven. How will you arrange the three modules in order to avoid jamming in all 20MHz channels not overlapped with 2.45GHz? (**4 pts**) What are the IIP_{IM3} and ISFDR of the RF front end now? (**6 pts**) The weakest input to be heard remains at –100dBm.

Now this is in-band jamming, so the nonlinearity for the whole block is still most important. We can however switch the Amp2 to front to make the NF of the filter less critical.



We can prove this point: (not required)

$$f_{cas} = f_1 + \frac{f_2 - 1}{g_1} + \frac{f_3 - 1}{g_1 g_2} = 10^{10/10} + \frac{10^{5/10} - 1}{10^{10/10}} + \frac{10^{2/10} - 1}{10^{10/10} \cdot 10^{-1/10}} = 10.3$$

NF_{cas} = 10.1dB. (The difference from (e) is small due to the large *NF* in Amp2)

Now we can calculate IIP_{IM3} for the whole module. We will ignore the contribution of BPF, as it can be treated as a linear module.

Amp1 has IIP_{IM3} = 30dBm – 4.77dB = 25.23dBm. OIP_{IM3} = 40.23dBm.
Amp2 has IIP_{IM3} = 15dBm – 4.77dB = 10.23dBm. OIP_{IM3} = 20.23dBm.

$$OIP_{IM3,cas} = \left(\frac{1}{g_2 \cdot OIP_{IM3,1}} + \frac{1}{OIP_{IM3,2}} \right)^{-1} = \left(\frac{1}{10^{15/10} \cdot 10^{20.23/10}} + \frac{1}{10^{40.23/10}} \right)^{-1} = 2533mW$$

= 34.0 dBm.
$gain_{cas}$ = 10dB + 15dB – 1dB = 24dB
$IIP_{IM3,cas}$ = 10.0 dBm

Assuming that IM3 is dominant here:
$ISFDR = P_{in\max} - P_{in\min} = \frac{2}{3}(IIP3 - P_{in\min}) = \frac{2}{3}(10.0dBm - (-100dBm))$ = 73dBm. This is relatively high as P_{inmin} is very small.

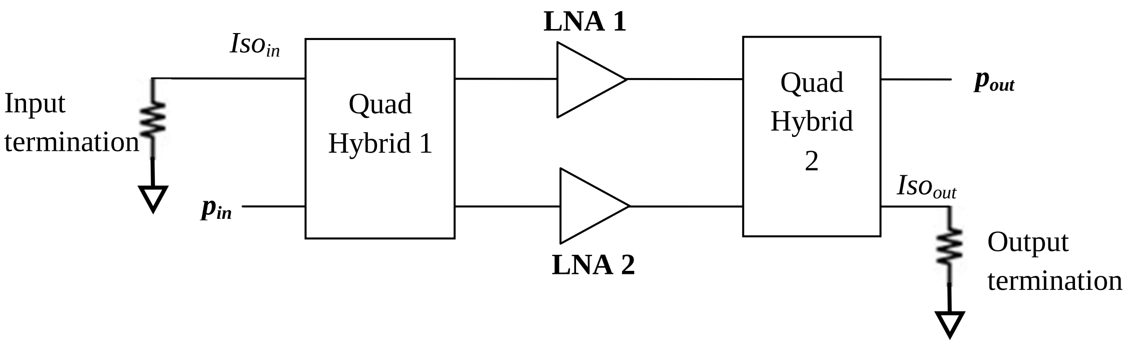
f) By using your configuration in e), what is the signal level in dBm of the 2.45GHz jamming signal that can desensitize any desirable signal within the Wi-Fi band? (**6 pts**) Use a simple estimate will be sufficient. No detailed calculation necessary.

$$B = \sqrt{\frac{1}{3}\left|\frac{a_1}{a_3}\right|} = \frac{1}{2} A_{IIPM3} = \frac{1}{2}\sqrt{2Z_0 \cdot 10^{9.0/10} \cdot 10^{-3}} = 0.45V$$

Or power of B for the jamming level: $p_B = \frac{\left(\frac{1}{2}\sqrt{2Z_0 \cdot p_{IIPIM3,cas}} \right)^2}{2Z_0} = \frac{p_{IIPIM3,cas}}{4}$

The jamming signal B thus have the power level at = 1.98mW = 2.98dBm, which is about 6dB lower than IIP_{IM3}. We can see this RF front end can be easily jammed due to the low IIP_{IM3} of Amp2.

3. (**Quad hybrid modules, 40 pts**) A quad hybrid architecture is used to implement an amplifier block out of two LNA. The overall system you design will have line impedance at 50Ω, and your quad hybrid unit is well matched to 50Ω. However, the individual LNA here has 75Ω Z_{in} and Z_{out}. The VTC of the individual LNA at matched input-output impedance conditions can be reasonably approximated by: $v_{out} = 20v_{in} - 2.5v_{in}^2 - 5v_{in}^3$ in volts and in a broad bandwidth.



a) By using a small testing p_{in} where the nonlinearity can be ignored, what is the S parameter matrix when the individual LNA is measured with a network analyzer with port impedance at 50Ω? (**6 pts**). Express your S parameters in dB.

From 50Ω to 75Ω, we have $\Gamma_L = \frac{V_-}{V_+} = \frac{Z_L/Z_o - 1}{Z_L/Z_o + 1} = 0.2 = -13.98dB$. The transmitted wave is then $T_L = \frac{2\sqrt{Z_L/Z_0}}{Z_L/Z_o + 1} = 0.98$. Check: $\Gamma_L^2 + T_L^2 = 1$. The transmitted wave is always 100%, and no point will be deduced if you do not include it.

From 75Ω to 50Ω, we have $T_L = \frac{2\sqrt{Z_L/Z_0}}{Z_L/Z_o + 1} = 0.98$ as well.

Linear voltage gain = 0.98×20×0.98 = 19.2 = 25.7dB.

By the definition of the unipolar amplifier, the S matrix with the parameter is:

$$\begin{bmatrix} 0.2 & 0 \\ 19.2 & 0.2 \end{bmatrix}$$

or in dB

$$\begin{bmatrix} -13.98 & -\infty \\ 25.7 & -13.98 \end{bmatrix}.$$

Notice that the negative infinite dB in S_{12} is often a very small number from actual measurement, limited by noise or reverse leakage.

b) Following a), when the quad-hybrid module is used with the small testing p_{in}, what is the expected S parameter matrix now? (**6 pts**) Express your S parameters in dB.

Notice that as long as LNA1 and LNA2 match well, then the splitting signals are cancelled at p_{in} and p_{out}. The linear gain remains the same, but with a 90^o phase shift. Therefore the S parameters now:

$$\begin{bmatrix} 0 & 0 \\ -19.2j & 0 \end{bmatrix}$$

Or in dB (notice now the magnitude is in dB and angle has to be explicit):

$$\begin{bmatrix} -\infty & -\infty \\ 25.7\angle -90^0 & -\infty \end{bmatrix}$$

c) Following b) but now LNA1 has $Z_{in} = Z_{out} = 75\Omega$ but LNA2 has $Z_{in} = Z_{out} = 50\Omega$, what is the expected S parameter matrix in dB for the quad-hybrid module? (**6 pts**)

Reflected voltage wave ratio at p_{in} is $S_{11} = \frac{-j}{\sqrt{2}} \cdot 0.2 \cdot \frac{-j}{\sqrt{2}} + 0 = -0.1$.

Reflected voltage wave ratio at p_{out} is the same: $S_{22} = -0.1$

The transmitted wave ratio is: $S_{21} = \frac{-j}{\sqrt{2}} \cdot 19.2 \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot 20 \cdot \frac{-j}{\sqrt{2}} = -19.6j$

The S matrix is now:

$$\begin{bmatrix} -0.1 & 0 \\ -19.6j & -0.1 \end{bmatrix}$$

or in dB:

$$\begin{bmatrix} -20\angle 180^0 & -\infty \\ 25.8\angle -90^0 & -20\angle 180^0 \end{bmatrix}$$

d) Estimate the IIP_{H2} and IIP_{H3} of the individual LNA when $Z_{in} = Z_{out} = 50\Omega$. (**6 pts**) Give your answers in dBm.

We have the VTC as: $v_{out} = 20v_{in} - 2.5v_{in}^2 - 5v_{in}^3$, i.e., $a_2 = -2.5\text{V}^{-1}$ and $a_3 = -5.0\text{V}^{-2}$.

$A_{IIPIM\,2} = \left|\frac{a_1}{a_2}\right| = 8$; IIP_{IM2} = 28.1dBm; IIP_{H2} = 34.1dBm.

$A_{IIPIM\,3} = \sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|} = 2.3$; IIP_{IM3} = 17.2dBm; IIP_{H3} = 22.0dBm.

e) When both the quadratic and cubic terms are important, estimate $I_{1dBcomp}$ in dBm of the individual LNA. (**8 pts**) Hint: There is a quadratic equation to be solved.

1dB compression is $10^{-1/20}$ = 0.89 times smaller.
As now we cannot assume which term is dominating, we have to use the entire VTC approximation as:

$$\frac{v_{out,realistic}}{v_{out,projected}} = \frac{20v_{in} - 2.5v_{in}^2 - 5.0v_{in}^3}{20v_{in}} = 0.89.$$

$$5.0v_{in}^2 + 2.5v_{in} - 2.17 = 0. \quad \Rightarrow \quad v_{in} = \frac{-2.5 + \sqrt{2.5^2 + 20 \cdot 2.17}}{10} = 0.43V.$$

Notice that in VTC, v_{in} is in magnitude, and has to be positive.

$I_{1dBcomp}$ = 1.85mW = 2.66dBm.

Notice if you assume only the third order dominates, you will obtain:
22.0dBm – 4.77dB – 9.64dB = 7.59dBm, which is rather off.

f) Estimate IIP_{H2} and in-band IIP_{H3} in dBm of the quad hybrid module when $Z_{in} = Z_{out}$= 50Ω. (**8 pts**)
Hint: Work out the voltage combination before converting to power in the harmonic frequency.

We now have no reflection and well matched LNA1 and LNA2. For a voltage input at p_{in}, for the third harmonic H3, we know that path 1 has –270^o phase shift while path 2 has –90^o. Therefore, H3 will cancel out, i.e., IIP_{H3} = ∞ (in theory).

We also know that for H2, path 1 has –180^o phase shift while path 2 has –90^o:

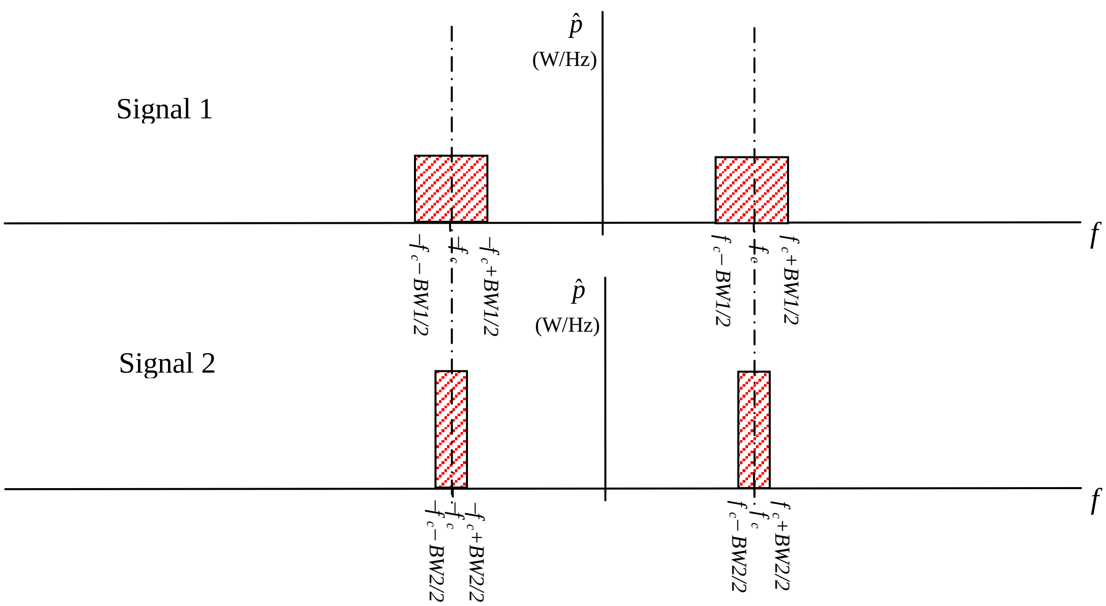
$$\left|v_{out,H2}\right| \propto \left|\frac{(-j)^2}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{-j}{\sqrt{2}}\right| = \left|\frac{1}{2}(-1-j)\right| = \frac{1}{\sqrt{2}} = 3\text{dB}.$$

Therefore the power will add up with a 3dB decrease.

Each LNA will have 3dB less input power, so another 3dB less H2 generated.

We can thus estimate $IIP_{H2,quad}$ = 34.1dBm + 6dB = 40.1dBm.

4. (**Intermodulation of two finite-bandwidth signals, 10 pts**) Two signals have the spectral components as shown below. They are received by the same antenna and fed into an amplifier low IIP_{IM2} and negligible IM3. Both Signals 1 and 2 are centered at f_c, and have bandwidths of *BW1* and *BW2*, respectively. We will assume *BW2* < *BW1* << f_c. Signal 2 has stronger power but smaller bandwidth than Signal 1.



a) What is the appropriate noise description? Choose the most appropriate *one* answer below. (**5 pts**)

(A) The shot noise is larger than the thermal noise.

(B) If the flicker noise ($1/f^{\alpha}$) is dominant, then Signal 2 has larger total noise power than Signal 1.

(C) If the thermal noise is dominant, then Signal 1 has larger total noise power than Signal 2.

(D) If the AC power line noise is dominant, then Signal 1 has larger total noise power than Signal 2.

(E) The blackbody radiation noise is strong around f_c.

Answer: (C). The total noise power for thermal noise at room temperature is –174dBm/Hz times the bandwidth, and Signal 1 has a larger bandwidth.

(A) is incorrect, as we do not know the proportion of thermal and shot noises where both have white spectrum. (B) is incorrect, as the flicker noise or the corner frequency cannot be identified clearly. (D) and (E) are incorrect, as the two signals are on the same line with the same external noises such as power line noise and black body radiation noise.

b) For the intermodulation signal of Signals 1 and 2, which of the following statement is correct? Choose the most appropriate *one* answer below. (**5 pts**)

(A) The intermodulation signal has an upper frequency bound of $2\times(f_c + BW1 + BW2)$.

(B) The intermodulation signal has negligible component around f_c.

(C) The intermodulation signal has negligible DC components.

(D) The intermodulation signal has a square shape.

(E) The intermodulation signal has a triangular shape.

(F) The intermodulation signal has a quadratic shape.

Answer: (B). As there is only 2nd-order intermodulation and negligible 3rd-order intermodulation, there is negligible component around f_c.

(A) is incorrect as the upper frequency bound would be $2f_c + BW1 + BW2$. (C) is incorrect, as the DC component is strong (about two times than those around $2f_c$). The intermodulation signal from IM2 has a trapezoidal shape out of the convolution of Signals 1 and 2.

(**Bonus question**) Which of the following actions by students in ECE 4880 is most detestable to Prof. Kan? (**0 pts**) Choose the most appropriate *one* answer below.

(A) Eating in class.

(B) Late for class.

(C) Copy the number from calculator with much more significant digits than feasible.

(D) Get the unit wrong for the final calculation results.

(E) Late homework submission.

If you have heard enough from me, the only answer is (D). I do not like (C), but not as offended as the wrong unit. I do not mind (A), (B), and (E) much, as long as you do not disturb the whole class.