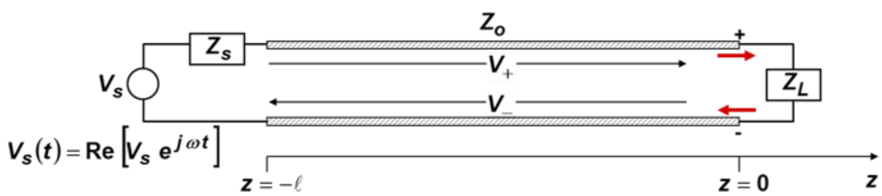
ECE 4880 RF Systems Fall 2016
Prelim Exam 1 Solution
Please BOX your number answers to help with an accurate grading!



We can thus define the **load reflection coefficient** Γ_L at the load as: $\Gamma_L = \dfrac{V_-}{V_+} = \dfrac{Z_L/Z_o - 1}{Z_L/Z_o + 1}$

We can express the forward and reverse traveling waves by using the reflection coefficient Γ_L:

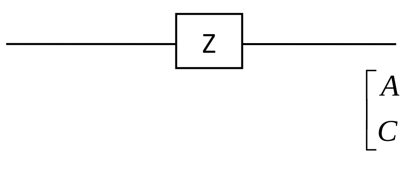
$$V(z) = V_+\left(e^{-jkz} + \Gamma_L\, e^{+jkz}\right); \qquad I(z) = \frac{V_+}{Z_o}\left(e^{-jkz} - \Gamma_L\, e^{+jkz}\right)$$

If we observe the impedance at position z, the impedance towards the load will be:

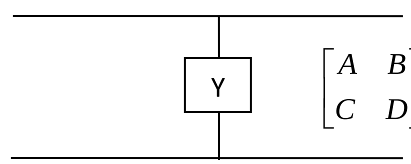
$$Z(z) = \frac{V(z)}{I(z)} = Z_o\,\frac{1+\Gamma_L\, e^{2jkz}}{1-\Gamma_L\, e^{2jkz}}$$

The Frii's line-of-sight (LoS) law between a transmitter and receiver pair:

$$\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi r}\right)^2 \Psi_T \Psi_R$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{A + BY_0 + C/Y_0 + D}\begin{bmatrix} A + BY_0 - C/Y_0 - D & 2(AD - BC) \\ 2 & -A + BY_0 - C/Y_0 + D \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{B}\begin{bmatrix} D & BC - AD \\ -1 & A \end{bmatrix} \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{Y_{21}}\begin{bmatrix} -Y_{22} & -1 \\ Y_{12}Y_{21} - Y_{11}Y_{22} & -Y_{11} \end{bmatrix}$$

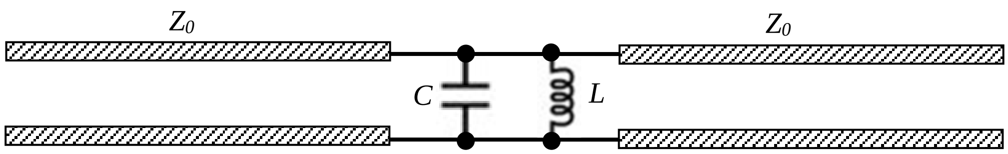
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{Y_0}{(1+S_{11})(1+S_{22}) - S_{12}S_{21}}\begin{bmatrix} (1-S_{11})(1+S_{22}) + S_{12}S_{21} & -2S_{12} \\ -2S_{21} & (1+S_{11})(1-S_{22}) + S_{12}S_{21} \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{(1+Z_0Y_{11})(1+Z_0Y_{22}) - Z_0^2Y_{12}Y_{21}}\begin{bmatrix} (1-Z_0Y_{11})(1+Z_0Y_{22}) + Z_0^2Y_{12}Y_{21} & -2Z_0Y_{12} \\ -2Z_0Y_{21} & (1+Z_0Y_{11})(1-Z_0Y_{22}) + Z_0^2Y_{12}Y_{21} \end{bmatrix}$$

1. (**Radio history**) When Edwin Armstrong proposed the superheterodyne radio architecture in 1918, which of the following element is NOT needed? (Only one correct answer) (**5 pts**)
   a) An RF antenna
   b) RF amplifiers
   c) Filters for band selection
   d) Mixer for frequency conversion
   e) Analog-to-digital data converter
   f) Local oscillator for frequency generation

Answer: e). The radio in the early 20th century is entirely analog. No digital data are involved. Digital computers emerged around 1920s, but were very immature without a working architecture

2. (**LC Filter in the Signal Chain**) A parallel LC network is inserted between two transmission lines with characteristic impedance of 50Ω. We have L = 1nH and C = 10pF. We will define $\omega_0 = \frac{1}{\sqrt{LC}}$.



   a) What is the ABCD matrix for the LC network as a function of frequency? (**5 pts**)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\omega C + \frac{1}{j\omega L} & 1 \end{bmatrix}$$

   b) What is the S matrix for the LC network as a function of frequency? (**5 pts**)

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{A + BY_0 + C/Y_0 + D} \begin{bmatrix} A + BY_0 - C/Y_0 - D & 2(AD - BC) \\ 2 & -A + BY_0 - C/Y_0 + D \end{bmatrix}$$

$$= \frac{1}{2 + Z_0\left(j\omega C + \frac{1}{j\omega L}\right)} \begin{bmatrix} -Z_0\left(j\omega C + \frac{1}{j\omega L}\right) & 2 \\ 2 & Z_0\left(j\omega C + \frac{1}{j\omega L}\right) \end{bmatrix}$$

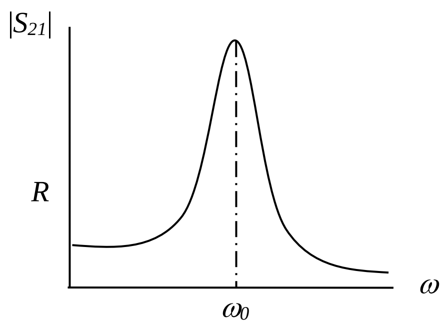
The S matrix is symmetric, as the network is the same looking from the left or from the right.

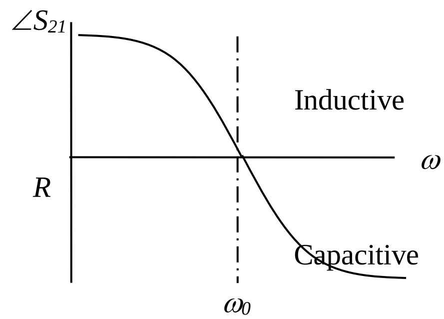
   c) Plot the magnitude and phase of S_{21} across a broad frequency around ω_0. (**5 pts**)

$$S_{21} = \frac{2}{2 + Z_0\left(j\omega C + \frac{1}{j\omega L}\right)}$$. At ω_0, we have S_{21} = 1.

When $\omega << \omega_0$, $S_{21} \cong \dfrac{2}{2+Z_0\left(\dfrac{1}{j\omega L}\right)} = \dfrac{2}{2+\dfrac{Z_0^2}{\omega^2 L^2}}\left(2+jZ_0\left(\dfrac{1}{\omega L}\right)\right)$, which has positive phase (inductive).

When $\omega >> \omega_0$, $S_{21} \cong \dfrac{2}{2+j\omega Z_0 C} = \dfrac{2}{2+\dfrac{Z_0^2}{\omega^2 L^2}}\left(2-j\omega Z_0 C\right)$, which has negative phase (capacitive).

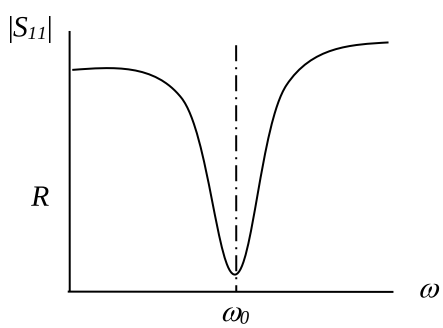


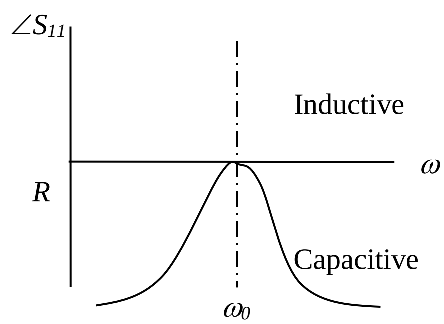


This is a bandpass filter. S_{21} is close to 1 in magnitude and 0 in phase.

d) Plot the magnitude and phase of S_{11} across a broad frequency around ω_0. (**5 pts**)

$$S_{11} = \frac{-Z_0\left(j\omega C+\dfrac{1}{j\omega L}\right)}{2+Z_0\left(j\omega C+\dfrac{1}{j\omega L}\right)};\ \lim_{\omega\to 0} S_{11} = -1;\ S_{11}\big|_{\omega=\omega_0} = 0\ \ \lim_{\omega\to\infty} S_{11} = -1$$





e) What is the passband S_{21} ratio between ω_0 and $\omega_0 - 100$MHz (in dB)? A number is required. (**5 pts**)

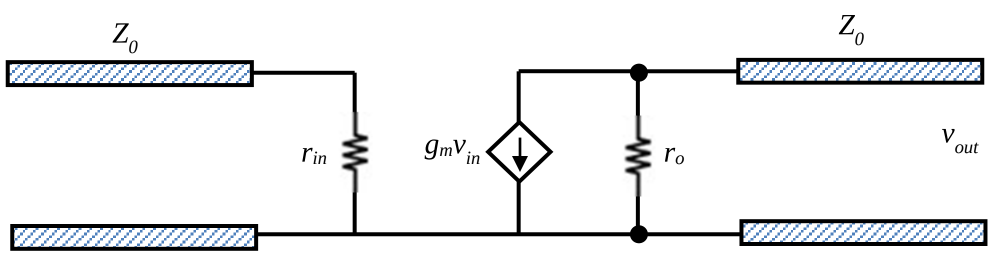
$$S_{21} = \frac{2}{2+Z_0\left(j\omega C+\frac{1}{j\omega L}\right)} = \frac{2}{2-j0.1}$$

. $|S_{21}|$ = –0.0087dB. We can see that when we are close to the resonant frequency, the pass band is significantly large.

f) What is the value of Y_{11} at ω_0? Explain the value in one sentence. (**5 pts**)

$Y_{11} = \infty$ at ω_0. This is a short circuit input, which means no voltage response can be observed as all signals are passed to Port 2.

3. (**BJT Power Amplifier**) A RF bipolar transistor amplifier within the voltage range of operation can be approximated by the π network shown below. The dependent current source has magnitude of $g_m v_{in}$ where v_{in} is the voltage drop in r_{in}. The input and output have transmission line of impedance of Z_0 = 50Ω.

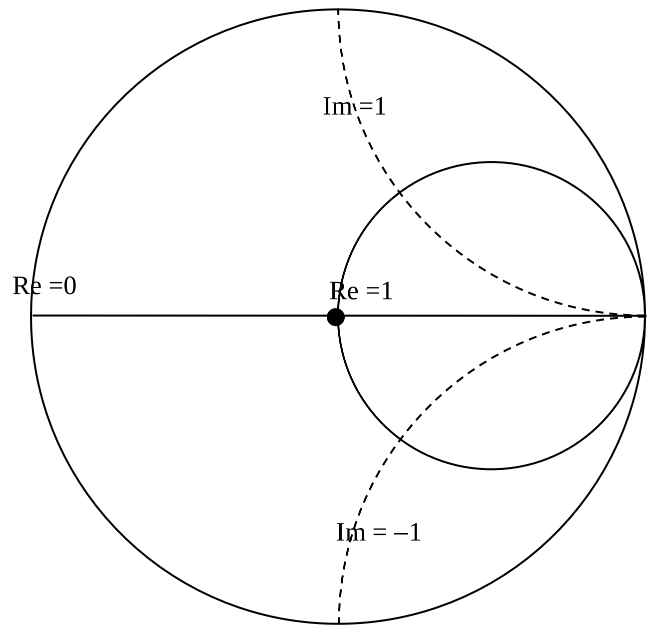


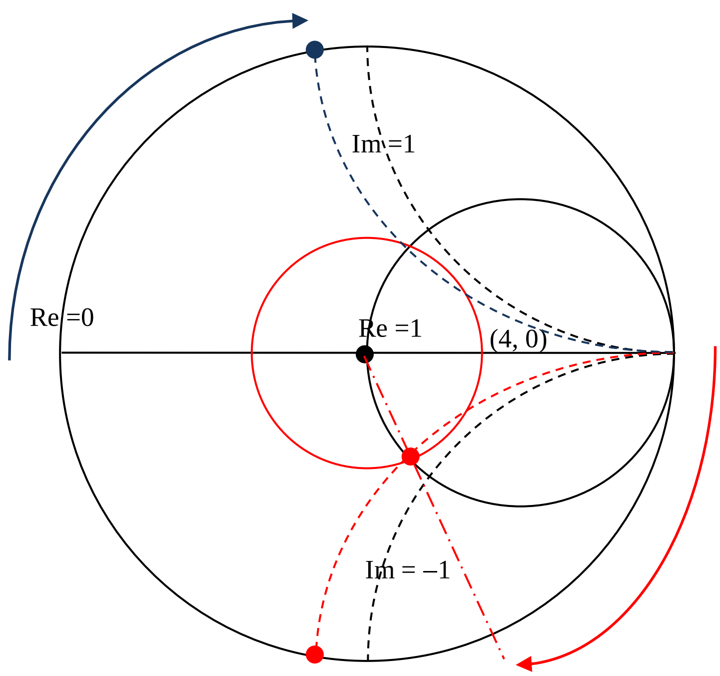
a) Write down the Y matrix for the π network. (**5 pts**)

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{r_{in}} & 0 \\ g_m & \frac{1}{r_o} \end{bmatrix}$$

b) We have r_{in} = 200Ω, which causes significant reflection. We decide to add an impedance match network by a transmission line and then a series stub line. Show the qualitative step in the simplified Smith chart below for the first step to transform the real part to 50Ω. Denote how the length of the transmission line should be determined. (**5 pts**) Which number below is closest to the transmission line length? 0, λ/10, λ/3, 4λ/5. Give one sentence to explain. **(5 pts)**

As the Smith Chart works on normalized unit, we will first identify (200/50, 0) = (4, 0) point. Use a compass to draw a centered circle that passes (4, 0). The intersection with the Re = 1 circle will be the impedance transform after the transmission line. Read the wavelength difference on the outer circle. We will see the transmission line should be close to λ/10 length, as the quarter circle is λ/8.

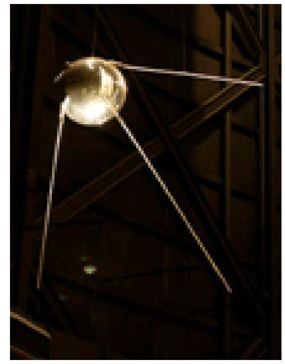




c) At the second step of the impedance matching, we will add a series stub line. Show the qualitative step in the Smith Chart above to determine the length of the series stub line. (**5 pts**) To minimize the length of the stub line, should we use an open-ended or short-ended stub line? **(5 pts)**

We will read the imaginary part of the intersection point above (negative), and find the corresponding positive imaginary part on the Re = 0 circle. This will be the inductive matching from the series stub line. To minimize the length of the stub line, we should use a short-ended stub.

4. (**Sputnik Radio Transmitter**) The first orbiting satellite is Sputnik 1 by the Soviet Union in 1957. The satellite orbits the earth in an ellipsoid of 1,450km and 223km axes. The satellite is basically a radio transmitter and weighs 83.6kg. The mission (in addition to the study of launching) is to transmit signals of 20MHz and 40MHz for 0.3s each, with intermittent pauses of 0.3s. This is a beacon signal that does not carry any data. The antennas on Sputnik 1 are about 2 – 3 m, as shown in the picture. Due to the battery limit (no solar cells on Sputnik 1), the transmitter is at 30 dBm. The RF receivers on earth have a sensitivity of –100dBm and the antenna gain for both frequencies are at 30 dBi. There are more than 3 receivers at different locations to trilaterate the satellite.



Replica of Sputnik 1 in DC Aerospace Museum

(a) If we have no control of the satellite orientation in space, what is the largest satellite antenna gain we can achieve? (**5 pts**)

We can only use a 0 dBi antenna for nearly isotropic radiation.

(b) Assume there is only free-space loss between the satellite and the receiving station on earth. How far can we listen to both frequencies at the line-of-sight? The speed of light is 3×10^8 m/s. We will use the best possible antenna in (a). (**5 pts**)

$$\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi r}\right)^2 \Psi_T \Psi_R = -30dBm - 100dBm = -130dB.$$

The wavelength at 20MHz is 15m, and at 40MHz is 7.5m.

The distance at 20MHz will be $10^{-13} = \left(\frac{\lambda}{4\pi r}\right)^2 10^3$; $r = 1.19\times10^8$m = 119,000km.

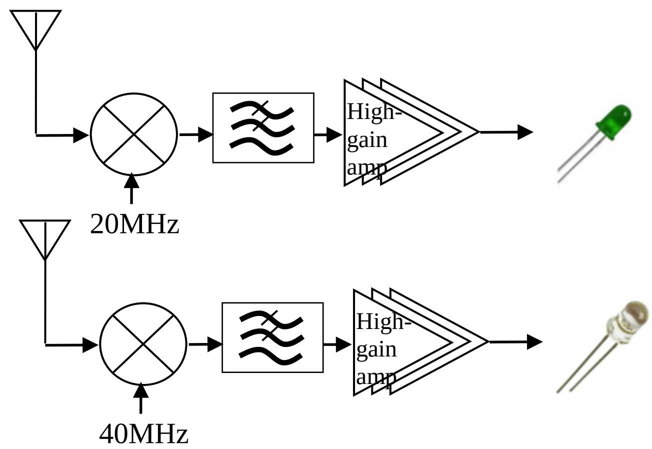
The distance at 40MHz will be 59,700km.

(c) The ion sphere will cause additional path loss (which we can use to study the composition of the ion sphere). If the maximum loss can be 15dB, how far we can receive the 40MHz signal? (**5 pts**)

5,970km $\times 10^{-15/20}$ = 10,600km.

(d) Design the block diagram of the receiver that can output the beacon signal by two LED lights of different colors that can be activated by 3.2V. (**5 pts**) What is the needed gain in dB in the receiver chain with the weakest signal? Assume the receiver antenna has an impedance of 50 Ω. (**5 pts**)

The incoming signal of -100dBm has voltage amplitude at 3.2μV. The voltage gain needed is then 10^6, which is about 120 dB. As we have no data contained in the beacon signal, the noise behavior is not important. No LNA is needed, and we can put most of the amplification in the base (quasi-DC) band. These low-frequency quasi-static amplifiers will need to be DC balanced, so that if the output of the mixer contains a shift from matching frequency, it can be properly amplified to the desired DC level whether positive or negative.



5. (**Walkie-Talkie**) You are a WWII communication officer. Before you are dispatched, you need to put together a point-to-point radio link (similar to a walkie-talkie) from scratch to communicate with your base unit. These are the components available to you (two of each to make two transceivers):
    - Oscillators at 100MHz with 0.1V voltage magnitude;
    - Antennas around 100MHz of 8 dBi;
    - Broadband mixers;
    - Microphones that can transduce voice to an electric signal of -10dBm and 4kHz bandwidth;
    - Speakers that can transduce an electric signal of 10dBm and 4kHz to an audible sound;
    - Amplifiers at the voice band (with little distortion);
    - Low-noise amplifiers at the 100MHz with high sensitivity and low noise;
    - Power amplifier at 100MHz with 20dB gain and maximum 30dBm output;
    - Bandpass filters around 100MHz; Lowpass filters around 8kHz;
    - Circulator at 100MHz with -60dB rejection.

    (a) If you need an operation range of 10km at line of sight, what is your receiver sensitivity? (**5 pts**)

    The wavelength at 100MHz is 3m (if a quarter-wavelength antenna is used, then it is about 0.75m long).

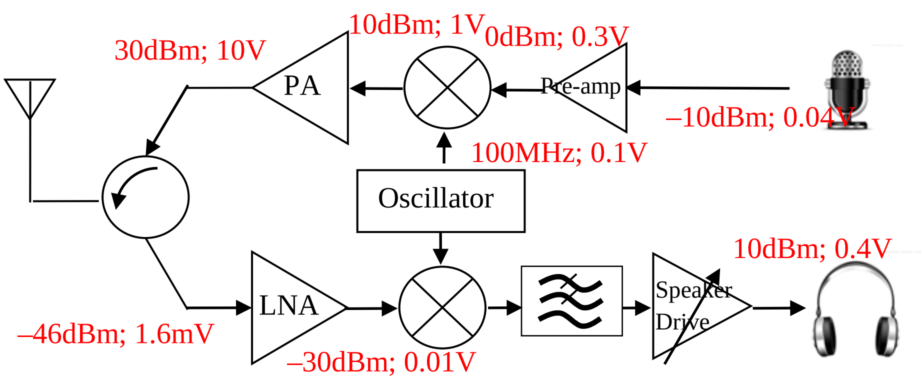
$$\left(\frac{\lambda}{4\pi r}\right)^2 = \left(\frac{3}{4\pi \cdot 10^4}\right)^2 = -92dB$$

$$P_R = \left(\frac{\lambda}{4\pi r}\right)^2 \Psi_T \Psi_R P_T = -92dB + 8dB + 8dB + 30dBm = -46dBm$$

    This is a very low requirement, and explains why Walkie-Talkie can be that cheap to make. For a reasonable receiver with –86dBm sensitivity, we can readily go for 1,000km (for sure, line of sight, where now we can be easily limited by the earth curvature).

    We also see that the self interference is rather large with just –60dB isolation in the circulator. The self interference signal is about 30dBm – 60dB = –30dBm, much stronger than -46dBm. The cheap walkie-talkie resolves this problem by not talking and listening simultaneously (time division).

(b) Draw the transceiver block diagram, and denote the power or voltage at each stage (treat mixers with a fixed gain). (**10 pts**)



Speakers and microphones often have impedance around 4 - 10Ω, instead of 50Ω, and we have used 8Ω for the above voltage amplitude calculation. However, this is not used for grading.

An important consideration is the variable gain in the receiving chain. When the distance is less than 10km and the incoming signal is much stronger, we cannot put too much gain in the speaker drive, or else it will totally saturate the speaker. This is often a low-frequency amplifier that is directly controlled by the "volume knob".