ECE 4880

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Chapter 9

Phase Noises in the Signal Chain

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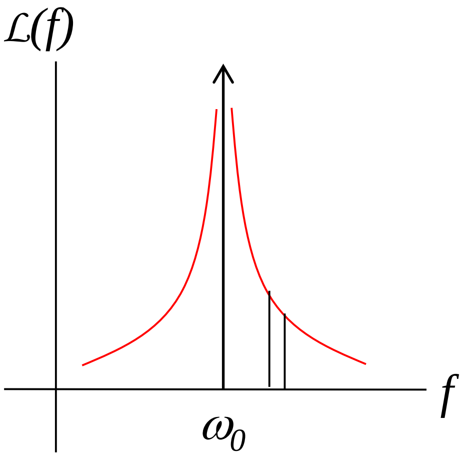
Goals

- Effects of amplitude and phase noises in RF transceivers
- Intuitive modeling of phase noises by LC resonators
- Empirical models for phase noises

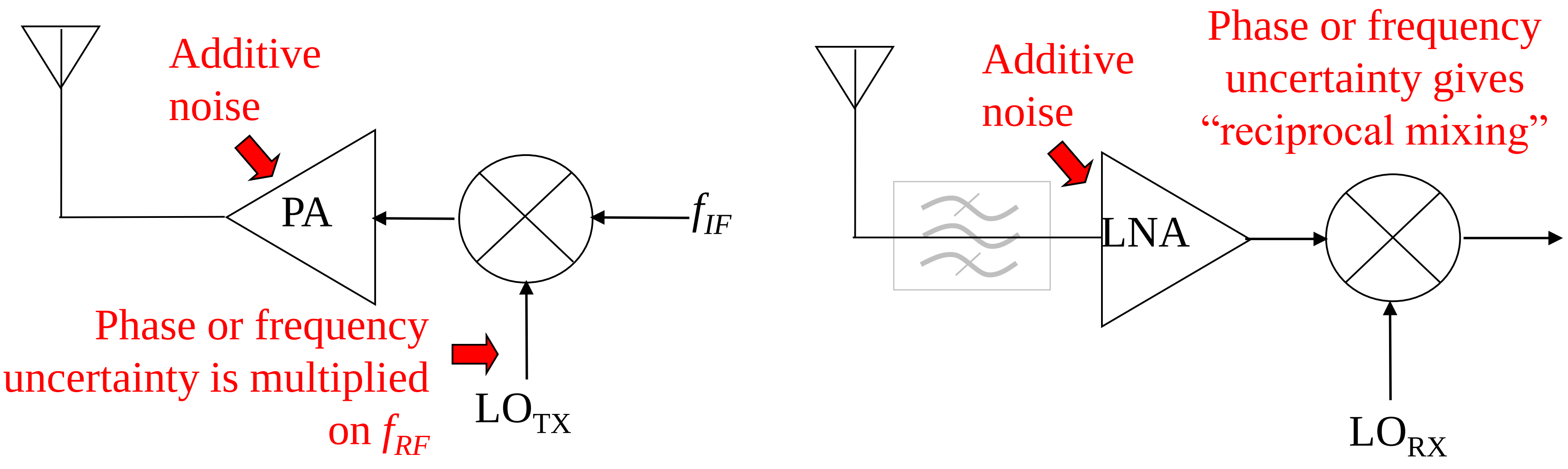
Phase Noise Definition

- Noise free signal: $v(t) = A\cos(\omega_0 t)$
- Amplitude noise (AM): stochastic process by : $v(t) = A(t)\cos(\omega_0 t)$
- Phase noise (PM): stochastic process by: $A\cos(\omega_0 t + \varphi(t))$

- The phase noise is often represented as one-sided spectral density of a signal's phase deviation or phase instability $S_\varphi(f)$
- Phase noise is often defined by $\mathcal{L}(f)$ (as script L of f), which represents the noise power relative to the carrier contained in a 1Hz bandwidth centered at the carrier frequency.
- $\mathcal{L}(f)$ has the unit of dBc/Hz.
- For a $1/f$ phase noise, $\mathcal{L}(f)$ can be –80dBc/Hz at 10kHz offset or –90dBc/Hz at 100kHz.

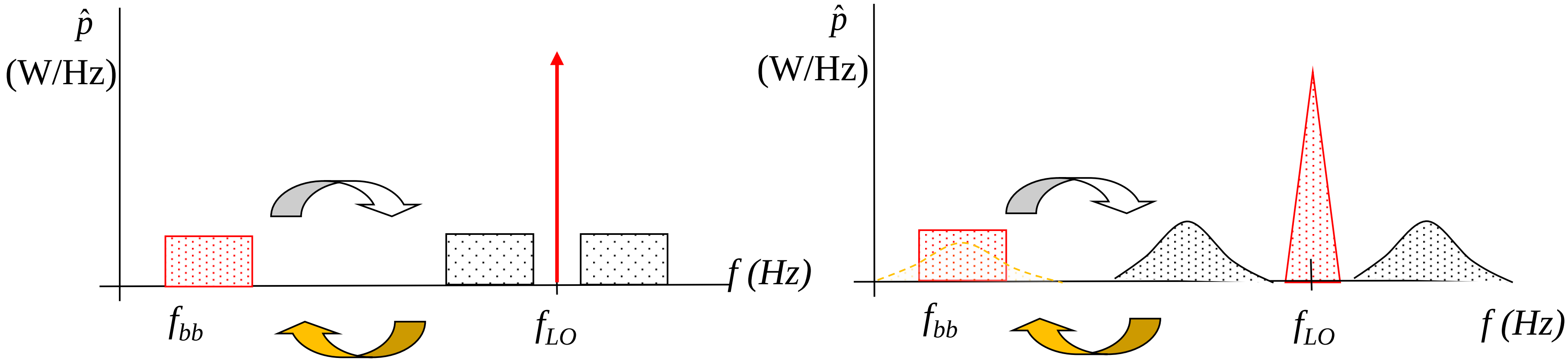


Phase Noise in Oscillators



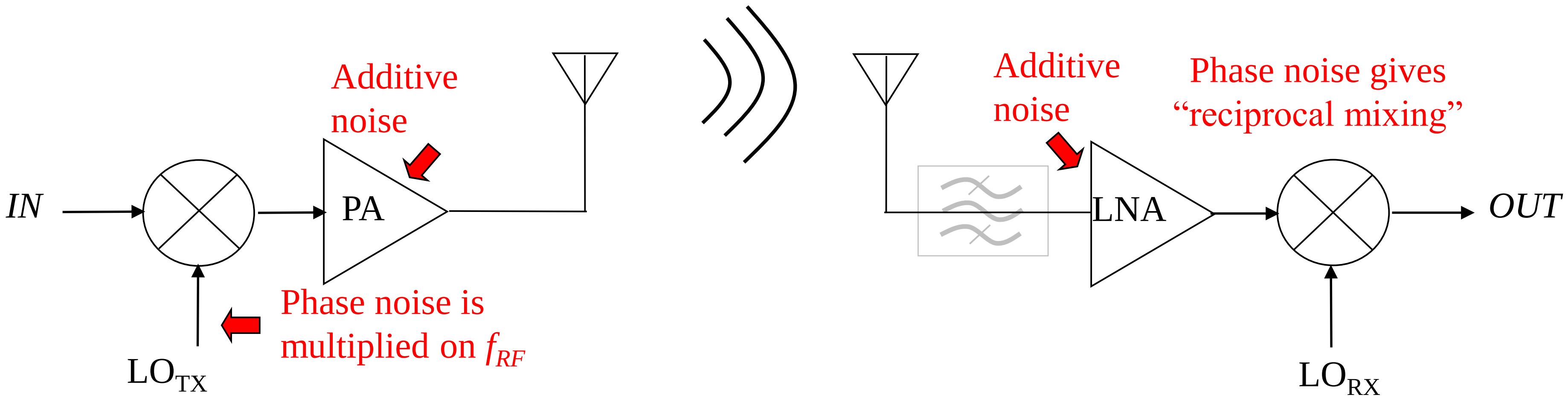
- Phase noises of LO will be multiplied to the signals in TX and RX (convolution in the frequency domain).
- LO_{TX} and LO_{RX} uncertainty will demand wider channel separation and larger separation of "symbols".
- Silver lining: the phase shift offset between LO_{TX} and LO_{RX} carries the time-of-flight information.

Phase Noise in Oscillators



- Perfect LO as a delta function in the spectrum will just perform frequency translation with NO additional components.
- Phase noise will convolute into the mixer output as amplitude (envelope) and phase (zero crossing) noises, generally termed as "reciprocal mixing" of AM and PM.
- Frequency (FM) and phase (PM) modulation can be different for long integration cycles.
- Phase noise is especially serious for timing-based modulation.
- Phase noise has been important for any clocks.

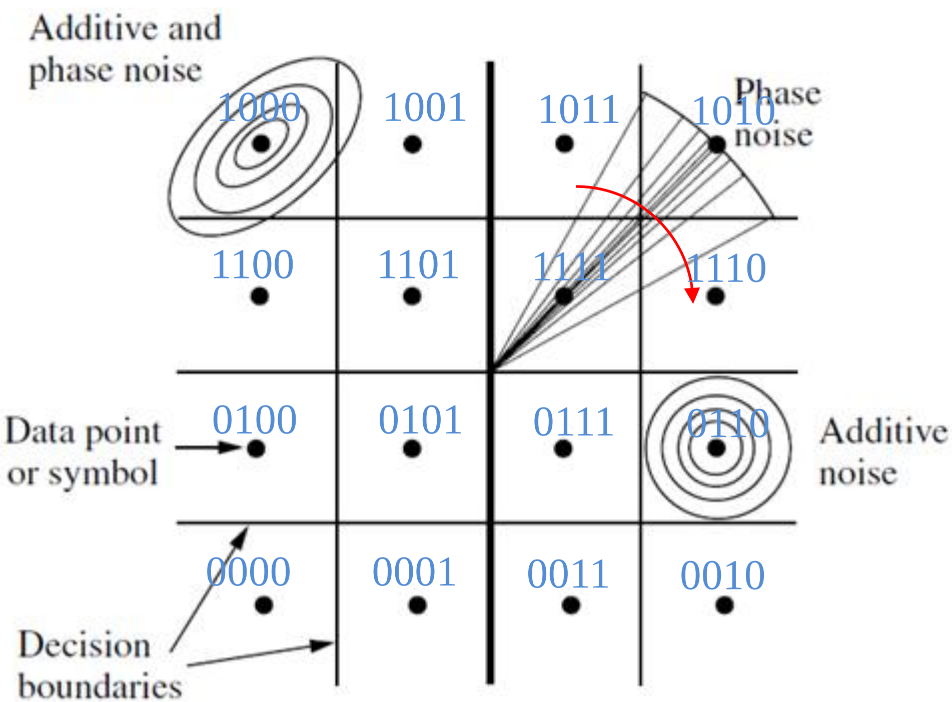
Wireless Link: Noises and Distortion



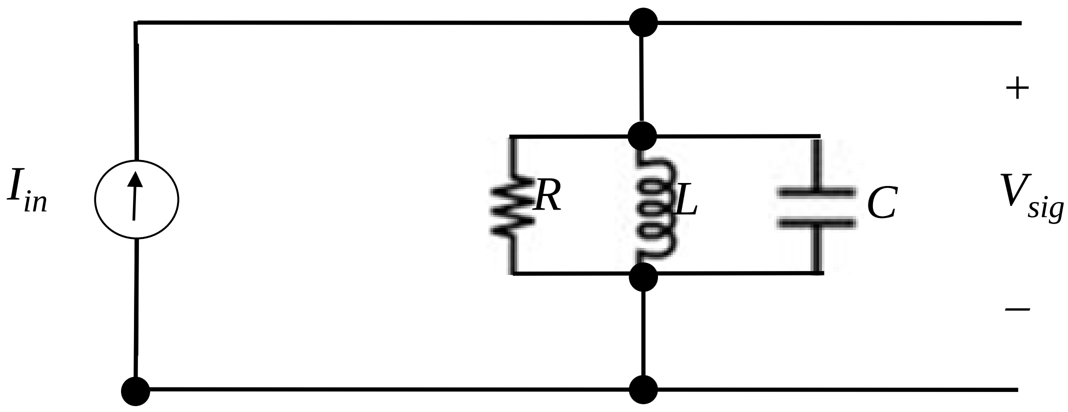
- The main purpose of wireless link is to retrieve information from IN to OUT, with the noise and nonlinearity of
  - Two mixers
  - Two LO generators
  - PA and LNA
  - Antenna
  - Free-space path

Effects of LO Phase Noise

- Receiver desensitization by the phase noises of LO_{TX} and LO_{RX}
- Jitter is the uncertainty in the synchronization (zero-crossing points) source:
  - AM: from uncertainty in loop gain (should be 1)
  - PM: from uncertainty in phase delay (should be $2n\pi$)
- Larger bit error rate in Q-ary modulation by phase error: only bordering domain has 1-bit error (easier for ECC).



Phase Noises in LC Oscillators

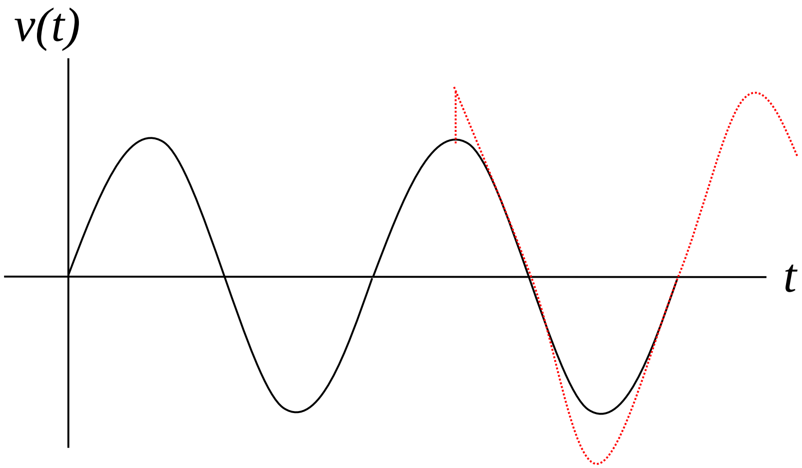


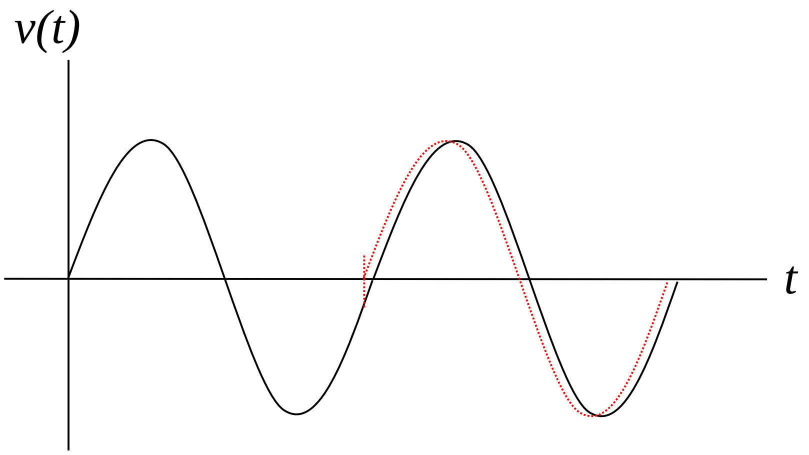
Stable solution: $V_A\cos(\omega_0 t)$

Average E_{stored}: $\dfrac{CV_A^2}{2}\omega_0$

- LC resonators can serve as an oscillator when R is compensated by an active I_{in}: electrostatic and magnetostatic energies balanced each other
- Colpitts or Clapp or cross-coupled oscillators
- Piezoelectric resonators
- Mechanical oscillators (pendulum with compensation): AM and PM can be easiest visualized.
- Loop gain: 1; phase loop delay: $2n\pi$.

Energy Injection for AM and PM





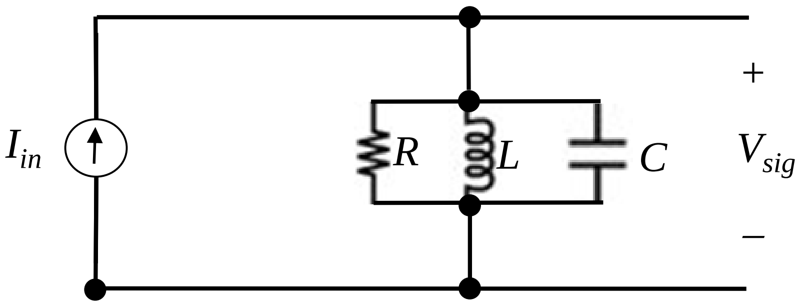
Injection at peak :

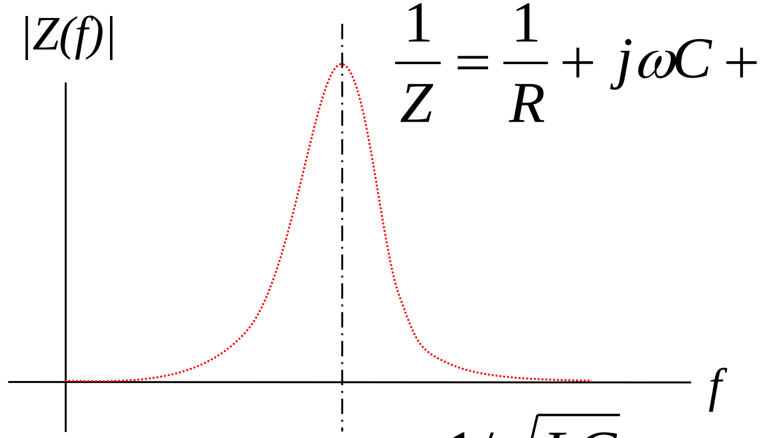
- Additional energy of $V_A \Delta I_{in}$
- Follow the solution of a higher amplitude (remember all amplitudes are allowed here)
- Zero crossing point will not change: only AM; no PM

Injection at zero crossing:

- No added energy
- Stay at the same magnitude V_A and frequency ω_0
- The local addition of charge will help the voltage reach its maximum faster: only PM; no AM!

Noises in the RLC Resonator





$$\frac{1}{Z} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$\overline{V_n}^2 = 4kTR \cdot BW = 4kTR \cdot \int_0^\infty \left|\frac{Z(f)}{R}\right|^2 df = 4kTR \cdot \frac{1}{4RC} = \frac{kT}{C}$$

$$Q \equiv \frac{P_{stored}}{P_{diss}} = \frac{\omega_0 E_{sig}}{P_{diss}} = \frac{R}{\omega_0 L} = \omega_0 RC$$

- *R* is the only noise source
- From 2nd law of thermodynamics, the noise energy is unretrievable, i.e., energy conservation system does not have noise

Noises Normalized to the Signal Carrier

- The energy stored in L and C is: $E_{sig} = C\overline{V}_{sig}^2$
- The noise-to-signal power ratio in an oscillator is (in dBc):

$$\frac{P_{noise}}{P_{signal}} = \frac{\overline{V}_n^2}{\overline{V}_{sig}^2} = \frac{kT}{E_{sig}}$$

- If the noise in R is equally partitioned in AM and PM:

$$P_{phase\_noise} = \frac{kT}{2E_{sig}}$$

- The phase noise will not be white, but focus around ω_0
- **No** clean analytical form is yet available.

A Common Derivation of $1/f^2$ for $\mathcal{L}(f)$

- Impedance: $\dfrac{1}{Z} = \dfrac{1}{R} + j\omega C + \dfrac{1}{j\omega L}$

- Close to $\omega_0 + \Delta\omega$:

$$\left|Z(\omega_0 + \Delta\omega)\right| \cong R \cdot \frac{\omega_0}{2Q\Delta\omega} \qquad \mathrm{Im}\big(Z(\omega_0 + \Delta\omega)\big) \cong -j \cdot \frac{\omega_0 L}{2\left(\dfrac{\Delta\omega}{\omega_0}\right)}$$

- If the total phase noise power is from the **thermal noise**:

$$P_{phase\_noise} = \frac{kT}{2E_{sig}} = 2\int_{\omega_0}^{\infty} df\, \mathcal{L}(f)$$

We must have: $\mathcal{L}(\Delta\omega) = \dfrac{2kT}{P_{sig}} \cdot \left(\dfrac{\omega_0}{2Q\Delta\omega}\right)^2$

Empirical Formula *for ℒ(f)*

- Leeson's approximation:

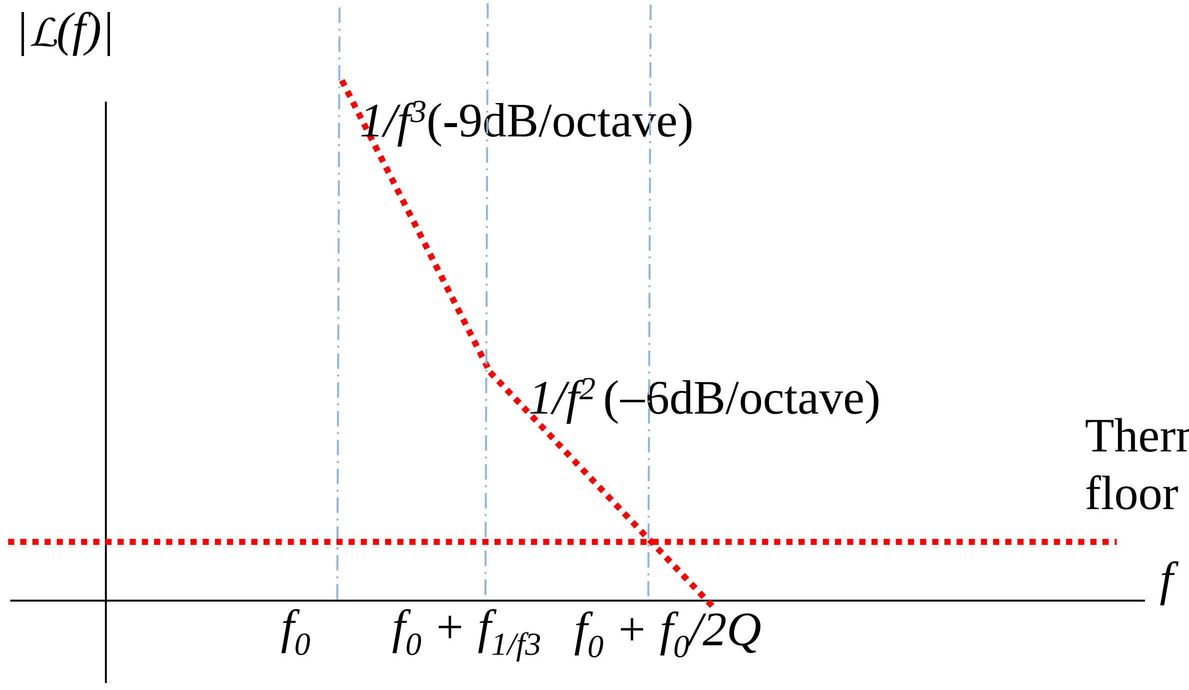
$$\mathcal{L}(\Delta\omega) = \frac{2FkT}{P_{sig}} \cdot \left[1 + \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right] \cdot \left(1 + \frac{\Delta\omega_{1/f^3}}{|\Delta\omega|}\right)$$

- Parameters: Q and $\Delta\omega_{1/f3}$
- Thermal noise and Flicker noise contributions

Leeson's Approximation *for* $\mathcal{L}(f)$

- $f_0/2Q > f_{1/f3}$ (higher thermal noise and lower Flicker noise)

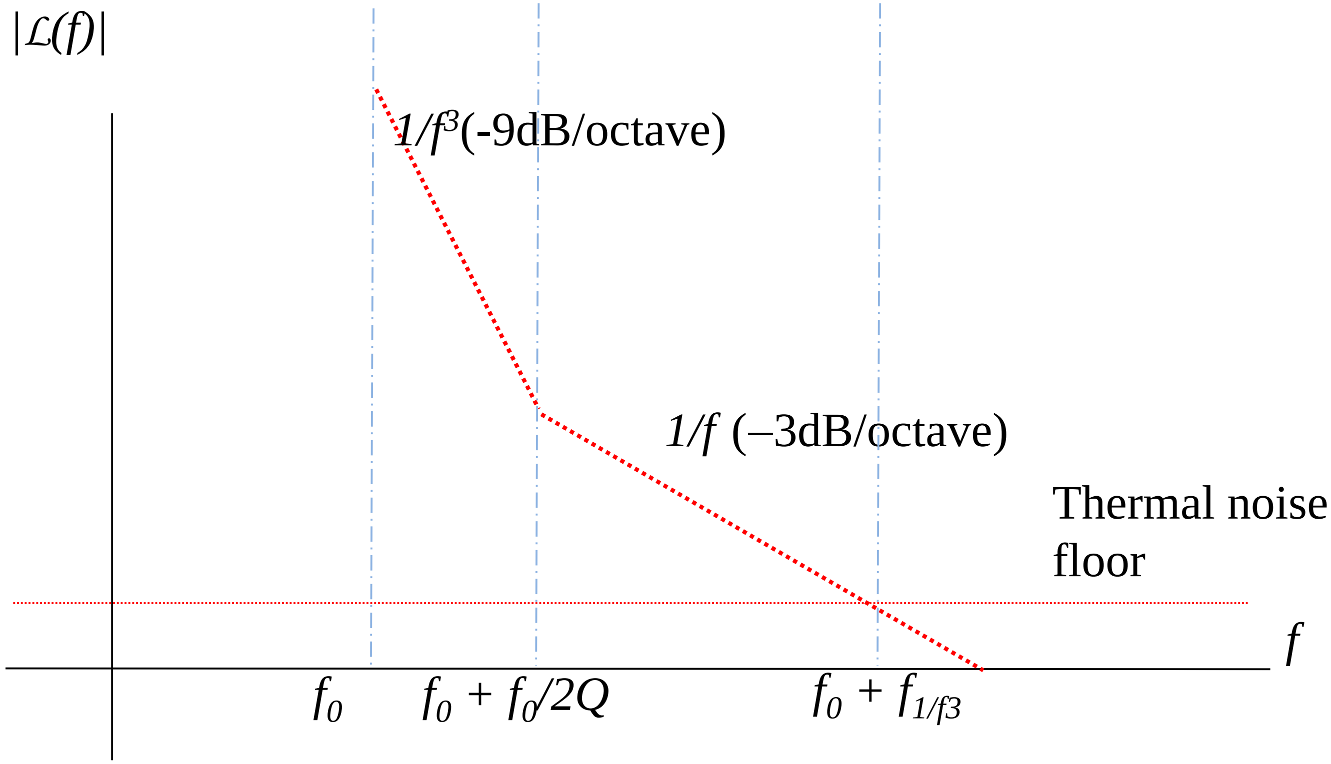
$$\mathcal{L}(\Delta\omega) = \frac{2FkT}{P_{sig}} \cdot \left[1 + \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right] \cdot \left(1 + \frac{\Delta\omega_{1/f^3}}{|\Delta\omega|}\right)$$



Leeson's Approximation *for* $\mathcal{L}(f)$

- $f_0/2Q < f_{1/f3}$ (lower thermal noise, i.e., large Q and higher Flicker noise)

$$\mathcal{L}(\Delta\omega) = \frac{2FkT}{P_{sig}} \cdot \left[1 + \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right] \cdot \left(1 + \frac{\Delta\omega_{1/f^3}}{|\Delta\omega|}\right)$$



What Have You Learned?

- The origin and effects of phase noises
- AM and PM distribution and effects
- Oscillator LC models
- Empirical PM models