ECE 4880

Fall 2016

Chapter 6

Interplay between Noise and Nonlinearity

Edwin C. Kan

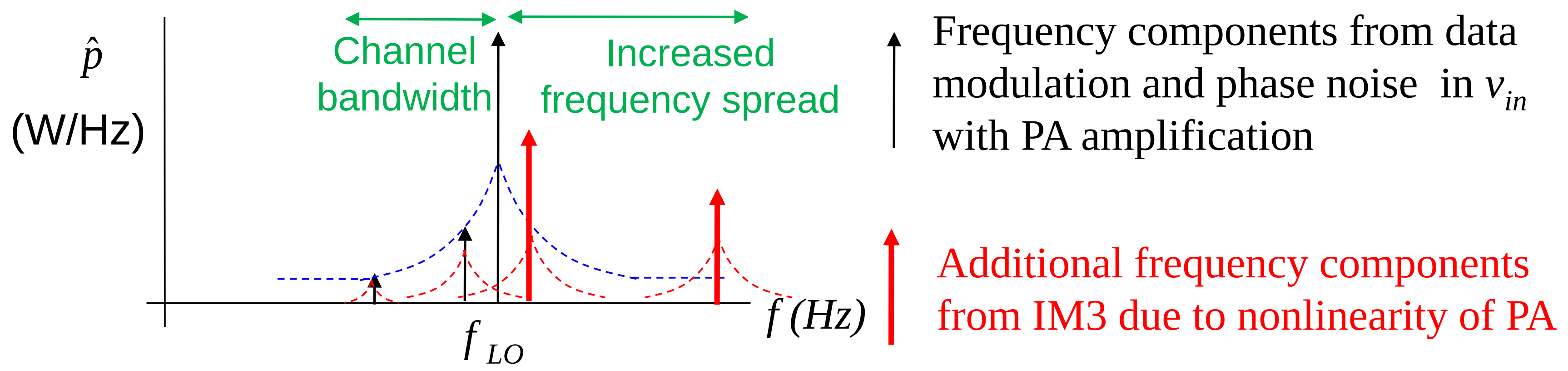
School of Electrical and Computer Engineering

Cornell University

Goals

- Understanding the problem of noise and nonlinearity qualitatively
- Intermodulation and convolution in the frequency domain
- Spurious-free dynamic range

Example of Effects of IM3 and Noises

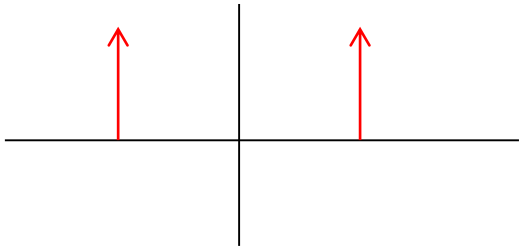


$$v_{out,IM3} = \frac{a_3}{4} 3\left[A^2 B\cos(2\varphi_a - \varphi_b) + AB^2 \cos(2\varphi_b - \varphi_a)\right]$$

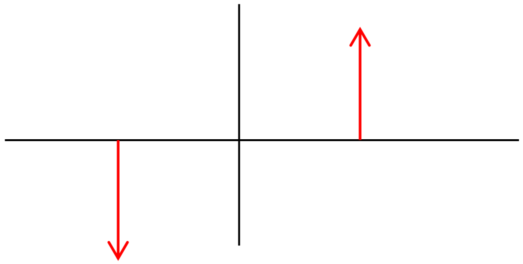
- Image frequency
- Larger frequency spread.
- The IM3 magnitude $3a_3/4 \times AB^2$ can be larger or smaller than the original component with *A*.

2^{nd}-Order Nonlinearity

$$\cos\left(\omega_a t+\theta_a\right)=\frac{e^{j\left(\omega_a t+\theta_a\right)}+je^{-j\left(\omega_a t+\theta_a\right)}}{2}$$



$$\sin\left(\omega_a t+\theta_a\right)=\frac{e^{j\left(\omega_a t+\theta_a\right)}-je^{-j\left(\omega_a t+\theta_a\right)}}{2}$$



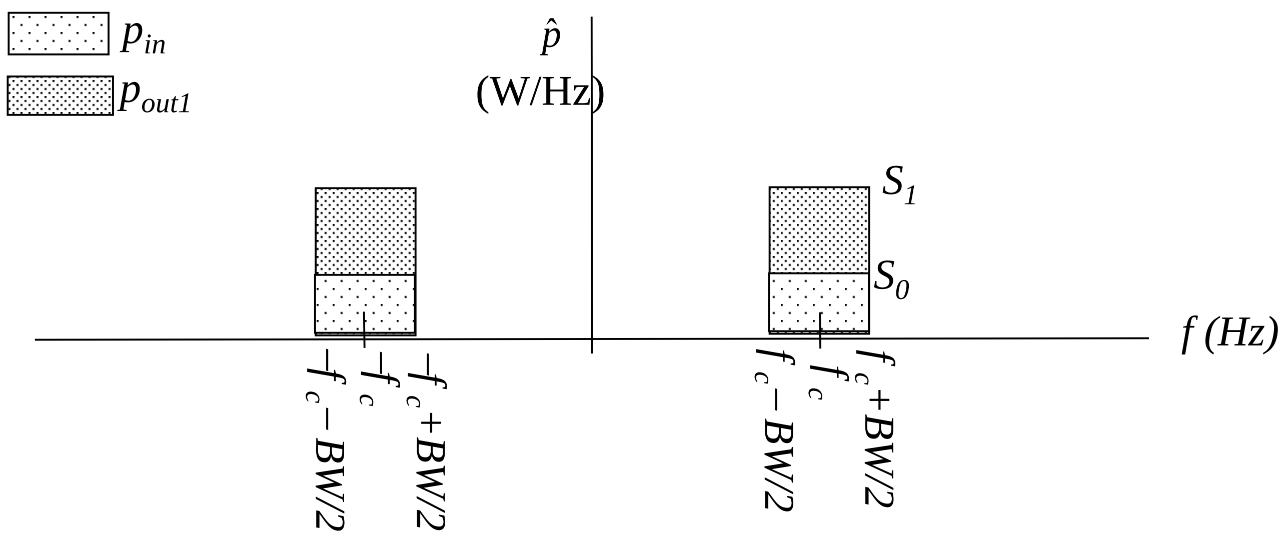
$$v_A(\omega) = A\cos\varphi_a = A\cos(\omega_a t+\theta_a)$$

$$v_B(\omega) = B\cos\varphi_b = B\cos(\omega_b t+\theta_b)$$

$$v_{out,2nd}=a_2 v_{in}^2=a_2\left\{\underbrace{\frac{A^2+B^2}{2}}_{DC}+\underbrace{\frac{A^2}{2}\cos 2\varphi_a+\frac{B^2}{2}\cos 2\varphi_b}_{H2}+\underbrace{AB\left[\cos\left(\varphi_a-\varphi_b\right)+\cos\left(\varphi_a+\varphi_b\right)\right]}_{IM2}\right\}$$

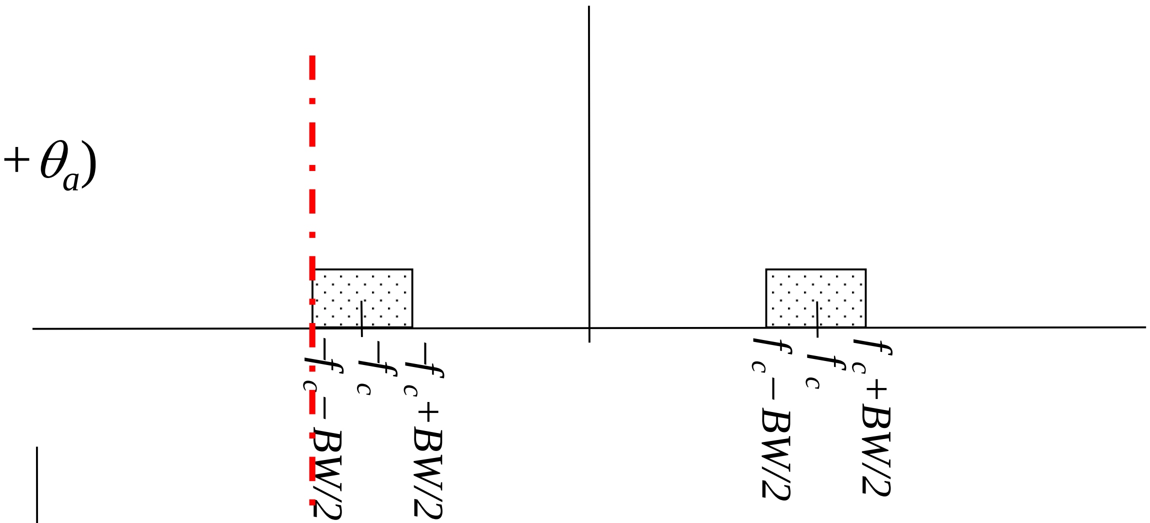
Multiplication in Time vs. Convolution in Frequency

$$v_{out,2nd}(f) = a_2 \cdot v_A(f) \otimes v_B(f) = a_2 \cdot \int_{-\infty}^{\infty} v_A(x) v_B(f - x)\,dx$$

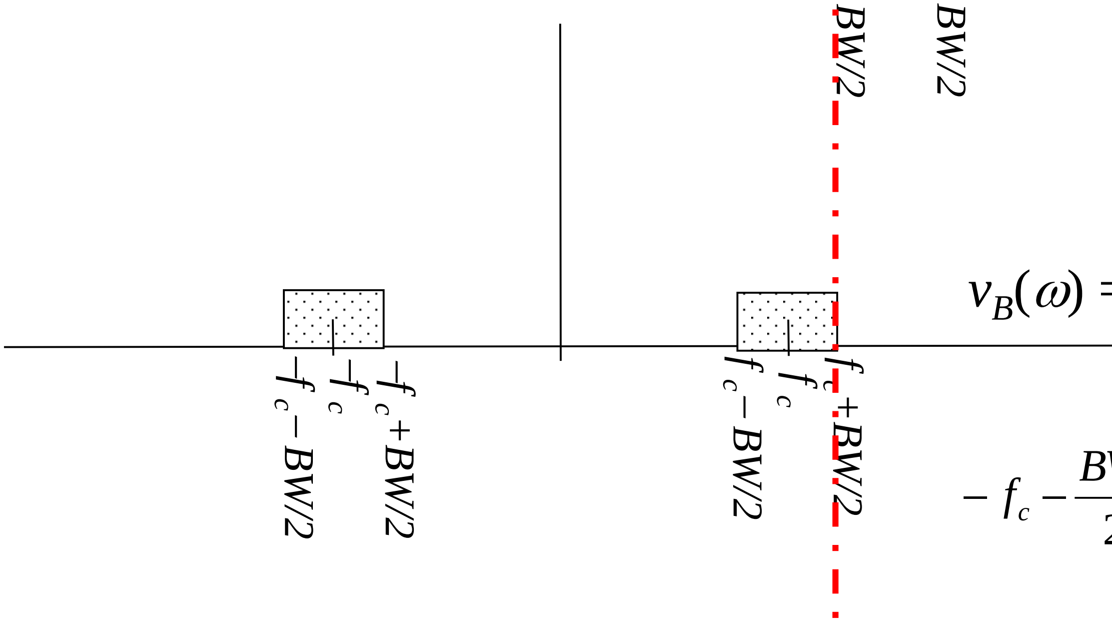


2^{nd}-Nonlinearity of Gaussian Signals

$$v_A(\omega) = A\cos\varphi_a = A\cos(\omega_a t + \theta_a)$$



$$v_B(\omega) = B\cos\varphi_b = B\cos(\omega_b t + \theta_b)$$

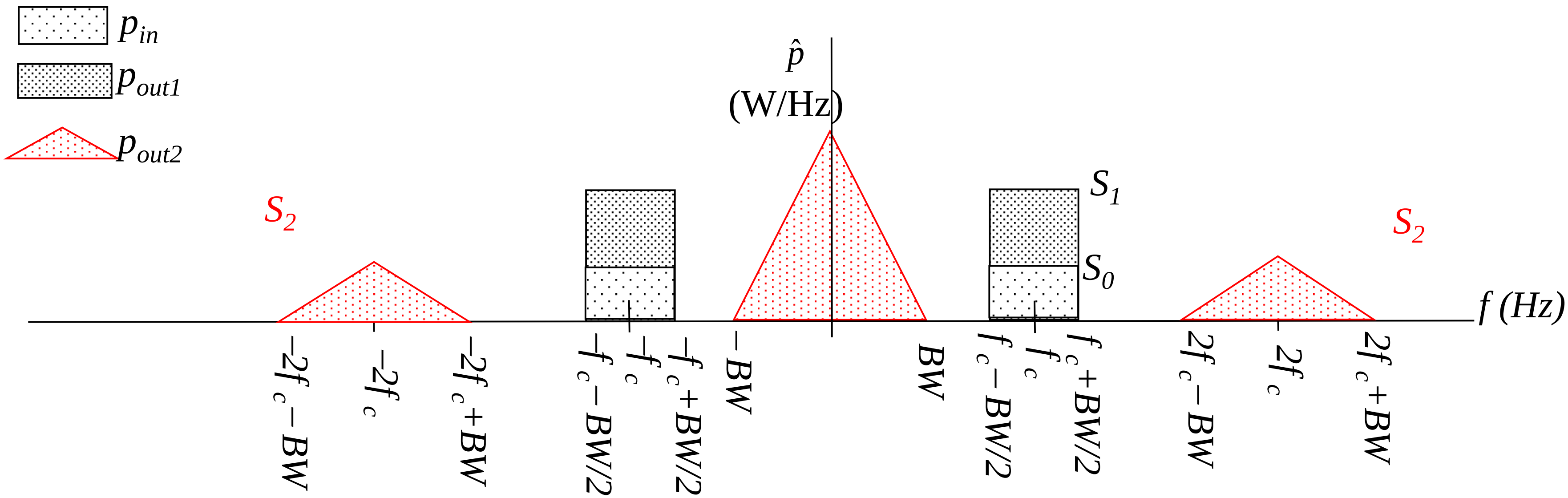


$$-f_c - \frac{BW}{2} - \left(f_c + \frac{BW}{2}\right) = -2f_c - BW$$

$$v_{out,2nd}(f) = a_2 \cdot v_A(f) \otimes v_B(f) = a_2 \cdot \int_{-\infty}^{\infty} v_A(x) v_B(f - x) dx$$

Multiplication in Time vs. Convolution in Frequency

$$v_{out,2nd}(f) = a_2 \cdot v_A(f) \otimes v_B(f) = a_2 \cdot \int_{-\infty}^{\infty} v_A(x) v_B(f-x)\, dx$$

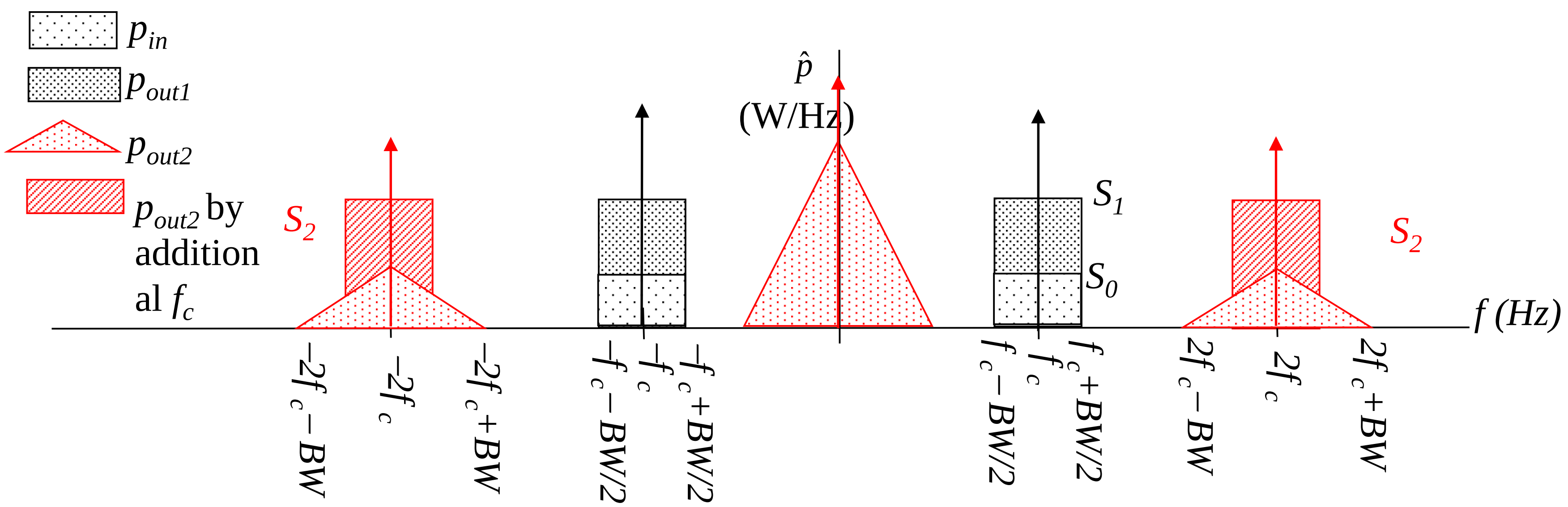


Effects of Convolution of Two Gaussian Signals

- Frequency spread.
- Triangular shaped spectral profiles vs. square spectral profiles from fundamentals.
- DC terms two times larger than terms around $2f_c$.

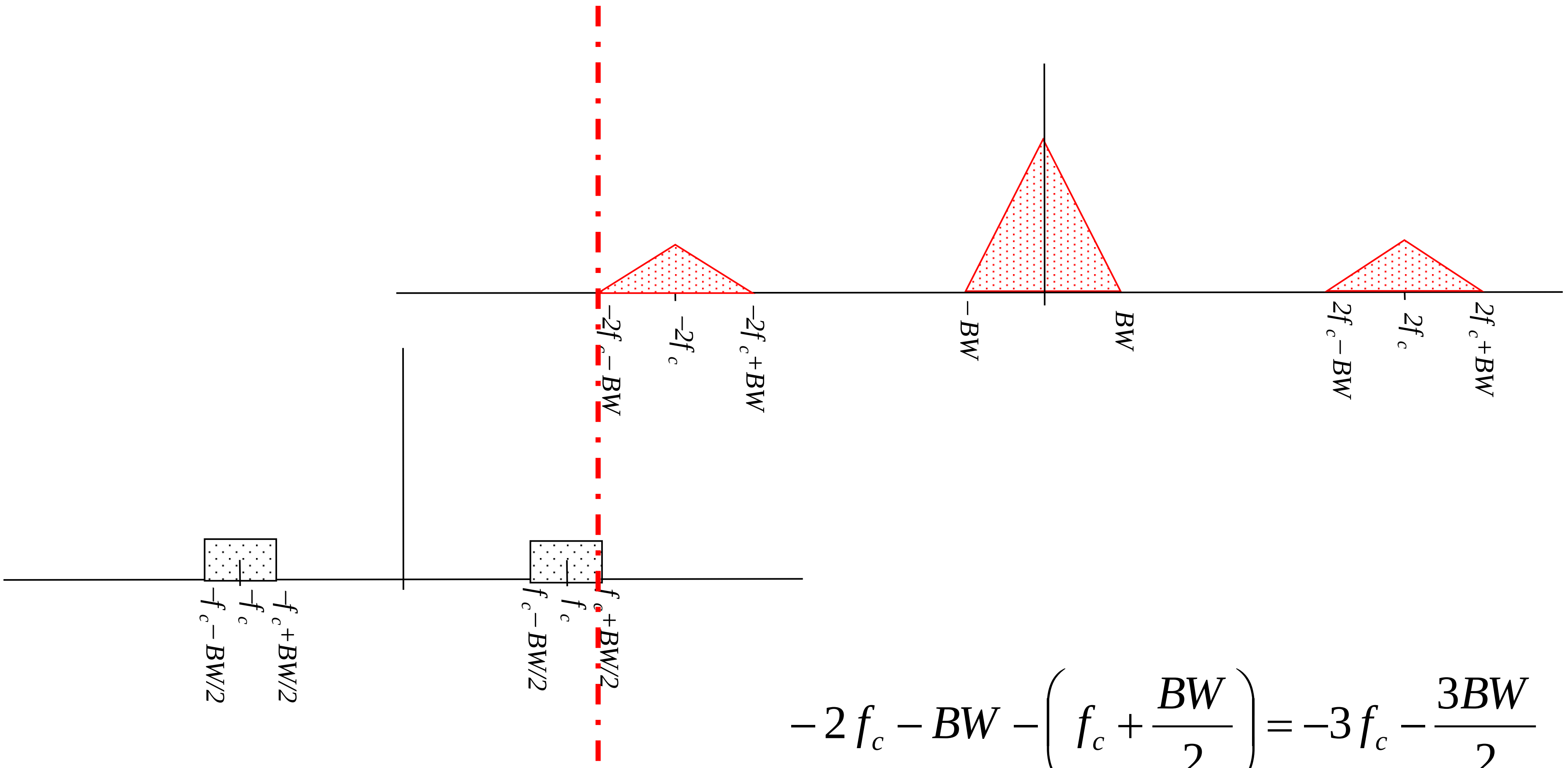
Additional Component at f_c

$$v_{out,2nd}(f) = a_2 \cdot v_A(f) \otimes v_B(f) = a_2 \cdot \int_{-\infty}^{\infty} v_A(x) v_B(f-x)\,dx$$



3^{rd}-Nonlinearity of Gaussian Signals

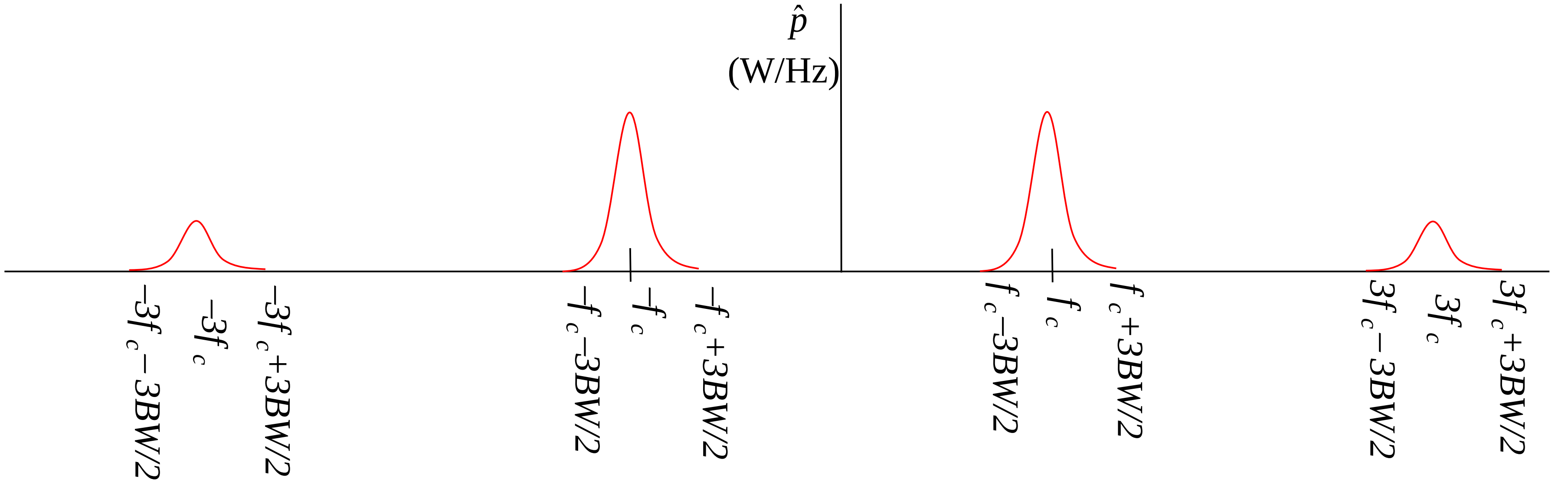
$$v_{out,3rd}(f) = a_3 \cdot [v_A(f) \otimes v_B(f)] \otimes v_C(f)$$



$$-2f_c - BW - \left(f_c + \frac{BW}{2}\right) = -3f_c - \frac{3BW}{2}$$

3rd-Nonlinearity of Gaussian Signals

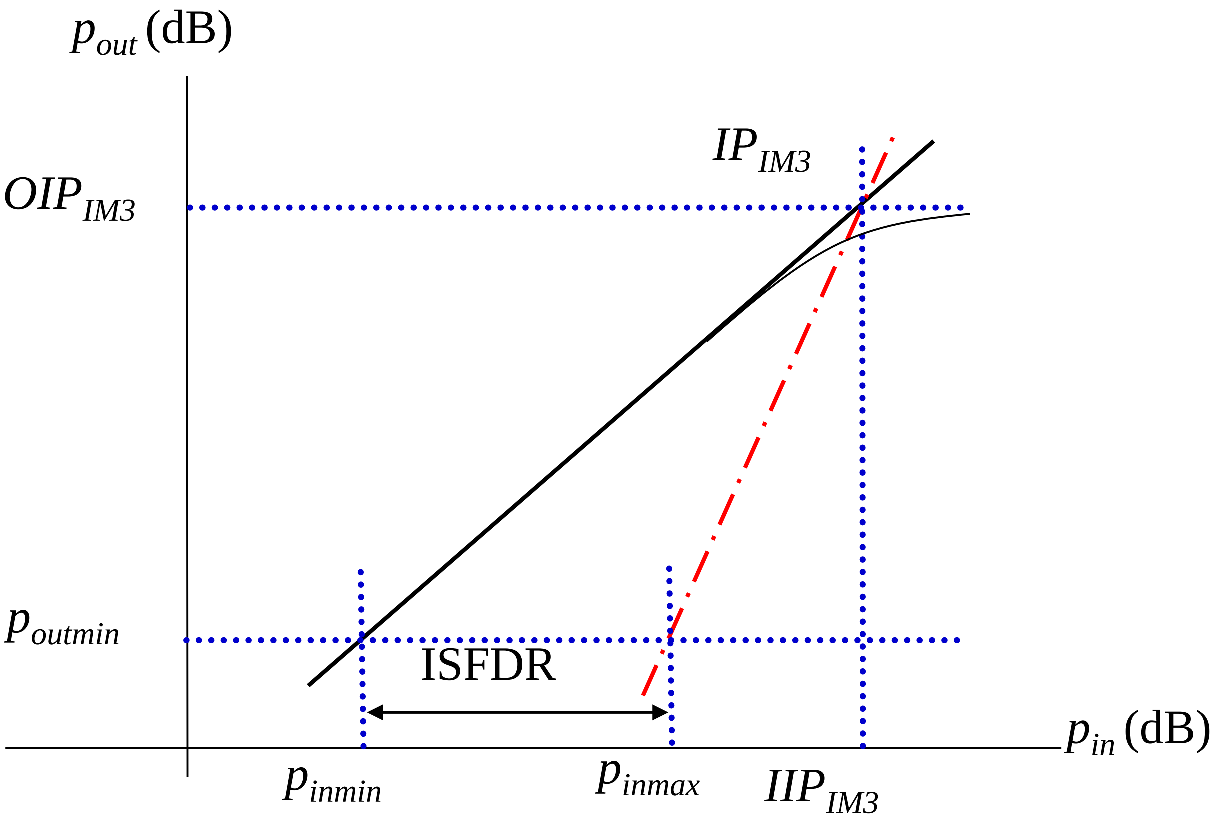
$$v_{out,3rd}(f) = a_3 \cdot [v_A(f) \otimes v_B(f)] \otimes v_C(f)$$



“Instantaneous” Spur-Free Dynamic Range (ISFDR)

- Instantaneous: no further techniques needed to deal with gain control, noise cancellation or signal orthogonality
- Spur-free: no serious spurs caused by nonlinearity
- No ambience interference is considered: Limit for the point-to-point single access transceivers

ISFDR with Only IM3



$$ISFDR = P_{in\max} - P_{in\min} = \frac{2}{3}(IIP3 - P_{in\min})$$

ISFDR Wi-Fi Example

- For a Wi-Fi receiver (IEEE 802.11n) with 40MHz bandwidth, cascade noise figure at 8dB and IIP3 = –3dBm (rather low, set usually by the LNA)

$$P_{in\,\mathrm{min}} = -174dBm + 76dB + 8dB = -90dBm$$

  - –174dBm: Thermal noise power per Hz at room temperature.
  - 76dB: $10\log_{10}(40\times10^6)$ from 40MHz bandwidth
  - 8dB: Additional noise by the cascade noise figure.

$$ISFDR = \frac{2}{3}(IIP3 - P_{in\,\mathrm{min}}) = 58dB$$

ISFDR GPS Example

- For a GPS receiver with 4kHz bandwidth, cascade noise figure at 8dB and IIP3 = –3dBm (notice that most RF components are similar, but the filter bandwidth is much smaller),

$$P_{in\min} = -174dBm + 36dB + 8dB = -130dBm$$

$$ISFDR = \frac{2}{3}(IIP3 - P_{in\min}) = 85dB$$

What Have You Learned?

- Interaction of nonlinearity with signals + noise profiles can generate many spurious terms
- Manual analyses will be too complicated: Simulation will be indispensible to track down the spurs
- Designer practice: Know when nonlinearity is NOT important yet $\rightarrow$ Spur-free dynamic range