ECE 4880 RF Systems Fall 2016
Homework 9 Solution

Reading before homework:

- Chaps. 8 and 9 of lecture notes

1. (**Classical AM radio**) AM bands are between 530kHz – 1610kHz with the base band BW_{bb} = 20kHz. Conventionally there are 27 stations within the same region, each separated by additional 20kHz.
   (a) When we choose f_{IF} = 455 kHz as a high-side injection, what is the range of f_{LO} for selecting all channel by high side injection? What is the ratio of the highest f_{LO} to lowest f_{LO}? Repeat for low-side injection. Can one station be the image interference of the other station? (**6 pts**)

High side injection requires f_{LO} changes between 985kHz and 2065kHz, where the ratio f_{LOmax}/f_{LOmin} = 2.1, an easily achievable ratio by the frequency synthesizer.

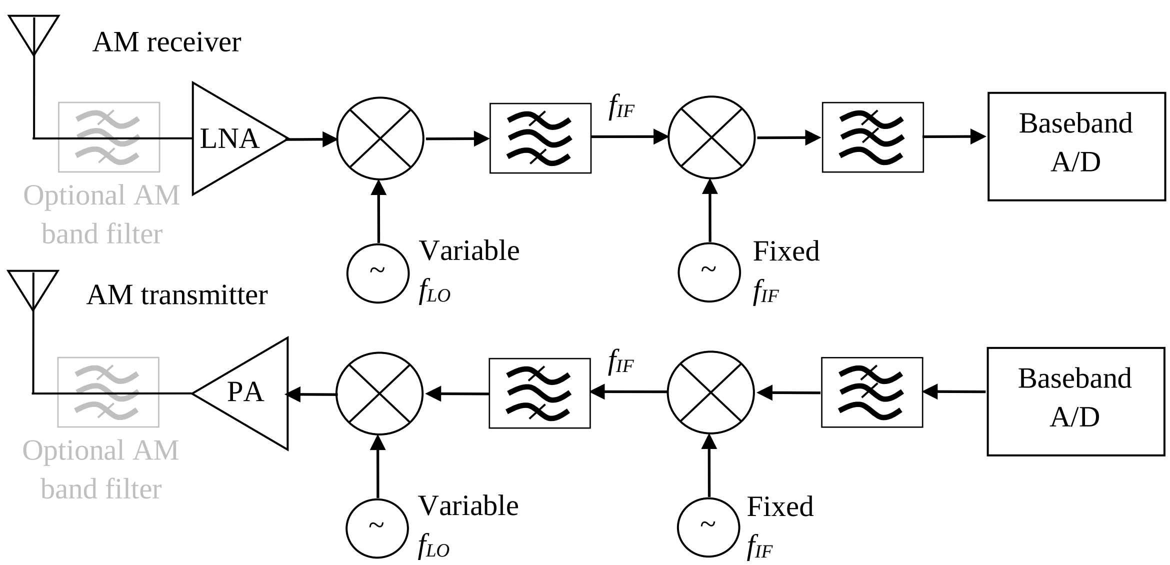
For low side injection, f_{LO} needs to change between 75kHz and 1155MHz, where the ratio f_{LOmax}/f_{LOmin} = 15.4, which is a tuning range too large for most good quality oscillators.

As f_{IF} is less than half of the total AM band, the station at higher frequency can be the image interference of the station at the lower frequency. This has to be resolved by either the "white" (or blank) space between the channels or with additional filters for different halves of the AM band.

   (b) Work out the superheterodyne up-conversion scheme with f_{IF} = 5.1MHz for both the transmitter modulation and recevier demodulation. Show the block diagrams for the transmitter and the receiver. Give the range of f_{LO} to receive all AM stations under the FCC regulation. (**8 pts**)

f_{IF} = 5.1MHz = f_{RF} + f_{LO} in the up-conversion scheme. f_{RF} = [530kHz, 1610kHz]. Therefore, f_{LO} = [3490kHz, 4570kHz].

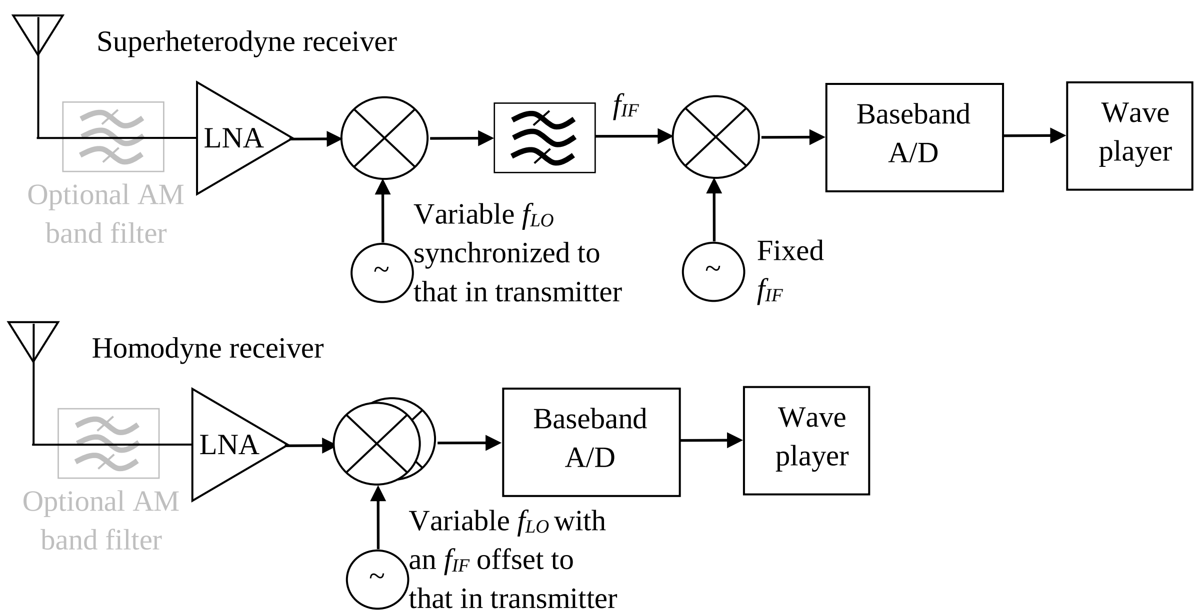
The transceiver for the up-conversion is similar to the regular transceiver, except the bandpass filtering around f_{IF} (cannot use low-pass filters).



As most superheterodyne transceivers, the transmitter is a mirror image of the receiver, independent of down conversion or up conversion, high side or low side injection, etc. The architecture is similar too, except for the filter range.

(c) If the transmitter has the classical choice of f_{IF} = 455kHz (as it is difficult to change the old radio tower) with high-side injection, present the radio receiver design in homodyne and superheterodyne architectures when you have an analog-to-digital converter (ADC) of 1MHz sampling with 12 bits per sample, followed by a .wav player. We will assume that if you need to sample a BW_{bb} = 20kHz, you will need to sample at least at 200kHz. You are free to use mixers, but no analog filters as they would be costly and bulky in this frequency range. You can assume that all digital functions such as filtering can be performed easily after ADC. (**8 pts**)

As the ADC is limited to 1MHz, we cannot sample f_{IF} correctly with sufficient waveform accuracy (too much aliasing). This gives two possible receiver designs. In superheterodyne, we still use two mixers (f_{LO} synchronized to that in the transmitter and a fixed f_{IF} at 455kHz) to down convert to BW_{bb}. In homodyne, we will use one mixer with f_{LO} generated at an f_{IF} offset to f_{LO} of the transmitter. We do draw two mixers with the quadrature scheme to deal with image cancellation, which can be performed digitally as well. See the diagram below.



We will not need the last low-pass filter in the superheterodyne scheme (whose cutoff frequency is very low and will be bulky for implementation), as we can use digital filtering when the baseband is sufficiently sampled. We do not need any filter for the homodyne scheme, as long as the desirable f_{LO} can be generated accurately.

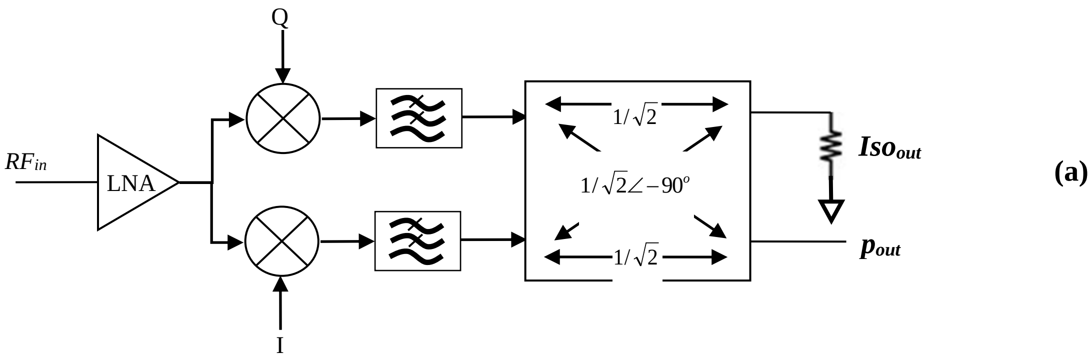
Notice that the digital functions need to be computationally efficient, or else it would put too much demand on the processor or the power consumption.

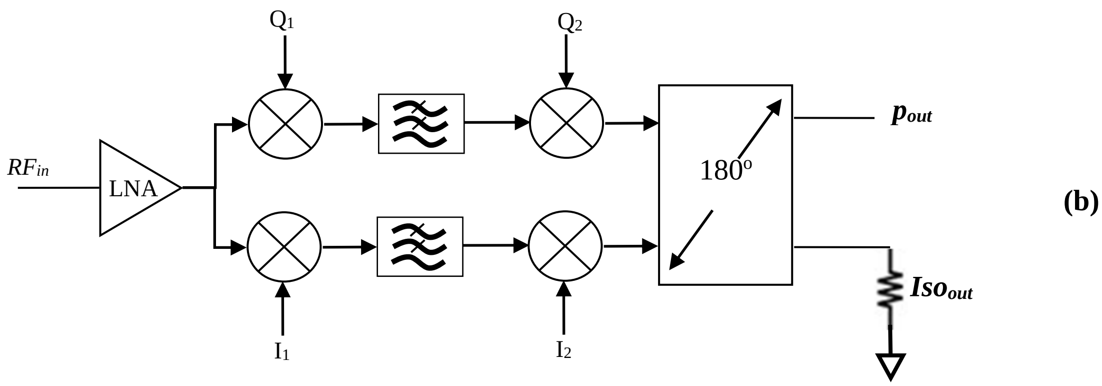
2. (**Parallel image cancellation**) For the two quadrature image rejection architectures as shown below, $RF_{in} = A\cos(\varphi_{LO} - \varphi_{IF}) + B\cos(\varphi_{LO} + \varphi_{IF})$, i.e., B is the image interference for A in the high-side

injection. The final hybrid combiner is for the intermediate frequency. Assume LNA has sufficient gain to just cancel the splitter loss. We will need to use the following trigonometry identities:

$$\cos a \cos b = \frac{1}{2}[\cos(a-b)+\cos(a+b)]; \qquad \sin a \sin b = \frac{1}{2}[\cos(a-b)-\cos(a+b)]$$

$$\sin a \cos b = \frac{1}{2}[\sin(a+b)+\sin(a+b)]; \qquad \cos a \sin b = \frac{1}{2}[\sin(a+b)-\sin(a-b)]$$





Prob. P9.2. (a) Image rejection mixer with a phase shifter. Signal: in-phase; image: 180° out of phase. (b) Weaver architecture for image rejection.

(a) For the architecture in Fig. P9.2(a), assume I_1 and Q_1 are synchronized to the frequency φ_{LO} in the coming signal. Show the wave functions immediately after the mixers and at p_{out} and Iso_{out}. (**10 pts**)

Assume I and Q will mix in the functions of $\cos\varphi_{LO}$ and $\sin\varphi_{LO}$, respectively. We will not write out the magnitude here explicitly for simpler observation:

	After mixer	After lowpass filter and hybrid
$A\cos(\varphi_{LO} - \varphi_{IF})$	I: $\cos(\varphi_{LO} - \varphi_{IF} + \varphi_{LO}) + \cos(\varphi_{IF})$ Q: $\sin(\varphi_{LO} - \varphi_{IF} + \varphi_{LO}) + \sin(\varphi_{IF})$	p_{out}: 0 Iso_{out}: $2\sin(\varphi_{IF})$
$B\cos(\varphi_{LO} + \varphi_{IF})$	I: $\cos(\varphi_{LO} + \varphi_{IF} + \varphi_{LO}) + \cos(\varphi_{IF})$ Q: $\sin(\varphi_{LO} + \varphi_{IF} + \varphi_{LO}) - \sin(\varphi_{IF})$	p_{out}: $2\cos(\varphi_{IF})$ Iso_{out}: 0

The intended signal will appear at p_{out} and the image is sent to Iso_{out}. Or more generally, the RF frequency lower than f_{LO} will appear at p_{out} and the RF frequency higher than f_{LO} is sent to Iso_{out}.

(b) For the architecture in Fig. P9.2(b), assume $\varphi_1 = \varphi_{LO}$ in I_1 and Q_1. Will I_1/Q_1 be different from I_2/Q_2? Show the wave functions immediately after the mixers and at p_{out} and Iso_{out}. (**10 pts**)

We will derive a general solution, and then make further observation. Assume I_1, Q_1, I_2 and Q_2 will mix in the functions of $\cos\varphi_1$, $\sin\varphi_1$, $\cos\varphi_2$ and $\sin\varphi_2$, respectively. We will not write out the magnitude here explicitly for simpler observation. Assume that I_1 and Q_1 are sufficiently high (often comparable to f_{LO}) that the low-pass filter after I_1/Q_1 mixer will rid of the higher frequency component successfully. We can then obtain the following functions after some functional manipulation:

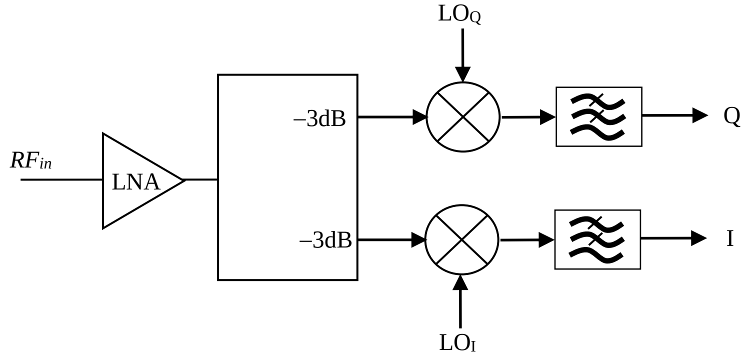
	After I_1/Q_1 and I_2/Q_2 mixers but before hybrid	For $\varphi_1 = \varphi_{LO}$
$A\cos(\varphi_{LO} - \varphi_{IF})$	I: $\cos(\varphi_{LO} - \varphi_{IF} - \varphi_1 + \varphi_2) + \cos(\varphi_{LO} - \varphi_{IF} - \varphi_1 - \varphi_2)$ Q: $\cos(\varphi_{LO} - \varphi_{IF} - \varphi_1 + \varphi_2) - \cos(\varphi_{LO} - \varphi_{IF} - \varphi_1 - \varphi_2)$	I: $\cos(\varphi_{IF} - \varphi_2) + \cos(\varphi_{IF} + \varphi_2)$ Q: $\cos(\varphi_{IF} - \varphi_2) - \cos(\varphi_{IF} + \varphi_2)$
$B\cos(\varphi_{LO} + \varphi_{IF})$	I: $\cos(\varphi_{LO} + \varphi_{IF} - \varphi_1 + \varphi_2) + \cos(\varphi_{LO} + \varphi_{IF} - \varphi_1 - \varphi_2)$ Q: $\cos(\varphi_{LO} + \varphi_{IF} - \varphi_1 + \varphi_2) - \cos(\varphi_{LO} + \varphi_{IF} - \varphi_1 - \varphi_2)$	I: $\cos(\varphi_{IF} + \varphi_2) + \cos(\varphi_{IF} - \varphi_2)$ Q: $\cos(\varphi_{IF} + \varphi_2) - \cos(\varphi_{IF} - \varphi_2)$

We can see that the Weaver mixer has good configurability for φ_1 and φ_2 to achieve the high-side or low-side selection, as well as choosing ready RF filters. A common choice is $\varphi_1 + \varphi_2 \cong \varphi_{LO} + \varphi_{IF}$ and $\varphi_1 > \varphi_2$. Further selection can be obtained by the final hybrid stage. For I + Q, $A\cos(\varphi_{IF} - \varphi_2)$ and $B\cos(\varphi_{IF} + \varphi_2)$ are shown at p_{out}, while for I – Q, $A\cos(\varphi_{IF} + \varphi_2)$ and $B\cos(\varphi_{IF} - \varphi_2)$ are shown at p_{out}. We can choose A (f_{RF} lower than f_{LO}) or B (f_{RF} higher than f_{LO}) by flipping or not flipping the sign of the quadrature (remember that A and B are image to each other)!

The Weaver mixer offers high flexibility in the frequency strategy, but was not popularly used. The main issue is that the quadrature phase information is used to reject the image, and not extracted after the two mixers. For any Q-ary modulation where the phase information is critical for larger number of bits per symbol, the one-quadrature mixer architecture is preferred. Indeed, most receiver today adopt the homodyne or low-f_{IF} one-quadrature architecture, and leaves the image rejection after I and Q go through ADC.

The hybrid structure is often implemented by passive transmission lines to achieve the phase delay. By definition, the bandwidth (line length in terms of wavelength) is rather limited. Therefore, unless for very high frequency or dedicated bands, the hybrid structure is also not popularly used in modern RF receivers where broadband or tunable band operation is implemented in cognitive radios (i.e., band selection is dynamically allocated after detection).

3. (**Direct conversion transceivers**) One of the main problems of direct-conversion homodyne is the excessive DC shift and drift. Owing to $f_{IF} = 0$, there is already a DC component after the mixer. A high-pass filter with very low frequency is very difficult to make (very large LC), unless we can do digital filtering without saturating the data converter. Moreover, there can be additional DC factors in the direct conversion receiver below. Assume that RF_{in} has a carrier at an angular frequency of ω = 1GHz and the power of the carrier is at –10dBm. All RF parts have impedance matched at 50Ω. LNA has a gain of 15dB with negligible phase delay and IIP_{H2}=30dBm. The power splitter has –3dB loss at each terminal with negligible phase delay. LO_I has 0dBm power at 1GHz and in phase with RF_{in}. LO_Q is the ideal quadrature of LO_I. The low pass filter is at 5MHz and is ideal.



(a) What is the output of the LO_I and LO_Q mixers? What is the output of I and Q after the LPF? Write the voltage waveforms at each stage. (**8 pts**) *Hint: Express known signals in the general form of Acos($\omega t + \theta$) and work out the functional forms.*

We can write down $RF_{in} = 0.1\cdot\cos(\omega t)$.

Input at the mixer is at 2dBm can be expressed as $0.40\cdot\cos(\omega t)$

$LO_I = 0.32\cdot\cos(\omega t)$. $LO_Q = 0.32\cdot\sin(\omega t)$.

We know: $\cos^2 u = \frac{1+\cos 2u}{2}$. Mixer output at I branch is $0.128\cos^2(\omega t) = 0.064+0.064\cos(2\omega t)$. After the low pass filter, Port I has only a DC level of 0.064V = 64mV.

We also know: $\cos u \sin u = \frac{\sin 2u}{2}$.

Mixer output at Q branch is $0.128\cos(\omega t)\sin(\omega t) = 0.064\sin(2\omega t)$. After the low pass filter, Q is identically at 0.

(b) Now we will consider the nonlinearity of LNA. Find a_2 (still assume negative) from IIP_{H2} and the resulting DC term after the LNA when RF_{in} = –10dBm. What is the DC level shift before the mixer? (**8 pts**)

IIP_{H2}=30dBm, and IIP_{IM2} = 24dBm. A_{IIPIM2} = 5.0 (V). $A_{IIPIM2} = \left|\frac{a_1}{a_2}\right|$. We thus obtain $a_2 = 6.32$ (V^{-1}).

$v_{2nd,DC} = \frac{a_2 A^2}{2} = 0.0316$V = 31.6mV. With a further -3dB or $1/\sqrt{2}$ loss, we have a DC shift at the input of the mixer for 22.3mV. Any of this DC level and the H_2 of LNA with the mixer H_2 can cause further effects of the DC level at I and Q.

(c) Assume LO_I leaks to its own RF port with –30dB loss. What is the resulting DC level from this self leakage at I? (**8 pts**)

$LO_I = 0.32\cdot\cos(\omega t)$. For the RF leak, when LO is coupled to RF_{in} with -30dB (often this is more realistic, as RF_{in} has a longer line to antenna, and hence more coupling to the LO line), at the input of the mixer, we will have a copy of LO after -30dB + 15dB (from LNA) – 3dB (from splitter) = -18dB. $RF_{leak} = 0.32\times10^{-18/20}\cos(\omega t)$. We will have a DC level at 40mV. This does not seem too large.

However, remember that this is independent of the signal level. When we have a low signal comes in, this DC pollution can be significant.

4. (**Spurious DC resolution in direct conversion**) For the direct conversion demodulator in Q. 3, assume I and Q will be directly fed into the ADC (analog-to-digital converter) and the spurious DC shift is mainly caused by (1) the LO coupling to RF_{in} and (2) the second-order nonlinearity of the LNA.

   (a) Will the feedthrough technique in Fig. 7.7 be helpful for the DC drift condition caused by LO coupling? By 2^{nd}-order nonlinearity in LNA? Give a brief explanation. Assume that you can subtract a nearly DC signal by a functional block of "subtractor" (which is typically a diff pair, but you do not need to give details). (**6 pts**)

   The feedthrough compensation can evaluate a copy of the nonlinear distortion to cancel it from the main path, but cannot eliminate LO coupling, as the amplifier a_1' will have similar LO coupling issues.

   (b) If a "zero" signal can be provided (such as the ground plane in a patch antenna), will that help the DC drift caused by LO coupling? By 2^{nd}-order nonlinearity of LNA? (**6 pts**)

   If we build an identical line with zero input to replace RF_{in}, we can estimate the LO coupling to subtract it. However, as zero input will not cause any nonlinear term in LNA, it would be ineffective to cancel the DC drift caused by the 2^{nd}-harmonic of the LNA, which derives from the intermodulation RF and LO.

   (c) If the data rate is at 5 million symbols per second, for a typical microcontroller of 200MHz clock cycle, is reset to zero on I and Q between symbols a practical technique to do? Assume that the original modulation has a "return-to-zero" (RTZ) scheme. (**6 pts**)

   For 5 million symbols per second, we will evaluate each symbol with 0.2μs integration budget. The reset by the microcontroller will only take about 5ns, which is only 1/40 of the sampling time. Therefore, RTZ is an effective way to avoid DC drift to accumulate in the DAC.

5. (**Phase noise in LO**) Assume that we use a LO with a phase noise profile as shown below:

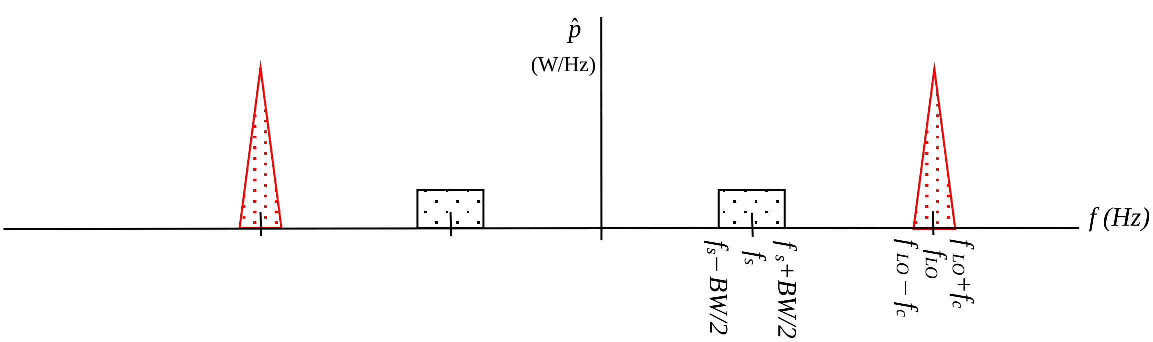
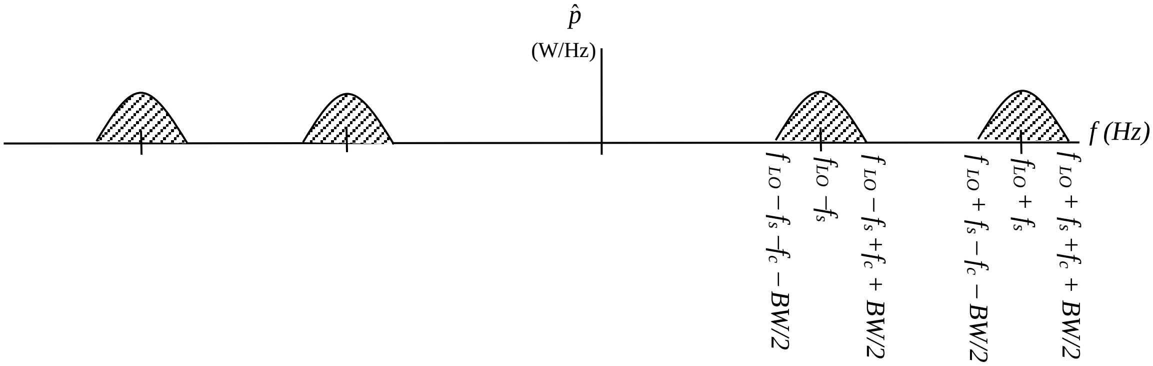


Fig. P9.5. Signals multiplied by LO with phase noise.

The phase noise of LO can be approximated with a $1/|f - f_{LO}|$ profile for the frequency of interest. Assume that the phase noise spectral power density will be equal to that of the background thermal noise at $f_{LO} \pm f_c$ (corner frequency). Surely the phase noise expression is not valid for $f \cong f_{LO}$, but we will not consider the information carried very close f_{LO}.

(a) Draw the spectral profile for an f_s with a BW bandwidth ($f_s < f_{LO}$ as high-side injection, and also assume $f_c < BW/2$) that is multiplied by f_{LO} with the above phase noise. Denote the frequency ranges for the multiplication product. (**8 pts**)

Multiplication in time is convolution in the frequency space.



(b) Assume the thermal noise floor has a spectral density of N_0 = –174dBm/Hz, for f_c = 10kHz, calculate the total noise power in dBm (thermal + phase) when BW = 200kHz and 2MHz. Assume all phase noise within 1Hz of LO can be discarded. (**8 pts**)

We will first find the phase noise power. As we know $N_{ph}(\Delta f = 10\text{kHz})$ = –174dBm/Hz or 4×10^{-21}W/Hz, we can find $N_{ph}(\Delta f = 1\text{Hz})$ = –134dBm/Hz or 4×10^{-17}W/Hz. The total phase noise will be:

$$\int_1^{10^4} N_{ph}(\Delta f)\frac{1}{\Delta f}df = 4\times10^{-17}\cdot\ln(\Delta f)\Big|_1^{10^4} = 3.68\times10^{-16}W = -124dBm.$$

For thermal noise at BW = 200kHz, N_{th} = 4×10^{-21}W/Hz × 2×10^5Hz = 8×10^{-16}W or -121dBm. N_{total} = 1.5×10^{-15}W or –119dBm, where the total thermal noise power is comparable to the total phase noise power.

For thermal noise at BW = 2MHz, N_{th} = 4×10^{-21}W/Hz × 2×10^6Hz = 8×10^{-15}W or -111dBm. N_{total} = 8.7×10^{-15}W or –111dBm, where the total thermal noise power is much larger than the total phase noise power.